

CHAPTER 5: INTEGRALS

SECTION 5.1: ANTIDERIVATIVES and INDEFINITE INTEGRALS

1)

a) $\frac{x^4}{2} - \frac{4x^{7/4}}{35} - \frac{x^6}{12} + \frac{4\sqrt{x}}{3} - 3x + C$, or $\frac{1}{2}x^4 - \frac{4}{35}x(\sqrt[4]{x^3}) - \frac{1}{12}x^6 + \frac{4}{3}\sqrt{x} - 3x + C$.

(Note: Many books don't even mention that we require $x > 0$.)

b) $\frac{2y^{5/2}}{5} + C$, or $\frac{2}{5}y^2\sqrt{y} + C$ (The restriction $y \geq 0$ is evident in the answer.)

c) $t^3 - t^2 + 2\sqrt{t} + C$. (Note: We technically require: $t > 0$.)

d) $\frac{w^3}{3} + \frac{7w^2}{2} + 12w + C$, or $\frac{1}{3}w^3 + \frac{7}{2}w^2 + 12w + C$, or $\frac{2w^3 + 21w^2 + 72w}{6} + C$

e) $\frac{81z^5}{5} + C$, or $\frac{81}{5}z^5 + C$

f) $-\frac{1}{t} - \frac{2}{t^3} - \frac{9}{5t^5} + C$, or $C - \frac{1}{t} - \frac{2}{t^3} - \frac{9}{5t^5}$, or $C - \frac{5t^4 + 10t^2 + 9}{5t^5}$

g) $\frac{x^3}{3} - x^2 + 4x + C$. (We require: $x \neq -2$.) Hint: Factor the numerator.

h) $-3\cos x + 5\sin x + C$, or $C - 3\cos x + 5\sin x$

i) $4\tan\theta + C$

j) $-\cot t + C$, or $C - \cot t$. (Note: We technically require: $\cos t \neq 0$.)

The restriction $\sin t \neq 0$ is essentially evident in the answer.)

k) $\sec r + C$.

l) $-\csc\theta + C$, or $C - \csc\theta$. Hint: Use a Pythagorean Identity.

m) $\pi^2 x + C$

n) C

o) $a^{10}t + \frac{abt^2}{2} + C$, or $\frac{2a^{10}t + abt^2}{2} + C$, or $\frac{at(2a^9 + bt)}{2} + C$

p) $\tan^5(x^4) + C$

2) $\sqrt{x^5 + x}$

3) a) $f(x) = 2x^3 + x^2 - x + 40$; b) $y = \frac{4}{3}x^{3/2} - 2$, or $y = \frac{4x\sqrt{x} - 6}{3}$

c) $f(x) = \frac{1}{2}x^3 + x^2 - 3x + \frac{7}{2}$, or $f(x) = \frac{x^3 + 2x^2 - 6x + 7}{2}$

d) $y = -7\sin x - 2\cos x + 11x + 12$

- 4) $s(t) = t^2 - t^3 - 5t + 4$
- 5) a) $s(t) = -16t^2 + 1600t$; b) $s(50) = 40,000$ feet
- 6) a) $s(t) = -16t^2 - 16t + 96$; b) 2 seconds; c) -80 feet per second
- 7) The reserves will be depleted in about 67.94 years.

SECTION 5.2: u SUBSTITUTIONS

1)

- a) $\frac{3(x^2 + 5)^4}{8} + C$, or $\frac{3}{8}(x^2 + 5)^4 + C$
- b) $\frac{(4p-5)^{3/2}}{6} + C$, or $\frac{1}{6}(4p-5)\sqrt{4p-5} + C$
- c) $\frac{\cos(\pi - 2\alpha^5)}{10} + C$, or $\frac{1}{10}\cos(\pi - 2\alpha^5) + C$, or $C - \frac{\cos(2\alpha^5)}{10}$ (the Unit Circle and the Difference Identities give us: $\cos(\pi - \theta) = -\cos\theta$)
- d) $\sqrt{r^4 + r^2} + C$, or $|r|\sqrt{r^2 + 1} + C$
- e) $\frac{1}{4}\sec^4 x + C$
- f) $C - \frac{1}{3}\cos(3\theta) + \frac{1}{2}\theta^2$, or $\frac{3\theta^2 - 2\cos(3\theta)}{6} + C$
- g) $C - \cot^2(\sqrt{x})$, or $C - \csc^2(\sqrt{x})$ (by Pythagorean IDs)
- h) $C - \frac{1}{x+1}$
- i) $C - \frac{1}{3(2+3\sec\beta)}$
- j) $\frac{2}{3}(x+1)^{3/2} - 2\sqrt{x+1} + C$, or $\frac{2\sqrt{x+1}(x-2)}{3}$. Hint: $u = x+1 \Rightarrow x = u-1$.
- k) $\frac{1}{2}x - \frac{1}{4}\sin(2x) + C$, or $\frac{2x - \sin(2x)}{4} + C$. Use a Power-Reducing Identity (PRI).
- 2) a) $\frac{1}{4}\sin(4x) + C$; b) $C - \frac{1}{3}\cos(3\theta)$; c) $2\tan\left(\frac{\theta}{2}\right) + C$; d) $\frac{1}{2}\sec(2\alpha) + C$;
- e) $C - \frac{1}{7}\csc(7\alpha)$

(Answers to Exercises for Chapter 5: Integrals) A.5.3

3) a) $\frac{2}{3}x^{3/2} - 2x + 2\sqrt{x} + C$; b) $\frac{2}{3}(\sqrt{x} - 1)^3 + C$

c) Hint: Expand $(\sqrt{x} - 1)^3$ from b), possibly by using the Binomial Theorem.

Remember that our answers to a) and b) can differ by a constant term.

4) $C - \frac{2\left(2 + \frac{1}{\sqrt{z}}\right)^5}{5}$, though a “simplified” version may be: $C - \frac{2\sqrt{z}(2\sqrt{z} + 1)^5}{5z^3}$.

5) $\frac{2}{7}(u-2)^{7/2} + \frac{8}{5}(u-2)^{5/2} + \frac{10}{3}(u-2)^{3/2} + C$, or $\frac{2}{105}(u-2)^{3/2}(15u^2 + 24u + 67)$.

6) $x + \frac{1}{x+1} + C$, or $\frac{x^2+x+1}{x+1} + C$, or $\frac{x^2+2x+2}{x+1} + C$, or $\frac{x^2}{x+1} + C$.

Hint: $\int \frac{x^2+2x}{x^2+2x+1} dx = \int \frac{x^2+2x+1-1}{x^2+2x+1} dx = \int \left[1 - \frac{1}{(x+1)^2}\right] dx$.

SECTIONS 5.3/5.4: AREA and DEFINITE INTEGRALS

(Observe that the given integrands are continuous on the intervals of interest.)

1) a) $\frac{14}{3}$; b) $-\frac{14}{3}$

2) a) 36; b) 25; c) 2.5; d) $\frac{9\pi}{4}$; e) $12 + 2\pi$

3) a) 30; b) 42; c) 36; d) 36

4) $\frac{49}{4}$, or 12.25.

SECTION 5.5: PROPERTIES OF DEFINITE INTEGRALS

(Observe that the given integrands are continuous on the intervals of interest.)

1) a) 78; b) $\frac{291}{2}$ or 145.5

2) True. $1 + \sin x \geq 0$ for all real x , particularly on the interval $[0, 4\pi]$.

3) True. $-\cos^2 x \leq 0$ for all real x , particularly on the interval $[3\pi, 5\pi]$.

4) $\int_{-3}^1 f(x) dx$

5) $\int_h^{c+h} g(t) dt$

6) a) 2; b) -1 (1 is technically not in the interval $(-2, 1)$.)

7) a) 6; b) 3

SECTION 5.6: FUNDAMENTAL THEOREM OF CALCULUS (FTC)

(Observe that the given integrands are continuous on the intervals of interest.)

1) a) 6; b) 5; c) $\frac{86}{3}$, or $28\frac{2}{3}$

d) $\frac{32}{5}$, or $6\frac{2}{5}$, or 6.4. The integrand is even: $\int_{-8}^8 \left(\sqrt[3]{x^2} - 2\right) dx = 2 \int_0^8 \left(\sqrt[3]{x^2} - 2\right) dx$.

e) -8; f) 0; g) 10; h) $\sqrt{2}$

i) $\frac{17}{6}$. Hint: $\int_0^2 f(x) dx = \int_0^1 f(x) dx + \int_1^2 f(x) dx = \frac{1}{2} + \frac{7}{3}$.

Observe that f is continuous on $[0, 2]$. In particular, it is continuous on $[0, 1]$ and on $[1, 2]$.

2) a) $\frac{204,085}{4}$, or $51,021\frac{1}{4}$, or 51,021.25

b) $\frac{14}{3}$, or $4\frac{2}{3}$; c) $\frac{1}{3}$; d) $\frac{5}{36}$; e) $\frac{3}{2}(\sqrt{3} - 1) \approx 1.10$; f) $1 - \sqrt{2} \approx -0.41$; g) 1

3) a) and b) $\frac{727}{3}$ or $242\frac{1}{3}$. On b), we have: $\int_{-2}^{-1} (8 - 5x)^2 dx = -\frac{1}{5} \int_{18}^{13} u^2 du$.

4) $\int_0^3 |x - 1| dx = \int_0^1 [-(x - 1)] dx + \int_1^3 (x - 1) dx = \frac{5}{2}$ or 2.5, the same as in Sections 5.3/5.4, Exercise 2c.

5) 0. The integrand is odd and continuous on $[-\pi, \pi]$, which is symmetric about 0.

6) a) $\frac{1}{2}$; b) $\sqrt{3}$ ($-\sqrt{3}$ is not in the interval $(0, 4)$.)

7) 0

8) $\sin^{10} x$

SECTION 5.7: NUMERICAL APPROXIMATION OF DEFINITE INTEGRALS

(Observe that the given integrands are continuous on the intervals of interest.)

1) a) 0.4728; b) 0.4637. Note: In fact, $\int_1^3 \frac{1}{1+x^2} dx \approx 0.4636$.

2) a) 2.2386; b) 2.3351. Note: In fact, $\int_0^\pi \sqrt{\sin x} dx \approx 2.3963$.