

# **CHAPTER 8: INVERSE TRIGONOMETRIC and HYPERBOLIC FUNCTIONS**

## **SECTION 8.1: INVERSE TRIGONOMETRIC FUNCTIONS**

1)

a)  $\frac{\pi}{6}$

b)  $-\frac{\pi}{3}$

c) undefined

d)  $\frac{3\pi}{4}$

e)  $-\frac{\pi}{4}$

f)  $\frac{\pi}{5}$

g)  $\frac{5\pi}{6}$

2)

a)  $\frac{x}{\sqrt{25-x^2}}$

b)  $\frac{1}{\sqrt{x^2+1}}$

## **SECTION 8.2: CALCULUS and INVERSE TRIGONOMETRIC FUNCTIONS**

1)

a) 
$$\frac{1}{2\sqrt{x}(1+x)}$$

b) 
$$2x \arcsin(3x) + \frac{3x^2}{\sqrt{1-9x^2}}, \text{ or } \frac{2x\sqrt{1-9x^2} \arcsin(3x) + 3x^2}{\sqrt{1-9x^2}}$$

c) 
$$-\frac{1}{x\sqrt{1-(\ln x)^2}}$$

d) 
$$5\left[e^t + \operatorname{arcsec}(t^4)\right]^4 \left[e^t + \frac{4}{t(\sqrt{t^8-1})}\right], \text{ or } \frac{5\left[e^t + \operatorname{arcsec}(t^4)\right]^4 \left(te^t\sqrt{t^8-1} + 4\right)}{t(t^8-1)}$$

e) 
$$\frac{1}{(1+x^2)\sqrt{1-(\arctan x)^2}}$$

2) a) and b) 1.

In a), use inverse properties. Observe that  $\arcsin(\sin x) = x$  if  $-\frac{\pi}{2} < x < \frac{\pi}{2}$ .

Note: There are other real values of  $x$  for which  $\arcsin(\sin x) \neq x$ ;

for example, consider  $x = \frac{5\pi}{6}$ .

In b), observe that  $\sqrt{1-\sin^2 x} = \sqrt{\cos^2 x} = |\cos x| = \cos x$ , because  $-\frac{\pi}{2} < x < \frac{\pi}{2}$ .

(Answers to Exercises for Ch.8: Inverse Trigonometric and Hyperbolic Functions) A.8.3

- 3) No. The “ $-1$ ” superscript is an exponent (indicating reciprocal or multiplicative inverse) in  $(\sin x)^{-1}$ , but it indicates an inverse function in  $\sin^{-1} x$ .

$$D_x \left[ (\sin x)^{-1} \right] = D_x \left[ \frac{1}{\sin x} \right] = D_x [\csc x] = -\csc x \cot x.$$

This is not equivalent to  $D_x (\sin^{-1} x)$ , which is  $\frac{1}{\sqrt{1-x^2}}$ .

4)

a)  $5\arcsin t + C$ , or  $5\sin^{-1} t + C$

b)  $\arcsin\left(\frac{x}{4}\right) + C$ , or  $\sin^{-1}\left(\frac{x}{4}\right) + C$

c)  $-\sqrt{16-x^2} + C$ , or  $C - \sqrt{16-x^2}$ . Hint: Use a  $u$ -sub instead of the techniques of this chapter.

d)  $\frac{1}{5}\arctan\left(\frac{x}{5}\right) + C$ , or  $\frac{1}{5}\tan^{-1}\left(\frac{x}{5}\right) + C$

e)  $\frac{1}{2}\text{arcsec}\left(\frac{x}{2}\right) + C$ , or  $\frac{1}{2}\sec^{-1}\left(\frac{x}{2}\right) + C$

f)  $\arcsin(e^x) + C$ , or  $\sin^{-1}(e^x) + C$

g)  $\frac{3}{20}\arctan\left(\frac{x^2}{10}\right) + C$ , or  $\frac{3}{20}\tan^{-1}\left(\frac{x^2}{10}\right) + C$

h)  $\frac{(\arctan x)^2}{2} + C$ , or  $\frac{(\tan^{-1} x)^2}{2} + C$

i)  $\frac{1}{12}\text{arcsec}\left(\frac{x^4}{3}\right) + C$ , or  $\frac{1}{12}\sec^{-1}\left(\frac{x^4}{3}\right) + C$ .

Hint: If  $u = x^4$ , then  $du = 4x^3 dx$ . Multiply the numerator and the denominator by  $4x^3$ .

5)  $\frac{\pi}{12}$

## **SECTION 8.3: HYPERBOLIC FUNCTIONS**

1)  $\sinh(1) \approx 1.175, \cosh(1) \approx 1.543, \tanh(1) \approx 0.7616$

2) Prove that  $D_x \left( \frac{e^x - e^{-x}}{2} \right) = \frac{e^x + e^{-x}}{2}$ .

3) Prove that  $\left( \frac{e^x + e^{-x}}{2} \right)^2 - \left( \frac{e^x - e^{-x}}{2} \right)^2 = 1$ .

4)

a)  $3\cosh(3x)$

b)  $3\sinh(3x)$

c)  $4e^x \operatorname{sech}^2(e^x)$

d)  $\ln(\operatorname{sech} x) - x \tanh x$

e)  $6x - 2^{\operatorname{csch} x} (\ln 2) \operatorname{csch} x \coth x$

f)  $-4 \sec t \tan t \coth^3(\sec t) \operatorname{csch}^2(\sec t)$

g) 
$$\frac{(1+x^2)\arctan x \sinh x - \cosh x}{(1+x^2)(\arctan x)^2}. \text{ Hint: } \frac{\arctan x \sinh x - \frac{\cosh x}{1+x^2}}{(\arctan x)^2} \text{ simplifies to this.}$$

5)

a)  $\frac{1}{3}\sinh(3x) + C$

b)  $\frac{1}{3}\cosh(3x) + C$

c)  $\frac{7}{8}\tanh(4x^2 - 1) + C$

d)  $-2\operatorname{sech}(\sqrt{x}) + C$

e)  $-\frac{1}{2}\coth^2(e^t) + C, \text{ or } C - \frac{\coth^2(e^t)}{2}, \text{ or } C - \frac{\operatorname{csch}^2(e^t)}{2}$