CALCULUS: THE ANSWERS

MATH 150: CALCULUS WITH ANALYTIC GEOMETRY I

VERSION 1.3

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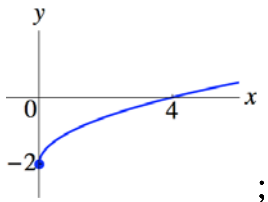
CHAPTER 1: REVIEW

1) $f(-4)=16$; $f(a+h)=(a+h)^2=a^2+2ah+h^2$
2) Polynomial: Yes, Rational: Yes, Algebraic: Yes
3) Polynomial: No, Rational: Yes, Algebraic: Yes
4) Polynomial: No, Rational: No, Algebraic: Yes
5) Polynomial: No, Rational: No, Algebraic: No
6) a) $(-\infty,\infty)$, b) $(-\infty,\infty)$, c) $\left(-\infty,-\frac{1}{2}\right)\cup\left(-\frac{1}{2},2\right)\cup(2,\infty)$, d) $[-2,5)\cup(5,\infty)$,

   e) $(-2,\infty)$, f) $(-\infty,7]$, g) $(-\infty,-1]\cup[3,\infty)$
7)



; Domain is $[0,\infty)$; Range is $[-2,\infty)$

8) the y-axis; the function is even
9) the origin; the function is odd
10) the function is even
11) the function is even
12) the function is neither even nor odd
13) $g(x)=x^4+x$, $f(u)=u^8$; there are other possibilities
14) $g(t)=\frac{1}{t}$, $f(u)=\sqrt[4]{u}$; there are other possibilities
15) $g(r)=r^2$, $f(u)=\sin u$; there are other possibilities
16) a) undefined, b) -1, c) $\sqrt{2}$, d) $-\frac{2\sqrt{3}}{3}$, e) $-\frac{\sqrt{3}}{2}$, f) $-\sqrt{3}$
17) Hint on a): Use a Double-Angle ID; Hint on b): Use a Pythagorean ID
18) a) $\left\{x\in\mathbb{R}\,\middle|\, x=-\frac{\pi}{6}+2\pi n,\ \text{or}\ x=\frac{7\pi}{6}+2\pi n,\ \text{or}\ x=\frac{3\pi}{2}+2\pi n\quad (n\in\mathbb{Z})\right\}$

    Note: $-\frac{\pi}{6}$ can be replaced by $\frac{11\pi}{6}$, $\frac{3\pi}{2}$ can be replaced by $-\frac{\pi}{2}$, etc.

    b) $\left\{x\in\mathbb{R}\,\middle|\, x=\pm\frac{\pi}{9}+\frac{2\pi n}{3}\quad (n\in\mathbb{Z})\right\}$, or, equivalently,

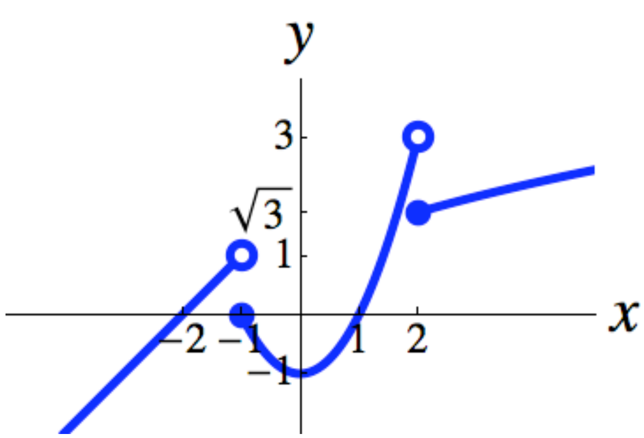
    $\left\{x\in\mathbb{R}\,\middle|\, x=\frac{\pi}{9}+\frac{2\pi n}{3},\ \text{or}\ x=\frac{5\pi}{9}+\frac{2\pi n}{3}\quad (n\in\mathbb{Z})\right\}$

CHAPTER 2: LIMITS AND CONTINUITY

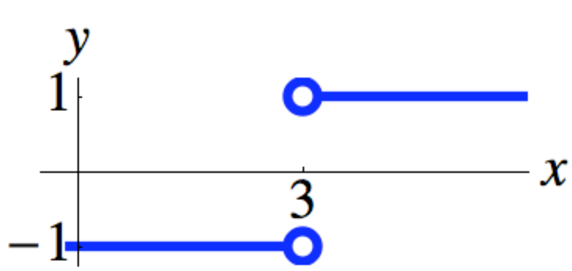
SECTION 2.1: AN INTRODUCTION TO LIMITS

1) 57
2) 11/7
3) 10
4) π^2
5) a) $f(1) = 0.9999$, $f(0.1) = 0.0999$, $f(0.01) = 0.0099$; b) -0.0001, No
6) a) 11/7, b) 11/7; as a consequence, the answer to Exercise 2 is the same.
7) No; a counterexample: $\lim_{x \to 0^+} \sqrt{x} = 0$, while $\lim_{x \to 0} \sqrt{x}$ does not exist (DNE).

   See also Example 8.
8) Yes
9) No; a counterexample: see Example 10 on $h(x) = \begin{cases} x+3, & x \neq 3 \\ 7, & x = 3 \end{cases}$
10) No; a counterexample: see Example 9 on $g(x) = x + 3,\ (x \neq 3)$.
11) a) b) 1, 0, DNE; c) 3, $\sqrt{3}$, DNE, 2



12) a) b) -1, 1, DNE



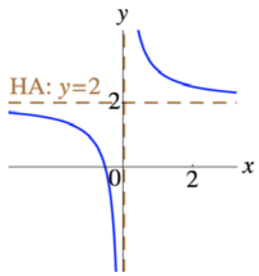
13) a) 0 (liters), which means that, if the gas's temperature approaches absolute zero (from above), its volume approaches zero (liters).
    b) DNE, because temperatures cannot go below absolute zero. The domain of V does not include values of T below absolute zero.

SECTION 2.2: PROPERTIES OF LIMITS and ALGEBRAIC FUNCTIONS

1) $\dfrac{39+18\sqrt{3}}{16}$, or $\dfrac{3}{16}\left(13+6\sqrt{3}\right)$
2) a) DNE, b) 0, c) DNE
3) a) 0, b) 0, c) 0
4) a) 0, b) DNE, c) DNE, d) 0, e) $\sqrt{5}$, f) DNE, g) 0
5) a) DNE, b) 0, c) DNE, d) 0, e) DNE, f) DNE, g) $\sqrt{6}$
6) Yes, by linearity of the limit operator.
7) No; a counterexample: $\lim\limits_{x\to 0}\sqrt{x^2}=0$. See also Example 5 on $\lim\limits_{x\to -7}\sqrt{(x+7)^2}$.

SECTION 2.3: LIMITS AND INFINITY I

1) a) b) 2, c) $y=2$



2) a) 0; b) 0 or 1; c) 0, 1, or 2
3) a) 0^+, b) 0^-, c) 0^+, d) $-\infty$, e) ∞, f) ∞, g) ∞, h) 0^+, i) 0
4) Yes
5) No; some counterexamples: Example 5 on $f(x)=\dfrac{\sin x}{x}$; also, $f(x)=\dfrac{\sin x+2}{\sin x+2}$.
6) a) DNE, b) 0 or 0^+, c) 0 or 0^-, d) 0 or 0^+, e) DNE, f) ∞
7) a) $\sqrt{\pi}$

   b) i. $\lim\limits_{x\to\infty}\left(x^5+3x^4-2\right)=\lim\limits_{x\to\infty}x^5=\infty$;

   ii. $\lim\limits_{x\to\infty}\left(x^5+3x^4-2\right)=\lim\limits_{x\to\infty}\underbrace{x^5}_{\to\infty}\underbrace{\left(1+\frac{3}{x}-\frac{2}{x^5}\right)}_{\to 1}=\infty$

   c) i. $\lim\limits_{x\to-\infty}\left(2x^3-6x^2+x\right)=\lim\limits_{x\to-\infty}2x^3=-\infty$;

   ii. $\lim\limits_{x\to-\infty}\left(2x^3-6x^2+x\right)=\lim\limits_{x\to-\infty}\underbrace{2x^3}_{\to-\infty}\underbrace{\left(1-\frac{3}{x}+\frac{1}{2x^2}\right)}_{\to 1}=-\infty$

   d) $\lim\limits_{w\to\infty}\left(5w-4w^4\right)=\lim\limits_{w\to\infty}\left(-4w^4\right)=-\infty$

8) a) i. 0, because g is a proper rational function, and we seek a "long-run" limit;

ii. $\lim_{r\to\infty} g(r) = \lim_{r\to\infty} \frac{3r^3}{2r^5} = \lim_{r\to\infty} \frac{3}{2r^2} = 0$;

iii. $\lim_{r\to\infty} g(r) = \lim_{r\to\infty} \dfrac{\frac{3r^3}{r^5} + \frac{r}{r^5} - \frac{4}{r^5}}{\frac{2r^5}{r^5} - \frac{7r^2}{r^5}} == \lim_{r\to\infty} \dfrac{\overbrace{\frac{3}{r^2}}^{\to 0} + \overbrace{\frac{1}{r^4}}^{\to 0} - \overbrace{\frac{4}{r^5}}^{\to 0}}{2 - \underbrace{\frac{7}{r^3}}_{\to 0}} = \frac{0}{2} = 0$, or

$$\lim_{r\to\infty} g(r) = \lim_{r\to\infty} \frac{3r^3 \overbrace{\left(1 + \frac{1}{3r^2} - \frac{4}{3r^3}\right)}^{\to 1}}{2r^5 \underbrace{\left(1 - \frac{7}{2r^3}\right)}_{\to 1}} = \lim_{r\to\infty} \frac{3r^3}{2r^5} = \lim_{r\to\infty} \frac{3}{2r^2} = 0;$$

iv. $\lim_{r\to -\infty} g(r) = 0$, also;

v. HA: $s = 0$

b) i. $\frac{7}{3}$, because the numerator and the denominator have the same degree, and we seek a "long-run" limit, so we take the ratio of the leading coefficients;

ii. $\lim_{x\to -\infty} \frac{7x^4 - 5x}{3x^4 + 2} = \lim_{x\to -\infty} \frac{7x^4}{3x^4} = \frac{7}{3}$;

iii. $\lim_{x\to -\infty} \frac{7x^4 - 5x}{3x^4 + 2} = \lim_{x\to -\infty} \dfrac{\frac{7x^4}{x^4} - \frac{5x}{x^4}}{\frac{3x^4}{x^4} + \frac{2}{x^4}} = \lim_{x\to -\infty} \dfrac{7 - \overbrace{\frac{5}{x^3}}^{\to 0}}{3 + \underbrace{\frac{2}{x^4}}_{\to 0}} = \frac{7}{3}$, or

$$\lim_{x\to -\infty} \frac{7x^4 - 5x}{3x^4 + 2} = \lim_{x\to -\infty} \frac{7x^4 \overbrace{\left(1 - \frac{5}{7x^3}\right)}^{\to 1}}{3x^4 \underbrace{\left(1 + \frac{2}{3x^4}\right)}_{\to 1}} = \lim_{x\to -\infty} \frac{7x^4}{3x^4} = \frac{7}{3};$$

iv. HA: $y = \frac{7}{3}$

c) i. $\lim_{x\to\infty}\dfrac{\sqrt{2}x^5+11x^8-\pi}{6x^5+x}=\lim_{x\to\infty}\dfrac{11x^8}{6x^5}=\lim_{x\to\infty}\dfrac{11}{6}x^3=\infty$;

ii. No, the graph has no HA;

iii. $-\infty$

d) 1/4

9) a) $f(x)=-3x+2+\dfrac{x^2}{x^3+1}$; b) $-\infty$; c) ∞; d) $y=-3x+2$, or $y=2-3x$

10) $\dfrac{\sqrt{3}}{7}$

11) a) $-\infty$

b) DNE

c) $\lim_{x\to\infty}\dfrac{\sqrt{4x^6+x^2}+2x^2}{5x^3-\sqrt[3]{x}}=\lim_{x\to\infty}\dfrac{\sqrt{4x^6}+2x^2}{5x^3}=\lim_{x\to\infty}\dfrac{2\left|x^3\right|+2x^2}{5x^3}=\lim_{x\to\infty}\dfrac{2x^3}{5x^3}=\dfrac{2}{5}$

d)

$$\lim_{x\to-\infty}\frac{\sqrt{4x^6+x^2}+2x^2}{5x^3-\sqrt[3]{x}}=\lim_{x\to-\infty}\frac{\sqrt{4x^6}+2x^2}{5x^3}=\lim_{x\to-\infty}\frac{2\left|x^3\right|+2x^2}{5x^3}$$

$$=\lim_{x\to-\infty}\frac{-2x^3}{5x^3}=-\frac{2}{5}$$

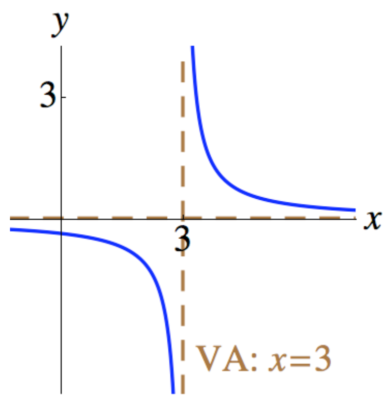
e) HAs: $y=\dfrac{2}{5}$ and $y=-\dfrac{2}{5}$

f) $\lim_{z\to\infty}\dfrac{\sqrt[3]{5z^{12}+7z^7}}{z^5+2}=\lim_{z\to\infty}\dfrac{\sqrt[3]{5z^{12}}}{z^5}=\lim_{z\to\infty}\dfrac{\sqrt[3]{5}z^4}{z^5}=\lim_{z\to\infty}\dfrac{\sqrt[3]{5}}{z}=0$

12) a) $p(t)=\dfrac{2500+100t}{2500+250t}\left(\dfrac{E.\ calculi\text{ bacteria in the dish}}{\text{total bacteria in the dish}}\right)$;

b) $\dfrac{2}{5}\left(\dfrac{E.\ calculi\text{ bacteria in the dish}}{\text{total bacteria in the dish}}\right)$; the proportion of the bacteria in the Petri dish that are *E. calculi* approaches $\dfrac{2}{5}$ in the long run, the same as for the incoming stream; this calculation assumes that the petri dish has infinite capacity and that infinitely many bacteria are available, which is unrealistic.

SECTION 2.4: LIMITS AND INFINITY II

1) a) 

b) ∞, c) $-\infty$, d) $x = 3$

2) a) 0
b) any nonnegative integer number
c) any nonnegative integer number, or infinitely many

3) a) ∞, b) $-\infty$, c) $-\infty$, d) ∞

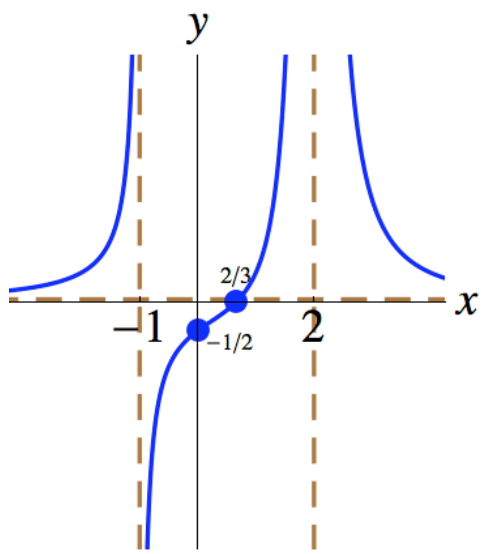
4) a) $f(x) = \dfrac{3x-2}{(x+1)(x-2)^2}$

b) i. $-\infty$. Work: $\displaystyle\lim_{x\to -1^+} f(x) = \lim_{x\to -1^+} \frac{\overbrace{3x-2}^{\to -5}}{\underbrace{(x+1)}_{\to 0^+}\underbrace{(x-2)^2}_{\to 9}}$ $\left(\text{Limit Form } \dfrac{-5}{0^+}\right) = -\infty$.

ii. ∞, iii. DNE, iv. ∞, v. ∞, vi. ∞, vii. $-\dfrac{1}{2}$

c) i. 0, ii. 0
d) HA: $y = 0$
e) VAs: $x = -1$ and $x = 2$
f) $\dfrac{2}{3}$, or $\left(\dfrac{2}{3}, 0\right)$
g) $-\dfrac{1}{2}$, or $\left(0, -\dfrac{1}{2}\right)$

h)



5) $f(x)=\dfrac{4x^2}{(x+2)(x-3)}$; there are other possibilities

6) a) VAs: $t=1$ and $t=3$;
for $t=1$, it is sufficient to show either $\lim_{t\to 1^+} g(t)=\infty$, or $\lim_{t\to 1^-} g(t)=-\infty$;
for $t=3$, it is sufficient to show either $\lim_{t\to 3^+} g(t)=-\infty$, or $\lim_{t\to 3^-} g(t)=\infty$.

b) HA: $w=-3$;

$$\lim_{t\to\infty} g(t)=\lim_{t\to\infty}\frac{\dfrac{t}{t^2}-\dfrac{6t^2}{t^2}}{\dfrac{2t^2}{t^2}-\dfrac{8t}{t^2}+\dfrac{6}{t^2}}=\lim_{t\to\infty}\frac{\overbrace{\dfrac{1}{t}}^{\to 0}-6}{2-\underbrace{\dfrac{8}{t}}_{\to 0}+\underbrace{\dfrac{6}{t^2}}_{\to 0}}=\frac{-6}{2}=-3;$$

also, $\lim_{t\to -\infty} g(t)=-3$ (observe that g is a rational function).

7) 0 VAs, 0 HAs

8) a) ∞, b) $-\infty$, c) ∞, d) ∞

9) a) ∞, which means that, if an object's speed approaches the speed of light (from below), its mass increases without bound
b) DNE, which makes sense because faster-than-light speed is impossible.

SECTION 2.5 : THE INDETERMINATE FORMS $\frac{0}{0}$ AND $\frac{\infty}{\infty}$

1) $f(x) = x$, $g(x) = x^2$; there are other possibilities

2) a) 6; see the equivalent function in Section 2.1, Example 9: $g(x) = x + 3$, $(x \neq 3)$.

b) $-\frac{19}{12}$, c) $\frac{1}{10}$; Hint: Factor the denominator, or rationalize the numerator.

d) $\frac{1}{8}$, e) $-\frac{1}{4}$

f) $\frac{17}{4}$; Hint: Factor the numerator by grouping, use Long Division, or use the Rational Zero Test (Rational Roots Theorem) and Synthetic Division from Section 2.3 of the Precalculus notes; for the last method, it helps to recognize: $2\left(x - \frac{1}{2}\right) = 2x - 1$.

3) a) $\text{Dom}(f) = \{x \in \mathbb{R} \mid x \neq -1 \text{ and } x \neq 4\} = (-\infty, -1) \cup (-1, 4) \cup (4, \infty)$

b) 3, or $(3, 0)$

c) $\frac{3}{4}$, or $\left(0, \frac{3}{4}\right)$

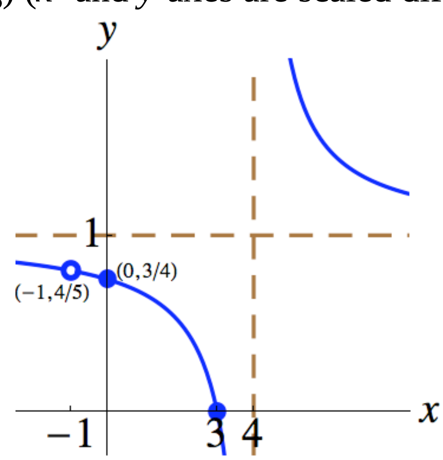
d) neither even nor odd

e) VA: $x = 4$, because $\lim_{x \to 4^+} f(x) = \infty$; also, $\lim_{x \to 4^-} f(x) = -\infty$.

HA: $y = 1$, because $\lim_{x \to \infty} f(x) = 1$.

f) $\left(-1, \frac{4}{5}\right)$, because $\lim_{x \to -1} f(x) = \frac{4}{5}$, and -1 is not in $\text{Dom}(f)$.

g) (x- and y-axes are scaled differently below)



SECTION 2.6: THE SQUEEZE (SANDWICH) THEOREM

1) Shorthand:

$$\text{As } x \to 0, \quad \underbrace{-x^2}_{\to 0} \le \underbrace{x^2 \sin\left(\frac{1}{x^2}\right)}_{\substack{\text{Therefore,}\\ \to 0}} \le \underbrace{x^2}_{\to 0} \quad (x \ne 0)$$

2) Note 1: The domain of $\cos\left(\dfrac{t+3}{\sqrt[3]{t^2 - t}}\right)$ is $\left\{t \in \mathbb{R} \,\middle|\, t \ne 0 \text{ and } t \ne 1\right\}$.

We can, for example, restrict our attention to the "punctured" t-interval $(-1, 1)$, $t \ne 0$.

Note 2: $t^4 + \sin^2 t > 0$ whenever $t \ne 0$.
Shorthand:

$$\text{As } t \to 0, \quad \underbrace{-\left(t^4 + \sin^2 t\right)}_{\to 0} \le \underbrace{\left(t^4 + \sin^2 t\right)\cos\left(\frac{t+3}{\sqrt[3]{t^2 - t}}\right)}_{\substack{\text{Therefore,}\\ \to 0}} \le \underbrace{t^4 + \sin^2 t}_{\to 0} \quad (t \ne 0, 1)$$

3) Shorthand:

$$\text{As } x \to 0, \quad \underbrace{cx^8}_{\to 0} \le \underbrace{x^8 f(x)}_{\substack{\text{Therefore,}\\ \to 0}} \le \underbrace{dx^8}_{\to 0} \quad (x \ne 0)$$

4) Shorthand:

$$\text{As } x \to 0, \quad \underbrace{-|x|}_{\to 0} \le \underbrace{x \cos\left(\frac{1}{x}\right)}_{\substack{\text{Therefore,}\\ \to 0}} \le \underbrace{|x|}_{\to 0} \quad (x \ne 0)$$

5) Shorthand:

$$\text{As } x \to \infty, \quad \underbrace{-\frac{1}{x^5}}_{\to 0} \le \underbrace{\frac{\cos x}{x^5}}_{\substack{\text{Therefore,}\\ \to 0}} \le \underbrace{\frac{1}{x^5}}_{\to 0} \quad (x > 0)$$

6) Note: If $\theta < 0$, then $4\theta^3 < 0$, and $-4\theta^3 > 0$.
Shorthand:

$$\text{As } \theta \to -\infty, \quad \underbrace{\frac{5}{4\theta^3}}_{\to 0} \le \underbrace{\frac{5\sin(3\theta)}{4\theta^3}}_{\substack{\text{Therefore,} \\ \to 0}} \le \underbrace{-\frac{5}{4\theta^3}}_{\to 0} \quad (\theta < 0)$$

7) 5.

8) No. The properties listed in Section 2.2, Part A are claimed to be true under the assumption that all of the indicated limits exist as real constants.
Here, $\lim_{x \to 0} \sin\left(\frac{1}{x^2}\right)$ does not exist (DNE).

SECTION 2.7: PRECISE DEFINITIONS OF LIMITS

1) $\left| f(x) - L \right| = \left| (3x - 7) - (-1) \right| = \left| 3x - 6 \right| = 3\left| x - 2 \right|$;

$\left| f(x) - L \right| < \varepsilon \iff 3\left| x - 2 \right| < \varepsilon \iff \left| x - 2 \right| < \frac{\varepsilon}{3}$; choose $\delta = \frac{\varepsilon}{3}$;

$0 < \left| x - a \right| < \delta \Rightarrow 0 < \left| x - 2 \right| < \frac{\varepsilon}{3} \Rightarrow 0 < 3\left| x - 2 \right| < \varepsilon \Rightarrow \left| f(x) - L \right| < \varepsilon$. Q.E.D.

2) $\delta = 0.2$, $\delta = 0.02$, $\delta = 0.002$

3) Hints: $\left| \frac{1}{4}x + 2 \right| = \frac{1}{4}\left| x - (-8) \right|$; choose $\delta = 4\varepsilon$.

4) All positive real numbers

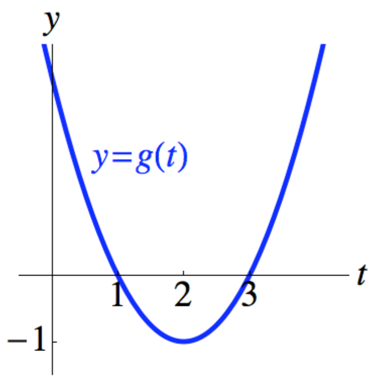
5) a) $9 - (2.9)^2 = 0.59$, b) $\sqrt[3]{8.01} - 2 \approx 0.00083299$

6) $\lim_{x \to a} f(x) = \infty \iff \forall M \in \mathbb{R}, \exists \delta > 0 \ni \left[0 < \left| x - a \right| < \delta \Rightarrow f(x) > M \right]$.

7) $\lim_{x \to a} f(x) = -\infty \iff \forall N \in \mathbb{R}, \exists \delta > 0 \ni \left[0 < \left| x - a \right| < \delta \Rightarrow f(x) < N \right]$.

SECTION 2.8: CONTINUITY

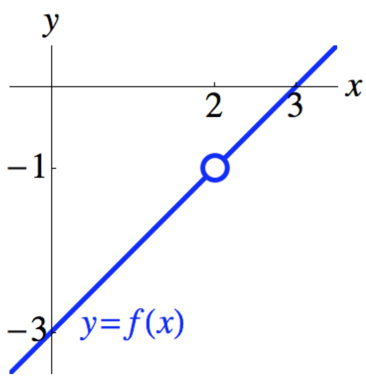
1) a) Discontinuities: None; Continuous on $(-\infty, \infty)$;



b) Observe: $f(x) = x - 3 \quad (x \neq 2)$;

Discontinuities: 2 (removable: $\lim_{x \to 2} f(x) = -1$, but f is undefined at 2);

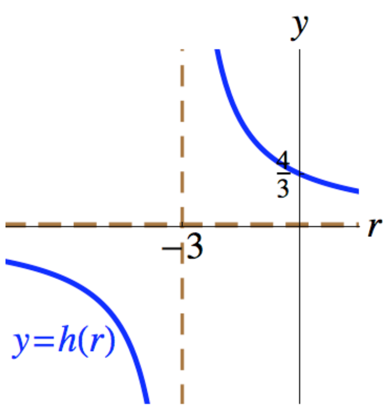
Continuous on $(-\infty, 2), (2, \infty)$;



c) Observe: $h(r) = \dfrac{4}{r+3}$;

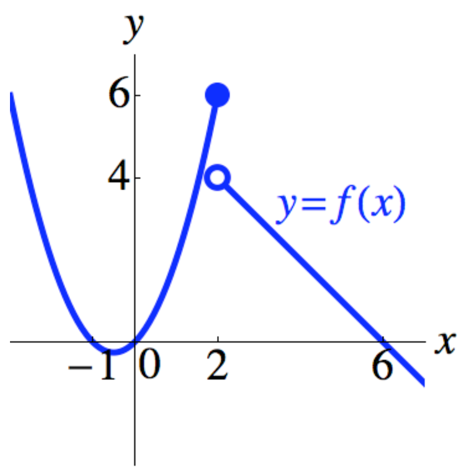
Discontinuities: -3 (infinite: $\lim_{r \to -3^+} h(r) = \infty$; also, $\lim_{r \to -3^-} h(r) = -\infty$);

Continuous on $(-\infty, -3), (-3, \infty)$;



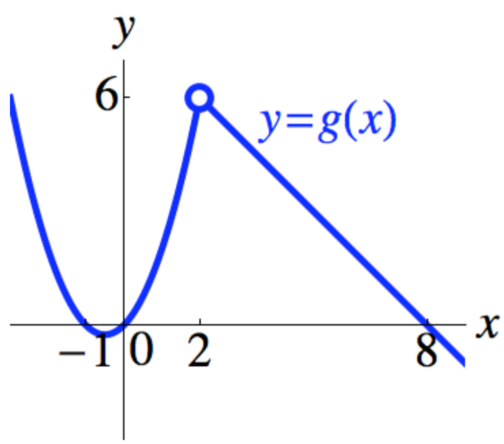
d) Discontinuities: 2 (jump: $\lim_{x\to 2^-} f(x) = 6$, and $\lim_{x\to 2^+} f(x) = 4$, but $6 \neq 4$);

Continuous on $(-\infty, 2), (2, \infty)$;



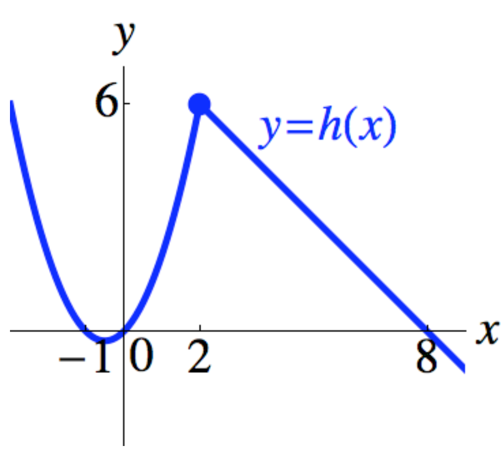
e) Discontinuities: 2 (removable: $\lim_{x\to 2^-} g(x) = 6$, and $\lim_{x\to 2^+} g(x) = 6$, so

$\lim_{x\to 2} g(x) = 6$, but f is undefined at 2); Continuous on $(-\infty, 2), (2, \infty)$;



f) Discontinuities: None; h is continuous at 2: $\lim_{x\to 2^-} h(x) = 6$, and $\lim_{x\to 2^+} h(x) = 6$,

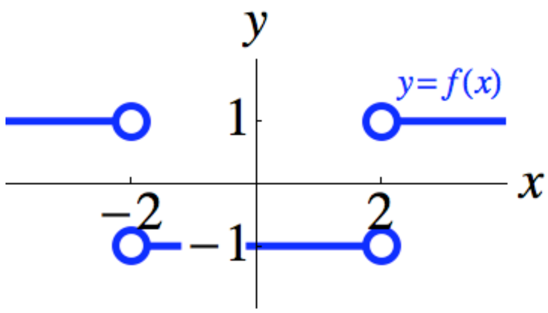
so $\lim_{x\to 2} h(x) = 6$, and $h(2) = 6$, so $\lim_{x\to 2} h(x) = h(2)$; Continuous on $(-\infty, \infty)$



g) Observe: $f(x)=\begin{cases}1, & \text{if } |x|>2 \\ -1, & \text{if } |x|<2\end{cases}$

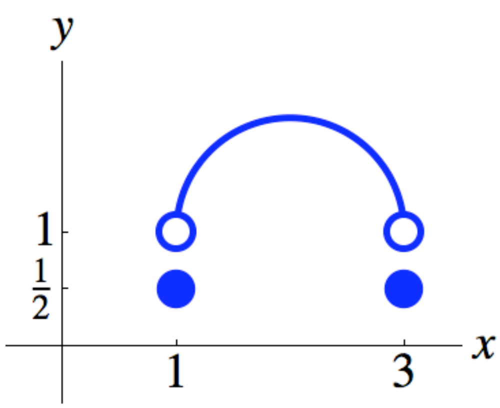
Discontinuities: -2 (jump: $\lim_{x\to -2^-} f(x)=1$, and $\lim_{x\to -2^+} f(x)=-1$, but $1\neq -1$), and 2 (jump: $\lim_{x\to 2^-} f(x)=-1$, and $\lim_{x\to 2^+} f(x)=1$, but $-1\neq 1$);

Continuous on $(-\infty,-2),(-2,2),(2,\infty)$;



2) Jump discontinuity

3) For example,



4) $A=10$

5) $[1,3),(3,4),(4,\infty)$

6) $\left\{x\in\mathbb{R}\,\middle|\, x\neq \frac{1}{\pi^2 n^2}\ \left(n\in\mathbb{Z}^+;\text{ i.e., } n \text{ is a positive integer}\right),\ \text{and } x>0\right\}$

7) a) Let $f(x) = 3x^3 - 2x^2 - 2x - 5$. f is continuous on $[1, 2]$, so the IVT applies.
$f(1) = -6$, and $f(2) = 7$. $0 \in [-6, 7]$, so, by the IVT, $\exists c \in [1, 2] \ni f(c) = 0$; such a value for c is a solution to the given equation.

b)

5/3	3	−2	−2	−5
		5	5	5
	3	3	3	0

8) a) s is continuous on $[0, 1]$, so the IVT applies. $s(0) = 4\,(\text{feet})$, and $s(1) = 18\,(\text{feet})$. $15 \in [4, 18]$, so, by the IVT, $\exists c \in [0, 1] \ni s(c) = 15$. Such a value for c is a time (in seconds within one second after the projectile is fired) that the projectile achieves a height of 15 feet.

b) $t = \dfrac{1}{2}$ of a second

9) f is continuous on $\mathbb{R}$; in particular, it is continuous on $[1, 3]$, so the IVT applies.
$f(1) = 6$, $f(3) = 14$. Let $d \in [6, 14]$, and let $c = \sqrt{d - 5}$; observe:

$$f(c) = d \text{ and } c \in [1, 3] \quad \Leftrightarrow \quad c^2 + 5 = d \text{ and } c \in [1, 3]$$

$$\Leftrightarrow \quad c^2 = d - 5 \text{ and } c \in [1, 3] \quad \Leftrightarrow \quad c = \sqrt{d - 5}, \text{ a value in } [1, 3].$$

Observe: $6 \le d \le 14 \quad \Leftrightarrow \quad 1 \le d - 5 \le 9 \quad \Leftrightarrow \quad 1 \le \sqrt{d - 5} \le 3$.

Then, $c \in [1, 3]$, and $f(c) = c^2 + 5 = \left(\sqrt{d - 5}\right)^2 + 5 = (d - 5) + 5 = d$.

Therefore, $\forall d \in [6, 14]$, $\exists c \in [1, 3] \ni f(c) = d$.

10) Hint 1: Use the Quadratic Formula. Hint 2: You will choose $c = \sqrt{d + 5} - 2$.