

CHAPTER 3: DERIVATIVES

SECTION 3.1: DERIVATIVES, TANGENT LINES, and RATES OF CHANGE

1) a)

$\frac{f(3.1) - f(3)}{3.1 - 3}$; this is $\frac{f(3+h) - f(3)}{h}$ with $h = 0.1$	30.5
$\frac{f(3.01) - f(3)}{3.01 - 3}$; this is $\frac{f(3+h) - f(3)}{h}$ with $h = 0.01$	30.05
$\frac{f(3.001) - f(3)}{3.001 - 3}$; this is $\frac{f(3+h) - f(3)}{h}$ with $h = 0.001$	30.005
$\frac{f(2.9) - f(3)}{2.9 - 3}$; this is $\frac{f(3+h) - f(3)}{h}$ with $h = -0.1$	29.5
$\frac{f(2.99) - f(3)}{2.99 - 3}$; this is $\frac{f(3+h) - f(3)}{h}$ with $h = -0.01$	29.95
$\frac{f(2.999) - f(3)}{2.999 - 3}$; this is $\frac{f(3+h) - f(3)}{h}$ with $h = -0.001$	29.995

b) No

c) 30

2) a) $f'(a) = \frac{3}{2\sqrt{3a-2}}$. Hint: Rationalize the numerator of the difference quotient.b) Point-Slope Form: $y - 5 = \frac{3}{10}(x - 9)$, Slope-Intercept Form: $y = \frac{3}{10}x + \frac{23}{10}$ c) Point-Slope Form: $y - 5 = -\frac{10}{3}(x - 9)$, Slope-Intercept Form: $y = -\frac{10}{3}x + 35$ 3) a) i. $-4.3 \frac{\text{cm}}{\text{sec}}$, ii. $-4.03 \frac{\text{cm}}{\text{sec}}$; b) $-4 \frac{\text{cm}}{\text{sec}}$

SECTION 3.2: DERIVATIVE FUNCTIONS and DIFFERENTIABILITY

1) Hint: $\frac{f(x+h)-f(x)}{h} = \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}; f'(x) = -\frac{2}{x^3}$

2) Hint: $\frac{r(x+h)-r(x)}{h} = \frac{(x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4) - x^4}{h}; r'(x) = 4x^3$

3) $f'(x) = \frac{6}{x^{1/3}}$, or $\frac{6}{\sqrt[3]{x}}$, $f''(x) = -\frac{2}{x^{4/3}}$, or $-\frac{2}{\sqrt[3]{x^4}}$, $f'''(x) = \frac{8}{3x^{7/3}}$, or $\frac{8}{3(\sqrt[3]{x^7})}$,

$$f^{(4)}(x) = -\frac{56}{9x^{10/3}}, \text{ or } -\frac{56}{9(\sqrt[3]{x^{10}})}$$

4) 0

5) a) $v(t) = 20t^4$

b) $v(1) = 20$ mph, $v(2) = 320$ mph, $v(-4.7) = 9759.362$ mph; mph = miles per hour

c) $a(t) = 80t^3$

d) $a(1) = 80 \frac{\text{mi}}{\text{hr}^2}$, $a(2) = 640 \frac{\text{mi}}{\text{hr}^2}$, $a(-4.7) = -8305.84 \frac{\text{mi}}{\text{hr}^2}$

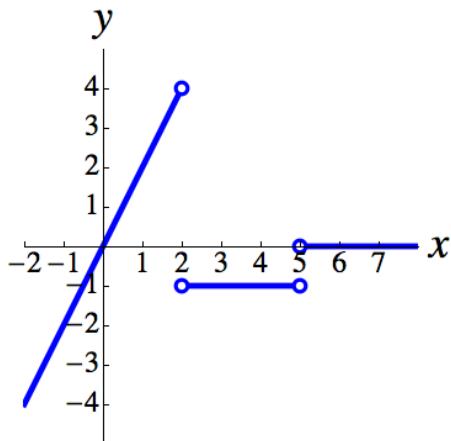
6) a) Yes; b) No; c) No; d) No (observe that p is discontinuous at -1); e) No; f) Yes

7) a) Yes, there is a vertical tangent line; a cusp

b) Yes, there is a vertical tangent line; neither a corner nor a cusp

c) No, there is not a vertical tangent line; a corner

8)



SECTION 3.3: TECHNIQUES OF DIFFERENTIATION

1) Hint: $\frac{g(w+h)-g(w)}{h} = \frac{\left[3(w+h)^2 - 5(w+h) + 4\right] - [3w^2 - 5w + 4]}{h};$
 $g'(w) = 6w - 5$

2) a) $15x^2 + \frac{6}{x^3} - \frac{1}{2x^{3/2}} + \frac{1}{18x^{2/3}}$, or $15x^2 + \frac{6}{x^3} - \frac{1}{2\sqrt{x^3}} + \frac{1}{18(\sqrt[3]{x^2})}$

b) $\frac{13}{(5-t)^2}$, or $\frac{13}{(t-5)^2}$

c) $4z^3 - 16z$; this can be factored as $4z(z+2)(z-2)$.

d) $(2w-3)(w^3-2) + (w^2-3w+1)(3w^2)$

e) i. $\frac{1}{x^3} - \frac{5}{2x^2}$ (Hint: First, reexpress using algebra.),

ii. $\frac{2-5x}{2x^3}$, which is equivalent to $\frac{1}{x^3} - \frac{5}{2x^2}$.

f) $-\frac{4(3+4x)}{(3x+2x^2)^2}$, which could be “simplified” to $-\frac{4(3+4x)}{x^2(3+2x)^2}$;

ask your instructor if s/he has a preference.

g) $18x$

h) $-\frac{6}{(3x+1)^3}$

3) a) $v(t) = 12t^2 + 30t - 18$

b) $v(1) = 24 \frac{\text{ft}}{\text{min}}$, $v(2) = 90 \frac{\text{ft}}{\text{min}}$, $v(-4.7) = 106.08 \frac{\text{ft}}{\text{min}}$

c) $a(t) = 24t + 30$

d) $a(1) = 54 \frac{\text{ft}}{\text{min}^2}$, $a(2) = 78 \frac{\text{ft}}{\text{min}^2}$, $a(-4.7) = -82.8 \frac{\text{ft}}{\text{min}^2}$

(Answers to Exercises for Chapter 3: Derivatives) A.3.4

4) a) Point-Slope Form: $y - 1 = -\frac{1}{2}(x - 2)$, Slope-Intercept Form: $y = -\frac{1}{2}x + 2$

b) Point-Slope Form: $y - 1 = 2(x - 2)$, Slope-Intercept Form: $y = 2x - 3$

5) Hint: $D_x[f(x)g(x)h(x)] = D_x([f(x)g(x)]h(x))$.

6) $3[f(x)]^2 f'(x)$. Hint: Assume that f, g , and h are equivalent functions.

7) a) $\left(-\frac{1}{3}, \frac{73}{54}\right)$ and $(2, -5)$

b) Point-Slope Form: $y - \left(-\frac{5}{2}\right) = -4(x - 1)$, Slope-Intercept Form: $y = -4x + \frac{3}{2}$

c) Point-Slope Form: $y - \left(-\frac{5}{2}\right) = \frac{1}{4}(x - 1)$, Slope-Intercept Form: $y = \frac{1}{4}x - \frac{11}{4}$

d) $\left(3, -\frac{1}{2}\right)$ and $\left(-\frac{4}{3}, -\frac{85}{27}\right)$

8) a) $(10, 200)$ and $(-10, 200)$. Hint: Find the point(s) $(a, f(a))$ on the flight path where the slope of the tangent line there equals the slope of the line connecting the point and the target.

b) $(-2, 104)$ and $(50, 2600)$

SECTION 3.4: DERIVATIVES OF TRIGONOMETRIC FUNCTIONS

1) a) $\frac{1}{4}$

b) $\frac{5}{3}$. Hint: $\lim_{x \rightarrow 0} \frac{\sin(5x)}{5x} = 1$.

c) 1000

d) 0. Hint: Factor the numerator.

2) a) $5x^4 \cos x - x^5 \sin x$, or $x^4(5\cos x - x \sin x)$

b) $\frac{1}{1 + \cos w}$

c) $\csc^2 r - (\csc r)(\cot r)$, or $(\csc r)(\csc r - \cot r)$

d) $7(\sec \alpha)(\tan \alpha) + 8\alpha$

e) $2\theta \tan \theta + \theta^2 \sec^2 \theta$, or $\theta(2 \tan \theta + \theta \sec^2 \theta)$

f) 0, $\text{Dom}(k') = \left\{ \beta \in \mathbb{R} \mid \beta \neq \frac{\pi}{2} + \pi n \quad (n \in \mathbb{Z}) \right\}$

3) $\left\{ x \in \mathbb{R} \mid x = \frac{\pi}{4} + 2\pi n, \text{ or } x = \frac{3\pi}{4} + 2\pi n \quad (n \in \mathbb{Z}) \right\}$

4) a) $\left\{ x \in \mathbb{R} \mid x = \pi n \quad (n \in \mathbb{Z}) \right\}$

b) $\left\{ x \in \mathbb{R} \mid x = \frac{4\pi}{3} + 2\pi n, \text{ or } x = \frac{5\pi}{3} + 2\pi n \quad (n \in \mathbb{Z}) \right\}$

c) Tangent line: $y = 3$, Normal line: $x = 0$

5) Tangent line:

$$\text{Point-Slope Form: } y - 2 = 3 \left(x - \left(-\frac{3\pi}{4} \right) \right),$$

$$\text{Slope-Intercept Form: } y = 3x + \frac{9\pi + 8}{4};$$

Normal line:

$$\text{Point-Slope Form: } y - 2 = -\frac{1}{3} \left(x - \left(-\frac{3\pi}{4} \right) \right),$$

$$\text{Slope-Intercept Form: } y = -\frac{1}{3}x + \frac{8 - \pi}{4}$$

$$6) \text{ Most efficiently: } \left\{ x \in \mathbb{R} \mid x = \frac{\pi}{2} + \frac{2\pi}{3}n, \text{ or } x = -\frac{\pi}{2} + 2\pi n \quad (n \in \mathbb{Z}) \right\}.$$

$$\text{Equivalently, } \left\{ x \in \mathbb{R} \mid x = \frac{\pi}{2} + \pi n, \text{ or } x = -\frac{\pi}{6} + 2\pi n, \text{ or } x = \frac{7\pi}{6} + 2\pi n \quad (n \in \mathbb{Z}) \right\}.$$

Hint 1: Use a Double-Angle ID.

Hint 2: $\sin^2 x = (\sin x)(\sin x)$.

$$7) \text{ Hint: } D_x(\cot x) = D_x \left(\frac{\cos x}{\sin x} \right)$$

$$8) \text{ Hint: } D_x(\csc x) = D_x \left(\frac{1}{\sin x} \right)$$

SECTION 3.5: DIFFERENTIALS and LINEARIZATION OF FUNCTIONS

$$1) 1.9975$$

$$2) -161.56$$

$$3) \frac{2\sqrt{3}}{3} + \frac{\pi}{180} = \frac{120\sqrt{3} + \pi}{180} \approx 1.172$$

SECTION 3.6: CHAIN RULE

1) a) $3(x^2 - 3x + 8)^2(2x - 3)$, or $3(2x - 3)(x^2 - 3x + 8)^2$

b) $-\frac{10m}{(m^2 + 4)^6}$

c) $6\left(x^2 - \frac{1}{x^2}\right)^5\left(2x + \frac{2}{x^3}\right)$, or $12\left(x + \frac{1}{x^3}\right)\left(x^2 - \frac{1}{x^2}\right)^5$, or $12\left(\frac{x^4 + 1}{x^3}\right)\left(\frac{x^4 - 1}{x^2}\right)^5$, or

$$\frac{12(x^4 + 1)(x^4 - 1)^5}{x^{13}}$$

d) $18[\tan^2(6t)][\sec^2(6t)]$

e) $3x^2 \sin^2(2x) + 4x^3 [\sin(2x)][\cos(2x)]$, or
 $[\sin^2(2x)][3\sin(2x) + 4x\cos(2x)]$, or $3x^2 \sin^2(2x) + 2x^3 \sin(4x)$, or
 $x^2 [3\sin^2(2x) + 2x\sin(4x)]$

f) $\frac{8t^2}{(8t^3 + 27)^{2/3}}$, or $\frac{8t^2}{\sqrt[3]{(8t^3 + 27)^2}}$, or $\frac{8t^2(\sqrt[3]{8t^3 + 27})}{8t^3 + 27}$

g) $7[\pi - 6\theta^5 + \csc(5\theta)]^6(-30\theta^4 - 5[\csc(5\theta)][\cot(5\theta)])$, or
 $-35[\pi - 6\theta^5 + \csc(5\theta)]^6(6\theta^4 + [\csc(5\theta)][\cot(5\theta)])$

h) $2[\sec(2w)][\tan(2w)] + 2\sec^2(2w)$, or $[2\sec(2w)][\tan(2w) + \sec(2w)]$

i) Same as h).

j) $-\frac{\sin(\sqrt{\phi})}{2\sqrt{\phi}} - \frac{\sin\phi}{2\sqrt{\cos\phi}}$, or $-\frac{1}{2} \left[\frac{\sin(\sqrt{\phi})}{\sqrt{\phi}} + (\sin\phi)\sqrt{\sec\phi} \right]$, or
 $-\frac{\sqrt{\phi}}{2} \left[\frac{\sin(\sqrt{\phi}) + (\sin\phi)\sqrt{\phi\sec\phi}}{\phi} \right]$

k) $18(6x - 7)^2(8x^2 + 9)^4 + 64x(6x - 7)^3(8x^2 + 9)^3$, which factors and simplifies as $2(264x^2 - 224x + 81)(6x - 7)^2(8x^2 + 9)^3$. By the Test for Factorability from Section 0.7 in the Precalculus notes, the discriminant of $(264x^2 - 224x + 81)$ is not a perfect square, so it cannot be factored further over the integers.

l) $\frac{x(x^2 - 3)(3x^2 + 23)}{(x^2 + 5)^{3/2}}$, or $\frac{x(x^2 - 3)(3x^2 + 23)}{\sqrt{(x^2 + 5)^3}}$, or $\frac{x(x^2 - 3)(3x^2 + 23)}{(x^2 + 5)\sqrt{x^2 + 5}}$, or

$$\frac{x(x^2 - 3)(3x^2 + 23)\sqrt{x^2 + 5}}{(x^2 + 5)^2}$$

2) $-\frac{6}{(3x + 1)^3}$

3) $3[f(x)]^2 f'(x)$, just like in Section 3.3, Exercise 4.

4) Most efficiently: $\left\{ x \in \mathbb{R} \mid x = \frac{\pi}{2} + \frac{2\pi}{3}n, \text{ or } x = -\frac{\pi}{2} + 2\pi n \quad (n \in \mathbb{Z}) \right\}$.

Equivalently, $\left\{ x \in \mathbb{R} \mid x = \frac{\pi}{2} + \pi n, \text{ or } x = -\frac{\pi}{6} + 2\pi n, \text{ or } x = \frac{7\pi}{6} + 2\pi n \quad (n \in \mathbb{Z}) \right\}$,

just like in Section 3.4, Exercise 6.

5) 42

6) a) $D_x \left[(x^3)^5 \right] = D_x [x^{15}] = 15x^{14}$

b) $D_x \left[(x^3)^5 \right] = 5(x^3)^4 (3x^2) = 15x^{14}$

c) $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = (5u^4)(3x^2) = 15x^2u^4 = 15x^2(x^3)^4 = 15x^{14}$

7) a) $D_x \left(\frac{1}{x^2 + 1} \right) = -\frac{D_x(x^2 + 1)}{(x^2 + 1)^2} = -\frac{2x}{(x^2 + 1)^2}$

b) $D_x \left(\frac{1}{x^2 + 1} \right) = D_x \left[(x^2 + 1)^{-1} \right] = -(x^2 + 1)^{-2}(2x) = -\frac{2x}{(x^2 + 1)^2}$

c) $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = \left(-\frac{1}{u^2} \right)(2x) = -\frac{2x}{u^2} = -\frac{2x}{(x^2 + 1)^2}$

8) Hints: How are slopes of perpendicular lines (or line segments) related?

$D_x \left(\sqrt{a^2 - x^2} \right) = -\frac{x}{\sqrt{a^2 - x^2}}$. Horizontal and vertical tangent lines correspond to special cases.

9) Hint: You will need the Cofunction Identities again: $\sec \left(\frac{\pi}{2} - x \right) = \csc x$ and

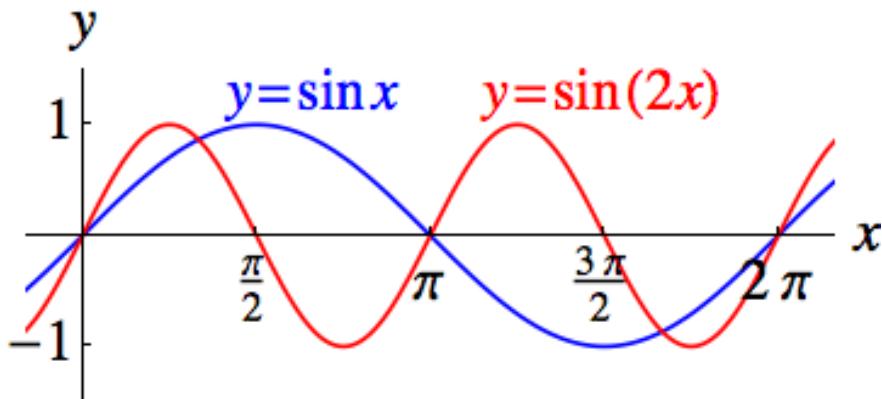
$$\tan \left(\frac{\pi}{2} - x \right) = \cot x .$$

10) a) $2 \cos(2x)$

b) $D_x [\sin(2x)] = D_x [2(\sin x)(\cos x)] = \dots = 2(\cos^2 x - \sin^2 x) = 2 \cos(2x)$

c) The range of $D_x [\sin(2x)]$ is $[-2, 2]$. The range of $D_x (\sin x)$ is $[-1, 1]$.

This tells us, among other things, that the steepest tangent lines to the graph of $y = \sin(2x)$ are twice as steep as the steepest tangent lines to the graph of $y = \sin x$. More incisively, the slope of the tangent line to the graph of $y = \sin(2x)$ at $x = a$ is twice the slope of the tangent line to the graph of $y = \sin x$ at $x = 2a$, where a is any real value.



SECTION 3.7: IMPLICIT DIFFERENTIATION

1) a) $\frac{dy}{dx} = -\frac{y^2}{2xy+1}$

b) $\left(\frac{1}{2}\right)(2)^2 + (2) = 4$

c) $-\frac{4}{3}$

d) Point-Slope Form: $y - 2 = -\frac{4}{3}\left(x - \frac{1}{2}\right)$, Slope-Intercept Form: $y = -\frac{4}{3}x + \frac{8}{3}$

2) a) $\frac{dy}{dx} = \frac{2y - 10x - 9x^2y^4}{2(6x^3y^3 - x - 4y)}$

b) $5(2)^2 - 2(2)(-1) + 3(2)^3(-1)^4 - 4(-1)^2 = 44$

c) $\frac{29}{46}$

d) Point-Slope Form: $y - (-1) = \frac{29}{46}(x - 2)$, Slope-Intercept Form: $y = \frac{29}{46}x - \frac{52}{23}$

3) a) $\frac{dy}{dx} = \frac{2\sqrt{y} \cos y}{2x\sqrt{y} \sin y - \cos(\sqrt{y})}$

b) $\sin(\sqrt{0}) + 3\cos 0 = 3$

c) 0

- 4) Hints: Consider the equation $x^2 + y^2 = a^2$, where $a > 0$. How are slopes of perpendicular lines (or line segments) related? Horizontal and vertical tangent lines correspond to special cases.

SECTION 3.8: RELATED RATES

1) $-\frac{24}{17}$

2) The radius is increasing at about $0.07458 \frac{\text{cm}}{\text{sec}}$. Note: $\frac{1.754}{4\pi(1.368)^2} \approx 0.07458$.

3) The top of the ladder is sliding down at about $5.728 \frac{\text{ft}}{\text{min}}$. Exact: $\frac{5\sqrt{21}}{4} \frac{\text{ft}}{\text{min}}$.

4) The distance is increasing at about $3.59 \frac{\text{in}}{\text{min}}$. Exact: $\frac{98\sqrt{745}}{745} \frac{\text{in}}{\text{min}}$.

5) The volume is decreasing at $4 \frac{\text{in}^3}{\text{min}}$.

6) The base radius is shrinking at $\frac{1}{4000\pi} \frac{\text{cm}}{\text{hr}}$.

7) The total resistance is increasing at about $0.007551 \frac{\text{ohm}}{\text{sec}}$. Exact: $\frac{37}{4900} \frac{\text{ohm}}{\text{sec}}$.

Hint: From the given equation, $R = \frac{12}{7}$ ohms at that moment.

8) The plane's speed is about $2303.8 \frac{\text{ft}}{\text{sec}}$, or about 1570.8 mph.

Exact: $\frac{2200\pi}{3} \frac{\text{ft}}{\text{sec}}$, or 500π mph.