

CALCULUS: THE ANSWERS

MATH 150: CALCULUS WITH ANALYTIC GEOMETRY I

VERSION 1.3
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CHAPTER 1: REVIEW

1) $f(-4)=16$; $f(a+h)=(a+h)^2=a^2+2ah+h^2$

2) Polynomial: Yes, Rational: Yes, Algebraic: Yes

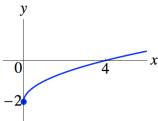
3) Polynomial: No, Rational: Yes, Algebraic: Yes

4) Polynomial: No, Rational: No, Algebraic: Yes

5) Polynomial: No, Rational: No, Algebraic: No

6) a) $(-\infty, \infty)$, b) $(-\infty, \infty)$, c) $\left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, 2\right) \cup (2, \infty)$, d) $[-2, 5] \cup (5, \infty)$,
e) $(-2, \infty)$, f) $(-\infty, 7]$, g) $(-\infty, -1] \cup [3, \infty)$

7)

; Domain is $[0, \infty)$; Range is $[-2, \infty)$ 8) the y -axis; the function is even

9) the origin; the function is odd

10) the function is even

11) the function is even

12) the function is neither even nor odd

13) $(f \circ g)(x)=x^8+2x^6+x^4+\frac{1}{x^4+x^2+1}$. $\text{Dom}(f \circ g)=\mathbb{R}$, or $(-\infty, \infty)$.

14) $g(x)=x^4+x$, $f(u)=u^8$; there are other possibilities

15) $g(t)=\frac{1}{t}$, $f(u)=\sqrt[4]{u}$; there are other possibilities

16) $g(r)=r^2$, $f(u)=\sin u$; there are other possibilities

17) a) undefined, b) -1 , c) $\sqrt{2}$, d) $-\frac{2\sqrt{3}}{3}$, e) $-\frac{\sqrt{3}}{2}$, f) $-\sqrt{3}$

18) Hint on a): Use a Double-Angle ID; Hint on b): Use a Pythagorean ID

19) a) $\left\{ x \in \mathbb{R} \mid x = -\frac{\pi}{6} + 2\pi n, \text{ or } x = \frac{7\pi}{6} + 2\pi n, \text{ or } x = \frac{3\pi}{2} + 2\pi n \quad (n \in \mathbb{Z}) \right\}$

Note: $-\frac{\pi}{6}$ can be replaced by $\frac{11\pi}{6}$, $\frac{3\pi}{2}$ can be replaced by $-\frac{\pi}{2}$, etc.

b) $\left\{ x \in \mathbb{R} \mid x = \pm \frac{\pi}{9} + \frac{2\pi n}{3} \quad (n \in \mathbb{Z}) \right\}$, or, equivalently,

$$\left\{ x \in \mathbb{R} \mid x = \frac{\pi}{9} + \frac{2\pi n}{3}, \text{ or } x = \frac{5\pi}{9} + \frac{2\pi n}{3} \quad (n \in \mathbb{Z}) \right\}$$

CHAPTER 2: LIMITS AND CONTINUITY

SECTION 2.1: AN INTRODUCTION TO LIMITS

1) 57

2) $11/7$

3) 10

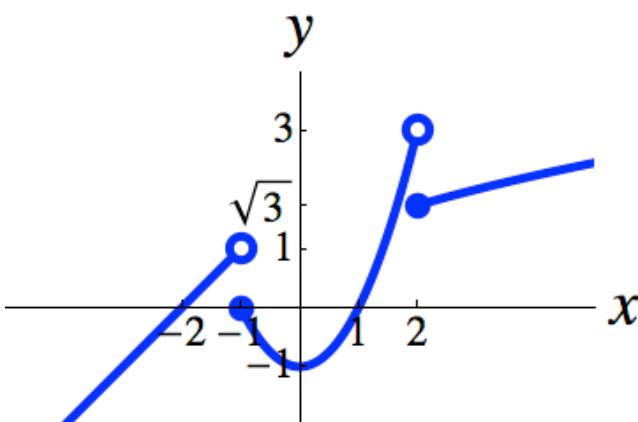
4) π^2 5) a) $f(1) = 0.9999$, $f(0.1) = 0.0999$, $f(0.01) = 0.0099$; b) -0.0001 , No6) a) $11/7$, b) $11/7$; as a consequence, the answer to Exercise 2 is the same.7) No; a counterexample: $\lim_{x \rightarrow 0^+} \sqrt{x} = 0$, while $\lim_{x \rightarrow 0} \sqrt{x}$ does not exist (DNE).

See also Example 6.

8) Yes

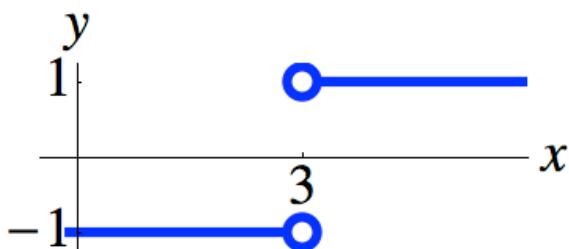
9) No; a counterexample: see Example 8 on $h(x) = \begin{cases} x + 3, & x \neq 3 \\ 7, & x = 3 \end{cases}$ 10) “might exist”; for example, see Example 7 on $g(x) = x + 3$, $(x \neq 3)$.

11) a)

b) 1, 0, DNE; c) 3, $\sqrt{3}$, DNE, 2

12) a)

b) -1, 1, DNE



13) a) 0 (liters), which means that, if the gas's temperature approaches absolute zero (from above), its volume approaches zero (liters).

b) DNE, because temperatures cannot go below absolute zero. The domain of V does not include values of T below absolute zero.

SECTION 2.2: **PROPERTIES OF LIMITS and ALGEBRAIC FUNCTIONS**

1) $\frac{39+18\sqrt{3}}{16}$, or $\frac{3}{16}(13+6\sqrt{3})$

2) a) DNE, b) 0, c) DNE

3) a) 0, b) 0, c) 0

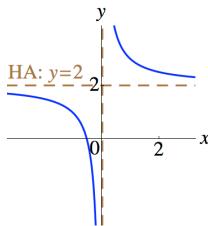
4) a) 0, b) DNE, c) DNE, d) 0, e) $\sqrt{5}$, f) DNE, g) 05) a) DNE, b) 0, c) DNE, d) 0, e) DNE, f) DNE, g) $\sqrt{6}$

6) Yes, by linearity of the limit operator.

7) “might exist”; for example, $\lim_{x \rightarrow 0} \sqrt{x^2} = 0$. See also Example 7 on $\lim_{x \rightarrow -7} \sqrt{(x+7)^2}$.

SECTION 2.3: LIMITS AND INFINITY I

1) a) $\lim_{x \rightarrow 0} \frac{\sin x}{x}$ b) 2, c) $y = 2$



2) a) 0; b) 0 or 1; c) 0, 1, or 2

3) In the table: 10 100 1000 10,000. The Limit Form yields ∞ .4) a) 0^+ , b) 0^- , c) 0^+ , d) $-\infty$, e) ∞ , f) ∞ , g) ∞ , h) 0^+ , i) 0, j) ∞ , k) 0^+

5) Yes

6) “might exist”; for example, Example 6 on $f(x) = \frac{\sin x}{x}$; also, $f(x) = \frac{\sin x + 2}{\sin x + 2}$.7) a) DNE, b) 0 or 0^+ , c) 0 or 0^- , d) 0 or 0^+ , e) DNE, f) ∞ 8) a) $\sqrt{\pi}$

b) i. $\lim_{x \rightarrow \infty} (x^5 + 3x^4 - 2) = \lim_{x \rightarrow \infty} x^5 = \infty$;

ii. $\lim_{x \rightarrow \infty} (x^5 + 3x^4 - 2) = \lim_{x \rightarrow \infty} \underbrace{x^5}_{\rightarrow \infty} \left(1 + \underbrace{\frac{3}{x}}_{\rightarrow 0} - \underbrace{\frac{2}{x^5}}_{\rightarrow 0} \right) = \infty$

c) i. $\lim_{x \rightarrow -\infty} (2x^3 - 6x^2 + x) = \lim_{x \rightarrow -\infty} 2x^3 = -\infty$;

ii. $\lim_{x \rightarrow -\infty} (2x^3 - 6x^2 + x) = \lim_{x \rightarrow -\infty} \underbrace{2x^3}_{\rightarrow -\infty} \left(1 - \underbrace{\frac{3}{x}}_{\rightarrow 0} + \underbrace{\frac{1}{2x^2}}_{\rightarrow 0} \right) = -\infty$

d) $\lim_{w \rightarrow \infty} (5w - 4w^4) = \lim_{w \rightarrow \infty} (-4w^4) = -\infty$

9) a) i. 0, because g is a proper rational function, and we seek a “long-run” limit;

$$\text{ii. } \lim_{r \rightarrow \infty} g(r) = \lim_{r \rightarrow \infty} \frac{3r^3}{2r^5} = \lim_{r \rightarrow \infty} \frac{3}{2r^2} = 0;$$

$$\text{iii. } \lim_{r \rightarrow \infty} g(r) = \lim_{r \rightarrow \infty} \frac{\frac{3r^3}{r^5} + \frac{r}{r^5} - \frac{4}{r^5}}{\frac{2r^5}{r^5} - \frac{7r^2}{r^5}} = \lim_{r \rightarrow \infty} \frac{\overbrace{\frac{3}{r^2} + \frac{1}{r^4} - \frac{4}{r^5}}^{\rightarrow 0}}{2 - \underbrace{\frac{7}{r^3}}_{\rightarrow 0}} = \frac{0}{2} = 0, \text{ or}$$

$$\lim_{r \rightarrow \infty} g(r) = \lim_{r \rightarrow \infty} \frac{3r^3 \left(1 + \underbrace{\frac{1}{3r^2} - \frac{4}{3r^3}}_{\rightarrow 1} \right)}{2r^5 \left(1 - \underbrace{\frac{7}{2r^3}}_{\rightarrow 1} \right)} = \lim_{r \rightarrow \infty} \frac{3r^3}{2r^5} = \lim_{r \rightarrow \infty} \frac{3}{2r^2} = 0;$$

$$\text{iv. } \lim_{r \rightarrow -\infty} g(r) = 0, \text{ also;}$$

v. HA: $s = 0$

b) i. $\frac{7}{3}$, because the numerator and the denominator have the same degree, and we seek a “long-run” limit, so we take the ratio of the leading coefficients;

$$\text{ii. } \lim_{x \rightarrow -\infty} \frac{7x^4 - 5x}{3x^4 + 2} = \lim_{x \rightarrow -\infty} \frac{7x^4}{3x^4} = \frac{7}{3};$$

$$\text{iii. } \lim_{x \rightarrow -\infty} \frac{7x^4 - 5x}{3x^4 + 2} = \lim_{x \rightarrow -\infty} \frac{\frac{7x^4}{x^4} - \frac{5x}{x^4}}{\frac{3x^4}{x^4} + \frac{2}{x^4}} = \lim_{x \rightarrow -\infty} \frac{7 - \underbrace{\frac{5}{x^3}}_{\rightarrow 0}}{3 + \underbrace{\frac{2}{x^4}}_{\rightarrow 0}} = \frac{7}{3}, \text{ or}$$

$$\lim_{x \rightarrow -\infty} \frac{7x^4 - 5x}{3x^4 + 2} = \lim_{x \rightarrow -\infty} \frac{7x^4 \left(1 - \underbrace{\frac{5}{7x^3}}_{\rightarrow 1} \right)}{3x^4 \left(1 + \underbrace{\frac{2}{3x^4}}_{\rightarrow 1} \right)} = \lim_{x \rightarrow -\infty} \frac{7x^4}{3x^4} = \frac{7}{3};$$

$$\text{iv. HA: } y = \frac{7}{3}$$

(Answers to Exercises for Chapter 2: Limits and Continuity) A.2.4

c) i. $\lim_{x \rightarrow \infty} \frac{\sqrt{2x^5 + 11x^8} - \pi}{6x^5 + x} = \lim_{x \rightarrow \infty} \frac{11x^8}{6x^5} = \lim_{x \rightarrow \infty} \frac{11}{6}x^3 = \infty;$

ii. No, the graph has no HA;

iii. $-\infty$

d) 1/4

10) a) $f(x) = -3x + 2 + \frac{x^2}{x^3 + 1}$; b) $-\infty$; c) ∞ ; d) $y = -3x + 2$, or $y = 2 - 3x$

11) $\frac{\sqrt{3}}{7}$

12) a) $-\infty$

b) DNE

c) $\lim_{x \rightarrow \infty} \frac{\sqrt{4x^6 + x^2} + 2x^2}{5x^3 - \sqrt[3]{x}} = \lim_{x \rightarrow \infty} \frac{\sqrt{4x^6} + 2x^2}{5x^3} = \lim_{x \rightarrow \infty} \frac{2|x^3| + 2x^2}{5x^3} = \lim_{x \rightarrow \infty} \frac{2x^3}{5x^3} = \frac{2}{5}$

d)

$$\begin{aligned} & \lim_{x \rightarrow -\infty} \frac{\sqrt{4x^6 + x^2} + 2x^2}{5x^3 - \sqrt[3]{x}} = \lim_{x \rightarrow -\infty} \frac{\sqrt{4x^6} + 2x^2}{5x^3} = \lim_{x \rightarrow -\infty} \frac{2|x^3| + 2x^2}{5x^3} \\ &= \lim_{x \rightarrow -\infty} \frac{-2x^3}{5x^3} = -\frac{2}{5} \end{aligned}$$

e) HAs: $y = \frac{2}{5}$ and $y = -\frac{2}{5}$

f) $\lim_{z \rightarrow \infty} \frac{\sqrt[3]{5z^{12} + 7z^7}}{z^5 + 2} = \lim_{z \rightarrow \infty} \frac{\sqrt[3]{5z^{12}}}{z^5} = \lim_{z \rightarrow \infty} \frac{\sqrt[3]{5}z^4}{z^5} = \lim_{z \rightarrow \infty} \frac{\sqrt[3]{5}}{z} = 0$

g) 5/2. Hint: Rationalize as in Example 19.

13) a) $p(t) = \frac{2500 + 100t}{2500 + 250t} \left(\frac{E. calculi \text{ bacteria in the dish}}{\text{total bacteria in the dish}} \right);$

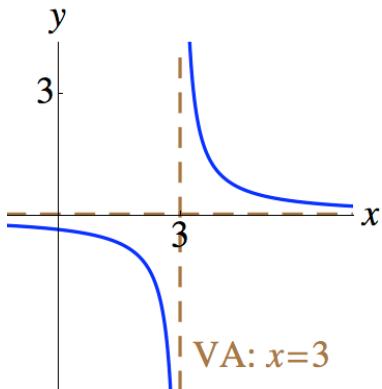
b) $\frac{2}{5} \left(\frac{E. calculi \text{ bacteria in the dish}}{\text{total bacteria in the dish}} \right)$; the proportion of the bacteria in the Petri

dish that are *E. calculi* approaches $\frac{2}{5}$ in the long run, the same as for the

incoming stream; this calculation assumes that the petri dish has infinite capacity and that infinitely many bacteria are available, which is unrealistic.

SECTION 2.4: LIMITS AND INFINITY II

1) a)

b) ∞ , c) $-\infty$, d) $x = 3$ 

2) a) 0

b) any nonnegative integer number

c) any nonnegative integer number, or infinitely many

3) a) ∞ , b) $-\infty$, c) $-\infty$, d) ∞

4) a) $f(x) = \frac{3x-2}{(x+1)(x-2)^2}$

b) i. $-\infty$. Work: $\lim_{x \rightarrow -1^+} f(x) = \lim_{x \rightarrow -1^+} \frac{\overbrace{3x-2}^{\rightarrow -5}}{\underbrace{(x+1)}_{\rightarrow 0^+} \underbrace{(x-2)^2}_{\rightarrow 9}}$ (Limit Form $\frac{-5}{0^+} = -\infty$).

ii. ∞ , iii. DNE, iv. ∞ , v. ∞ , vi. ∞ , vii. $-\frac{1}{2}$

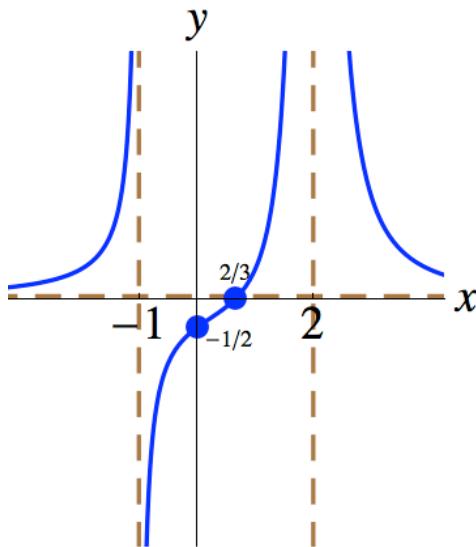
c) i. 0, ii. 0

d) HA: $y = 0$ e) VAs: $x = -1$ and $x = 2$

f) $\frac{2}{3}$, or $\left(\frac{2}{3}, 0\right)$

g) $-\frac{1}{2}$, or $\left(0, -\frac{1}{2}\right)$

h)



5) $f(x) = \frac{4x^2}{(x+2)(x-3)}$; there are other possibilities

6) a) VAs: $t = 1$ and $t = 3$;

for $t = 1$, it is sufficient to show either $\lim_{t \rightarrow 1^+} g(t) = \infty$, or $\lim_{t \rightarrow 1^-} g(t) = -\infty$;

for $t = 3$, it is sufficient to show either $\lim_{t \rightarrow 3^+} g(t) = -\infty$, or $\lim_{t \rightarrow 3^-} g(t) = \infty$.

b) HA: $w = -3$;

$$\lim_{t \rightarrow \infty} g(t) = \lim_{t \rightarrow \infty} \frac{\frac{t}{t^2} - \frac{6t^2}{t^2}}{\frac{2t^2}{t^2} - \frac{8t}{t^2} + \frac{6}{t^2}} = \lim_{t \rightarrow \infty} \frac{\frac{1}{t} - 6}{2 - \frac{8}{t} + \frac{6}{t^2}} \stackrel[t \rightarrow 0]{\overbrace{\frac{1}{t}}}{} \stackrel[t \rightarrow 0]{\overbrace{\frac{6}{t^2}}}{} = \frac{-6}{2} = -3;$$

also, $\lim_{t \rightarrow -\infty} g(t) = -3$ (observe that g is a rational function).

7) 0 VAs, 0 HAs

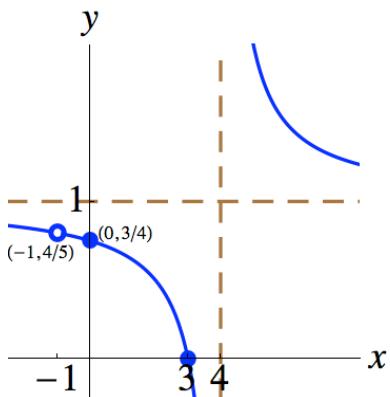
8) a) ∞ , b) $-\infty$, c) ∞ , d) ∞

9) a) ∞ , which means that, if an object's speed approaches the speed of light (from below), its mass increases without bound

b) DNE, which makes sense because faster-than-light speed is impossible.

SECTION 2.5 : THE INDETERMINATE FORMS $\frac{0}{0}$ AND $\frac{\infty}{\infty}$

- 1) $f(x) = x$, $g(x) = x^2$; there are other possibilities.
 - 2) $f(x) = x^2$, $g(x) = x$; there are other possibilities.
 - 3) a) 6; see the equivalent function in Section 2.1, Example 9: $g(x) = x + 3$, ($x \neq 3$).
 - b) $-\frac{19}{12}$, c) $\frac{1}{10}$; Hint: Factor the denominator, or rationalize the numerator.
 - d) $\frac{1}{8}$, e) $-\frac{1}{4}$
 - f) $\frac{17}{4}$; Hint: Factor the numerator by grouping, use Long Division, or use the Rational Zero Test (Rational Roots Theorem) and Synthetic Division from Section 2.3 of the Precalculus notes – observe: $2\left(x - \frac{1}{2}\right) = 2x - 1$.
 - g) 12; Hint: Factor the numerator.
- 4) a) $\text{Dom}(f) = \left\{x \in \mathbb{R} \mid x \neq -1 \text{ and } x \neq 4\right\} = (-\infty, -1) \cup (-1, 4) \cup (4, \infty)$
- b) 3, or $(3, 0)$
- c) $\frac{3}{4}$, or $\left(0, \frac{3}{4}\right)$
- d) neither even nor odd
- e) VA: $x = 4$, because $\lim_{x \rightarrow 4^+} f(x) = \infty$; also, $\lim_{x \rightarrow 4^-} f(x) = -\infty$.
HA: $y = 1$, because $\lim_{x \rightarrow \infty} f(x) = 1$.
- f) $\left(-1, \frac{4}{5}\right)$, because $\lim_{x \rightarrow -1} f(x) = \frac{4}{5}$, and -1 is not in $\text{Dom}(f)$.
- g) (x - and y -axes are scaled differently below)



SECTION 2.6: THE SQUEEZE (SANDWICH) THEOREM

1) Shorthand:

$$\text{As } x \rightarrow 0, \quad \underbrace{-x^2}_{\substack{\rightarrow 0 \\ \text{Therefore,}}} \leq \underbrace{x^2 \sin\left(\frac{1}{x^2}\right)}_{\substack{\rightarrow 0 \\ \text{Therefore,}}} \leq \underbrace{x^2}_{\substack{\rightarrow 0 \\ \text{Therefore,}}} \quad (\forall x \neq 0)$$

2) Note 1: The domain of $\cos\left(\frac{t+3}{\sqrt[3]{t^2-t}}\right)$ is $\{t \in \mathbb{R} \mid t \neq 0 \text{ and } t \neq 1\}$.

We can, for example, restrict our attention to t -values in $(-1, 1) \setminus \{0\}$, a punctured neighborhood of 0.

Note 2: $t^4 + \sin^2 t > 0$ whenever $t \neq 0$.

Shorthand:

$$\text{As } t \rightarrow 0, \quad \underbrace{-(t^4 + \sin^2 t)}_{\substack{\rightarrow 0 \\ \text{Therefore,}}} \leq \underbrace{(t^4 + \sin^2 t) \cos\left(\frac{t+3}{\sqrt[3]{t^2-t}}\right)}_{\substack{\rightarrow 0 \\ \text{Therefore,}}} \leq \underbrace{t^4 + \sin^2 t}_{\substack{\rightarrow 0 \\ \text{Therefore,}}} \quad (\forall t \neq 0, 1)$$

3) Shorthand:

$$\text{As } x \rightarrow 0, \quad \underbrace{cx^8}_{\substack{\rightarrow 0 \\ \text{Therefore,}}} \leq \underbrace{x^8 f(x)}_{\substack{\rightarrow 0 \\ \text{Therefore,}}} \leq \underbrace{dx^8}_{\substack{\rightarrow 0 \\ \text{Therefore,}}} \quad (\forall x \neq 0)$$

4) Shorthand:

$$\text{As } x \rightarrow 0, \quad \underbrace{-|x|}_{\substack{\rightarrow 0 \\ \text{Therefore,}}} \leq \underbrace{x \cos\left(\frac{1}{x}\right)}_{\substack{\rightarrow 0 \\ \text{Therefore,}}} \leq \underbrace{|x|}_{\substack{\rightarrow 0 \\ \text{Therefore,}}} \quad (\forall x \neq 0)$$

5) Shorthand:

$$\text{As } x \rightarrow \infty, \quad \underbrace{-\frac{1}{x^5}}_{\substack{\rightarrow 0 \\ \text{Therefore,}}} \leq \underbrace{\frac{\cos x}{x^5}}_{\substack{\rightarrow 0 \\ \text{Therefore,}}} \leq \underbrace{\frac{1}{x^5}}_{\substack{\rightarrow 0 \\ \text{Therefore,}}} \quad (\forall x > 0)$$

- 6) Note: If $\theta < 0$, then $4\theta^3 < 0$, and $-4\theta^3 > 0$.

Shorthand:

$$\text{As } \theta \rightarrow -\infty, \quad \underbrace{\frac{5}{4\theta^3}}_{\substack{\rightarrow 0 \\ \text{Therefore,}}} \leq \underbrace{\frac{5\sin(3\theta)}{4\theta^3}}_{\substack{\rightarrow 0}} \leq -\underbrace{\frac{5}{4\theta^3}}_{\substack{\rightarrow 0}} \quad (\forall \theta < 0)$$

- 7) 5.

- 8) No. The properties listed in Section 2.2, Part A are claimed to be true under the assumption that all of the indicated limits exist as real constants.

Here, $\lim_{x \rightarrow 0} \sin\left(\frac{1}{x^2}\right)$ does not exist (DNE).

SECTION 2.7: PRECISE DEFINITIONS OF LIMITS

1) $|f(x) - L| = |(3x - 7) - (-1)| = |3x - 6| = 3|x - 2|$;

$$|f(x) - L| < \varepsilon \Leftrightarrow 3|x - 2| < \varepsilon \Leftrightarrow |x - 2| < \frac{\varepsilon}{3}; \text{ choose } \delta = \frac{\varepsilon}{3};$$

$$0 < |x - a| < \delta \Rightarrow 0 < |x - 2| < \frac{\varepsilon}{3} \Rightarrow 0 < 3|x - 2| < \varepsilon \Rightarrow |f(x) - L| < \varepsilon. \text{ Q.E.D.}$$

2) $\delta = 0.2, \delta = 0.02, \delta = 0.002$

3) Hints: $\left|\frac{1}{4}x + 2\right| = \frac{1}{4}|x - (-8)|$; choose $\delta = 4\varepsilon$.

4) All positive real numbers

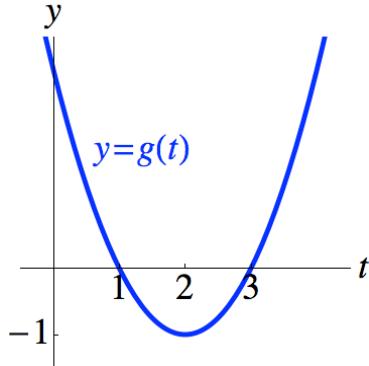
5) a) $9 - (2.9)^2 = 0.59$, b) $\sqrt[3]{8.01} - 2 \approx 0.00083299$

6) $\lim_{x \rightarrow a} f(x) = \infty \Leftrightarrow \forall M \in \mathbb{R}, \exists \delta > 0 \ \exists [0 < |x - a| < \delta \Rightarrow f(x) > M]$.

7) $\lim_{x \rightarrow a} f(x) = -\infty \Leftrightarrow \forall N \in \mathbb{R}, \exists \delta > 0 \ \exists [0 < |x - a| < \delta \Rightarrow f(x) < N]$.

SECTION 2.8: CONTINUITY

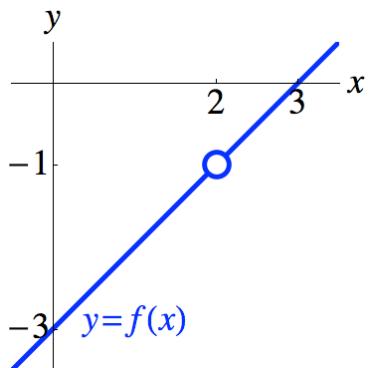
- 1) a) Discontinuities: None; Continuous on $(-\infty, \infty)$;



- b) Observe: $f(x) = x - 3$ ($x \neq 2$);

Discontinuities: 2 (removable: $\lim_{x \rightarrow 2} f(x) = -1$, but f is undefined at 2);

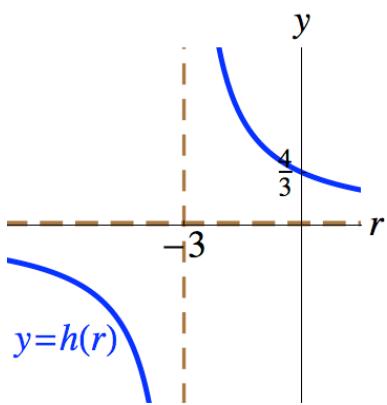
Continuous on $(-\infty, 2), (2, \infty)$;



- c) Observe: $h(r) = \frac{4}{r+3}$;

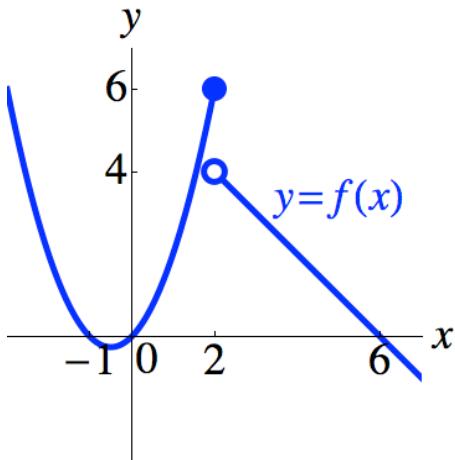
Discontinuities: -3 (infinite: $\lim_{r \rightarrow -3^+} h(r) = \infty$; also, $\lim_{r \rightarrow -3^-} h(r) = -\infty$);

Continuous on $(-\infty, -3), (-3, \infty)$;



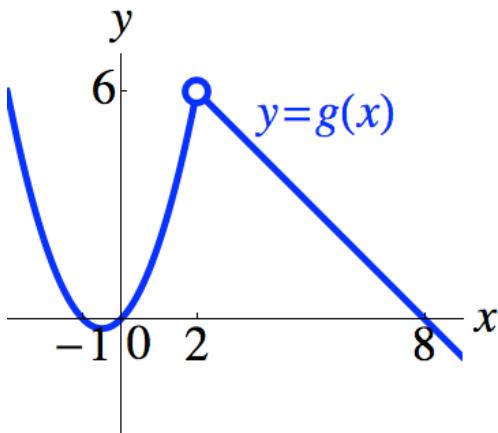
d) Discontinuities: 2 (jump: $\lim_{x \rightarrow 2^-} f(x) = 6$, and $\lim_{x \rightarrow 2^+} f(x) = 4$, but $6 \neq 4$);

Continuous on $(-\infty, 2], (2, \infty)$;



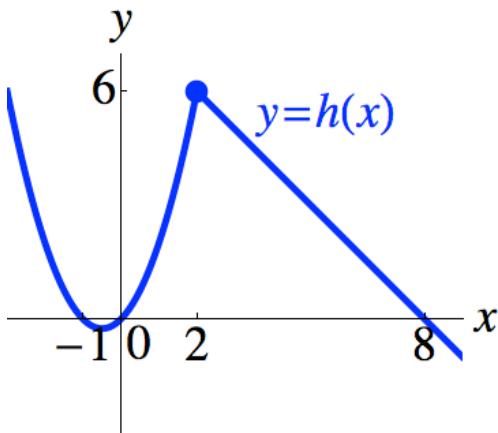
e) Discontinuities: 2 (removable: $\lim_{x \rightarrow 2^-} g(x) = 6$, and $\lim_{x \rightarrow 2^+} g(x) = 6$, so

$\lim_{x \rightarrow 2} g(x) = 6$, but g is undefined at 2); Continuous on $(-\infty, 2), (2, \infty)$;



f) Discontinuities: None; h is continuous at 2: $\lim_{x \rightarrow 2^-} h(x) = 6$, and $\lim_{x \rightarrow 2^+} h(x) = 6$,

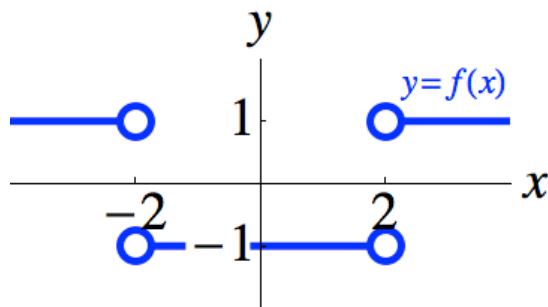
so $\lim_{x \rightarrow 2} h(x) = 6$, and $h(2) = 6$, so $\lim_{x \rightarrow 2} h(x) = h(2)$; Continuous on $(-\infty, \infty)$



g) Observe: $f(x) = \begin{cases} 1, & \text{if } |x| > 2 \\ -1, & \text{if } |x| < 2 \end{cases}$

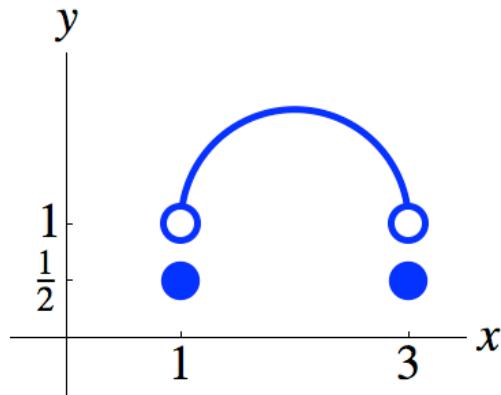
Discontinuities: -2 (jump: $\lim_{x \rightarrow -2^-} f(x) = 1$, and $\lim_{x \rightarrow -2^+} f(x) = -1$, but $1 \neq -1$), and 2 (jump: $\lim_{x \rightarrow 2^-} f(x) = -1$, and $\lim_{x \rightarrow 2^+} f(x) = 1$, but $-1 \neq 1$);

Continuous on $(-\infty, -2), (-2, 2), (2, \infty)$;



2) Jump discontinuity

3) For example,



4) $A = 10$

5) $[1, 3), (3, 4), (4, \infty)$

6) $\left\{ x \in \mathbb{R} \mid x \neq \frac{1}{\pi^2 n^2} \left(n \in \mathbb{Z}^+; \text{i.e., } n \text{ is a positive integer} \right), \text{ and } x > 0 \right\}$

(Answers to Exercises for Chapter 2: Limits and Continuity) A.2.13.

7) a) Let $f(x) = 3x^3 - 2x^2 - 2x - 5$. f is continuous on $[1, 2]$, so the IVT applies.

$f(1) = -6$, and $f(2) = 7$. $0 \in [-6, 7]$, so, by the IVT, $\exists c \in [1, 2] \ni f(c) = 0$; such a value for c is a solution to the given equation.

b)

$$\begin{array}{r|rrrr} 5/3 & 3 & -2 & -2 & -5 \\ \hline & 5 & 5 & 5 \\ \hline 3 & 3 & 3 & | & 0 \end{array}$$

8) a) s is continuous on $[0, 1]$, so the IVT applies. $s(0) = 4$ (feet), and

$s(1) = 18$ (feet). $15 \in [4, 18]$, so, by the IVT, $\exists c \in [0, 1] \ni s(c) = 15$. Such a value for c is a time (in seconds within one second after the projectile is fired) that the projectile achieves a height of 15 feet.

b) $t = \frac{1}{2}$ of a second

9) f is continuous on \mathbb{R} ; in particular, it is continuous on $[1, 3]$, so the IVT applies.

$f(1) = 6$, $f(3) = 14$. Let $d \in [6, 14]$, and let $c = \sqrt{d-5}$; observe:

$$f(c) = d \text{ and } c \in [1, 3] \Leftrightarrow c^2 + 5 = d \text{ and } c \in [1, 3]$$

$$\Leftrightarrow c^2 = d - 5 \text{ and } c \in [1, 3] \Leftrightarrow c = \sqrt{d-5}, \text{ a value in } [1, 3].$$

$$\text{Observe: } 6 \leq d \leq 14 \Leftrightarrow 1 \leq d - 5 \leq 9 \Leftrightarrow 1 \leq \sqrt{d-5} \leq 3.$$

$$\text{Then, } c \in [1, 3], \text{ and } f(c) = c^2 + 5 = (\sqrt{d-5})^2 + 5 = (d-5) + 5 = d.$$

$$\text{Therefore, } \forall d \in [6, 14], \exists c \in [1, 3] \ni f(c) = d.$$

10) Hint 1: Use the Quadratic Formula. Hint 2: You will choose $c = \sqrt{d+5} - 2$.