

CHAPTER 7: LOGARITHMIC and EXPONENTIAL FUNCTIONS

SECTION 7.1: INVERSE FUNCTIONS

1) a) 3; b) $f^{-1}(x) = \frac{x-4}{3}$, or $\frac{1}{3}x - \frac{4}{3}$; c) $\frac{1}{3}$, which is the reciprocal of 3

2) a) 12; b) $g(x) = \sqrt[3]{x}$, or $x^{1/3}$; c) $\frac{1}{12}$, which is the reciprocal of 12

SECTION 7.2: $\ln x$

1)

a) $\frac{15x^2 - 1}{5x^3 - x + 1}$

b) $\frac{3x+2}{x^2+x}$, or $\frac{3x+2}{x(x+1)}$. (Remember to simplify!)

c) $\frac{3}{3x+7}$

d) $\frac{-40}{7-4t}$, or $\frac{40}{4t-7}$. Hint: Use the Power Rule of Logarithms first.

e) $\frac{3+3(\ln x)^2}{x}$, or $\frac{3\left[1+(\ln x)^2\right]}{x}$.

Hint: Use the Power Rule of Logarithms on the first term.

f) $-2w$. Hint: $\ln\left(\frac{1}{w}\right) = \ln(w^{-1}) = -\ln w$.

g)
$$\frac{(1+\ln w)(2w\ln w+w)-(w^2\ln w)\left(\frac{1}{w}\right)}{(1+\ln w)^2} = \frac{w\left[1+2\ln w+2(\ln w)^2\right]}{(1+\ln w)^2}$$

2) $\frac{12x^3}{x^4+1} + \frac{1}{2x} - \frac{15}{3x-4}$, or $\frac{12x^3}{x^4+1} + \frac{1}{2x} + \frac{15}{4-3x}$, or $\frac{45x^5 - 100x^4 - 27x - 4}{2x(x^4+1)(3x-4)}$.

Hint: $\ln\left[\frac{(x^4+1)^3(\sqrt{x})}{(3x-4)^5}\right] = 3\ln(x^4+1) + \frac{1}{2}\ln x - 5\ln(3x-4)$ by laws of logarithms.

(Answers to Exercises for Chapter 7: Logarithmic and Exponential Functions) A.7.2

3) $\tan x$. We then (finally) have: $\int \tan x \, dx = \ln |\sec x| + C$.

4) a) and b) $-7(\sin \theta)\cos^6 \theta$. Note: You may have obtained $-7(\tan \theta)\cos^7 \theta$ for b).

This is equivalent to $-7(\sin \theta)\cos^6 \theta$, where $\cos \theta \neq 0$. Logarithmic Differentiation does not apply for values of θ that make $\cos \theta = 0$ in this problem.

5)

$$\text{a) } \frac{3(51x^2 + 80x + 2)(3x^2 + 2)^3}{2\sqrt{3x+5}}; \text{ b) } \left[\frac{24x}{3x^2 + 2} + \frac{3}{2(3x+5)} \right] (3x^2 + 2)^4 (\sqrt{3x+5});$$

c) Your answer should effectively be the same as your answer to part a).

6)

a) $(1, \infty)$. Hint: We require $x > 0$ and $\ln x > 0$.

b) $\frac{1}{x \ln x}$

c) $-\frac{1 + \ln x}{x^2 (\ln x)^2}$, or $-\frac{1 + \ln x}{(x \ln x)^2}$

d) On $\text{Dom}(f) = (1, \infty)$, $f'(x) > 0$ and $f''(x) < 0$. Therefore, f is increasing and the graph of f is concave down on the x -interval $(1, \infty)$.

e) Point-Slope Form: $y - \ln 2 = \frac{1}{2e^2}(x - e^2)$,

Slope-Intercept Form: $y = \frac{1}{2e^2}x + \left(\ln 2 - \frac{1}{2}\right)$, or $y = \frac{1}{2e^2}x + \frac{2\ln 2 - 1}{2}$

7) • $D_x(x) = 1$, and $D_x(\ln x) = \frac{1}{x} < 1$ whenever $x > 1$.

• Note that $1 > 0$, and also $\frac{1}{x} > 0$ whenever $x > 1$; therefore, x and $\ln x$ are increasing with respect to x on the interval $(1, \infty)$.

• Alternately, because $D_x(x - \ln x) = 1 - \frac{1}{x} > 0$ whenever $x > 1$, we can conclude that the “gap” $x - \ln x$ is increasing on the interval $(1, \infty)$, and therefore x is increasing faster than $\ln x$ is.

8) Hints: Let $y = x^n$. Apply Implicit Differentiation to both sides of $\ln y = \ln(x^n)$.

SECTION 7.3: e^x

1)

a) $-8e^{-8x}$, or $-\frac{8}{e^{8x}}$

b) $\frac{1+6e^{4x}}{\sqrt{1+2x+3e^{4x}}}$, or $\frac{6e^{4x}+1}{\sqrt{3e^{4x}+2x+1}}$

c) $(12x^2+1)e^{4x^3+x}$

d) e^{e^t+t} . Hint: $e^t e^{e^t}$ simplifies to this.

e) $\frac{xe^x(2e^x+x+2)}{(e^x+1)^2}$. Hint: $\frac{(e^x+1)(2xe^x+x^2e^x)-(x^2e^x)(e^x)}{(e^x+1)^2}$ simplifies to this.

f) $-\frac{e^{\frac{1}{x}}}{x^2}-e^{-x}$, or $-\frac{e^{\frac{1}{x}}}{x^2}-\frac{1}{e^x}$, or $-\frac{e^{x+\frac{1}{x}}+x^2}{x^2e^x}$, or $-\frac{e^{\frac{x^2+1}{x}}+x^2}{x^2e^x}$

g) $e^{x \ln x}(\ln x + 1)$, or $x^x(\ln x + 1)$

h) $\cos \theta - \sin \theta$

i) $e^x [\sec(e^x)] \tan(e^x)$

j) $120e^{6r} [\sec^2(4e^{6r})] \tan^4(4e^{6r})$

k) $-6[\csc(2x)][\cot(2x)]e^{3\csc(2x)+1}$

l) $-e^{-\theta} \cot(e^{-\theta})$, or $-\frac{\cot\left(\frac{1}{e^\theta}\right)}{e^\theta}$

m) $4e^{4x} \cot(\sqrt{x}) - \frac{e^{4x} \csc^2(\sqrt{x})}{2\sqrt{x}}$, or $\frac{e^{4x} [8(\sqrt{x}) \cot(\sqrt{x}) - \csc^2(\sqrt{x})]}{2\sqrt{x}}$

n) 0

2)

a) $\frac{2y(x^2-1)}{x(12y^6+1)}$, or $\frac{2x^2y-2y}{12xy^6+x}$, or $\frac{2y(x+1)(x-1)}{x(12y^6+1)}$,

b) $\ln[(e^3)^2 e] + 2e^6 - (e^3)^2 = 7 + e^6$; c) $\frac{2(e^6-1)}{e^2(1+12e^6)}$, or $\frac{2e^6-2}{e^2+12e^8}$

(Answers to Exercises for Chapter 7: Logarithmic and Exponential Functions) A.7.4

3) $-\frac{ye^{xy}}{xe^{xy} - (\sec y)\tan y}$, or $\frac{ye^{xy}}{(\sec y)\tan y - xe^{xy}}$, or $\frac{y}{\tan y - x}$ (because $\sec y = e^{xy}$; this is more easily seen if we had taken the natural logarithm ("ln") of both sides)

4) Hint: The rate of change of f with respect to t is given by $f'(t)$. The rate of decay is given by $-f'(t)$. Show that $-f'(t)$ is equal to a positive real constant times $f(t)$. In particular, $-f'(t) = b(ae^{-bt}) = b \cdot f(t)$.

5)

$$\text{Dom}(f) = (-\infty, \infty).$$

$f(x) > 0$ for all real values of x . Observe that $e^{-\frac{x^2}{2}} = \frac{1}{e^{\frac{x^2}{2}}} > 0$ for all real x .

f is even, so its graph is symmetric about the y -axis.

HA: only $y = 0$, because $\lim_{x \rightarrow \infty} f(x) = 0$, and $\lim_{x \rightarrow -\infty} f(x) = 0$.

$f'(x) = -\frac{1}{\sqrt{2\pi}} xe^{-\frac{x^2}{2}}$. Observe that $e^{-\frac{x^2}{2}} > 0$ for all real values of x .

CN: 0. Point at critical number: $\left(0, \frac{1}{\sqrt{2\pi}}\right)$, a local maximum point.

f is increasing on $(-\infty, 0]$.

f is decreasing on $[0, \infty)$.

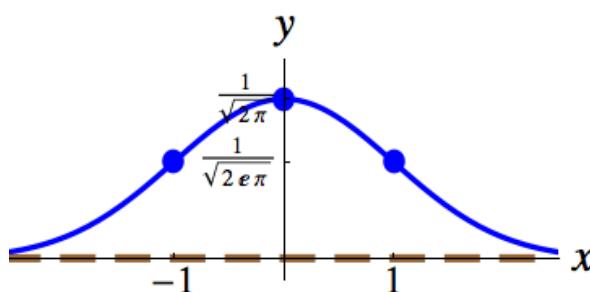
$f''(x) = \frac{1}{\sqrt{2\pi}} (x^2 - 1) e^{-\frac{x^2}{2}}$. Observe that $e^{-\frac{x^2}{2}} > 0$ for all real values of x .

PINs: -1 and 1.

Concave up on $(-\infty, -1] \cup [1, \infty)$.

Concave down on $[-1, 1]$.

Both PINs correspond to IPs: $\left(-1, \frac{1}{\sqrt{2e\pi}}\right)$ and $\left(1, \frac{1}{\sqrt{2e\pi}}\right)$.



SECTION 7.4: INTEGRATION and LOG / EXP. FUNCTIONS

1)

- a) $\frac{1}{2} \ln|2x - 3| + C$, or $\ln\sqrt{|2x - 3|} + C$. (The Power Rule for Logarithms can be used to reexpress some of the other expressions in these answers.)
- b) $\frac{1}{7}e^{7x} - \frac{1}{7}e^{-7x} + C$, or $\frac{1}{7}(e^{7x} - e^{-7x}) + C$, or $\frac{1}{7}\left(e^{7x} - \frac{1}{e^{7x}}\right) + C$, or $\frac{e^{14x} - 1}{7e^{7x}} + C$
- c) $- \frac{1}{3} \ln|\cos(3x)| + C$, or $C - \frac{1}{3} \ln|\cos(3x)|$, or $\frac{1}{3} \ln|\sec(3x)| + C$
- d) $5 \ln\left|\sin\left(\frac{x}{5}\right)\right| + C$
- e) $\frac{5}{8} \ln(4x^2 + 3) + C$. Note: $\frac{5}{8} \ln|4x^2 + 3| + C = \frac{5}{8} \ln(4x^2 + 3) + C$, because $4x^2 + 3 > 0$ for all real values of x .
- f) $\frac{1}{15}e^{5x^3} + C$
- g) $3 \ln|\sec(\theta^2 + e) + \tan(\theta^2 + e)| + C$
- h) $\frac{1}{2} \ln(x^2 - 4x + 5) + C$. Note: $\frac{1}{2} \ln|x^2 - 4x + 5| + C = \frac{1}{2} \ln(x^2 - 4x + 5) + C$, because $x^2 - 4x + 5 = (x^2 - 4x + 4) + 1 = (x - 2)^2 + 1 > 0$, $\forall x \in \mathbb{R}$.
- i) $\frac{t^2}{2} + 6t + 9 \ln|t| + C$, or $\frac{t^2 + 12t + 18 \ln|t|}{2} + C$, or $\frac{t^2 + 12t + \ln(t^{18})}{2} + C$.
Hint: Expand the numerator by performing the indicated square.
- j) $-\cot x + 8 \ln|\csc x - \cot x| + 16x + C$, or $C - \cot x + 8 \ln|\csc x - \cot x| + 16x$, or $C - \cot x - 8 \ln|\csc x + \cot x| + 16x$
- k) $-\cos(\ln x) + C$, or $C - \cos(\ln x)$
- l) $\frac{2\pi e^{\sqrt{x}}}{7} + C$
- m) $\ln|x + \tan x| + C$

(Answers to Exercises for Chapter 7: Logarithmic and Exponential Functions) A.7.6

n) $e^x + 2x - e^{-x} + C$, or $e^x + 2x - \frac{1}{e^x} + C$, or $\frac{e^{2x} + 2xe^x - 1}{e^x} + C$.

Hint: Expand the numerator by performing the indicated square.

o) $-\frac{1}{e^x - e^{-x}} + C$, or $C - \frac{e^x}{e^{2x} - 1}$, or $\frac{e^x}{1 - e^{2x}} + C$

p) $\ln(e^x + 1) + C$. Note: $\ln|e^x + 1| + C = \ln(e^x + 1) + C$, because $e^x + 1 > 0$ for all real values of x .

q) $\frac{1}{3} \ln|\sec(3e^x - e) + \tan(3e^x - e)| + C$

r) $e^{\sin x} + C$. Hint: Use a Reciprocal Identity.

s) $\ln|\csc \theta - \cot \theta| + \cos \theta + C$, or $-\ln|\csc \theta + \cot \theta| + \cos \theta + C$.

Hint: Use a Pythagorean ID.

t) $\frac{1}{2} \ln|1 + \csc(e^{-2x})| + C$, or $\frac{1}{2} \ln\left|1 + \csc\left(\frac{1}{e^{2x}}\right)\right| + C$

2) a) 0.697, b) $\ln 2 \approx 0.693$

3) (Left to the reader.)

4) Hint: $-\ln|\csc x + \cot x| = \ln(|\csc x + \cot x|^{-1}) = \ln\left(\frac{1}{|\csc x + \cot x|} \cdot \frac{|\csc x - \cot x|}{|\csc x - \cot x|}\right)$.

5) $s(t) = 2e^{2t} - \frac{3}{2}e^{-2t} + \frac{7}{2}$ in feet, or $s(t) = \frac{4e^{4t} + 7e^{2t} - 3}{2e^{2t}}$ in feet.

6) $\frac{\pi(e-1)}{e} m^3$. This is about 1.986 m^3 .

Hint: Setup is: $\int_0^1 2\pi x e^{-x^2} dx$ from the Cylinder / Cylindrical Shell Method (6.3).

Note 1: Observe that $e^{-x^2} = \frac{1}{e^{x^2}} > 0$ for all real values of x .

Note 2: $\pi(1 - e^{-1}) = \frac{\pi(e-1)}{e}$.

7) $\frac{\pi}{2} \ln 2 m^3$. Observe that both $-\pi \ln\left(\frac{\sqrt{2}}{2}\right) m^3$ and $\pi \ln(\sqrt{2}) m^3$ are equivalent to

this. This is about 1.089 m^3 .

SECTION 7.5: BEYOND e – NONNATURAL BASES

1)

a) $ex^{e-1} + e^x + 2^x \ln 2 - 2^{3x^4+x} (\ln 2)(12x^3 + 1)$

b) $\frac{1}{x} + \frac{1}{x \ln 2}$, or $\frac{1}{x} + \frac{1}{(\ln 2)x}$, or $\frac{\ln 2 + 1}{x \ln 2}$

c) $\frac{4(7x^6 - 12x^2)}{(x^7 - 4x^3 + 2)\ln 10}$, or $\frac{4x^2(7x^4 - 12)}{(x^7 - 4x^3 + 2)\ln 10}$

d) $x^{\pi-1}\pi^{x+1} + x^\pi\pi^x \ln \pi$, or $x^{\pi-1}\pi^x(\pi + x \ln \pi)$.

e) $5 \cdot 3^{\sec(5t)} (\ln 3) [\sec(5t)] \tan(5t)$, or $(\ln 243) 3^{\sec(5t)} [\sec(5t)] \tan(5t)$

f) $\frac{1}{(\ln 6)r \ln r}$

g) $\left[2x \ln(x+3) + \frac{x^2}{x+3} \right] (x+3)^{x^2}$, which can simplify to

$x \left[2(x+3) \ln(x+3) + x \right] (x+3)^{x^2-1}$.

h) $\left[(\sec^2 x) \ln x + \frac{\tan x}{x} \right] x^{\tan x}$, which can simplify to

$\left[x (\sec^2 x) \ln x + \tan x \right] x^{\tan x-1}$.

2) (Your answer should be equivalent to the one for Exercise 1h.)

3)

a) $\frac{x^{e+1}}{e+1} + e^e x + e^x + \frac{2^x}{\ln 2} - \frac{9\pi^x}{\ln \pi} + C$

b) $\frac{7^{5x+3}}{5 \ln 7} + C$, or $\frac{7^{5x+3}}{\ln 7^5} + C = \frac{7^{5x+3}}{\ln 16,807} + C$

c) $(\ln 10) \ln |\log x| + C$, or $(\ln 10) \ln |\ln x| + C$. (These are equivalent by the Change-of-Base Formula and the Quotient Rule for Logarithms. Remember that C can “absorb” constant terms.)

4) $\frac{\ln 11 - \ln 2}{\ln 10}$, or $\frac{\ln \left(\frac{11}{2} \right)}{\ln 10}$, or $\log \left(\frac{11}{2} \right)$ by the Change-of-Base Formula.