CHAPTER 8: INVERSE TRIG and HYPERBOLIC FUNCTIONS

SECTION 8.1: INVERSE TRIG FUNCTIONS

1)

a) $\frac{\pi}{6}$

b) $-\frac{\pi}{3}$

c) undefined

d) $\frac{3\pi}{4}$

e) $-\frac{\pi}{4}$

f) $\frac{\pi}{5}$

g) $\frac{5\pi}{6}$

2)

a) $\frac{x}{\sqrt{25-x^2}}$, or $\frac{x\sqrt{25-x^2}}{25-x^2}$

b) $\frac{1}{\sqrt{x^2+1}}$, or $\frac{\sqrt{x^2+1}}{x^2+1}$

SECTION 8.2: CALCULUS and INVERSE TRIG FUNCTIONS

1)

a) $\dfrac{1}{2\sqrt{x}(1+x)}$, or $\dfrac{\sqrt{x}}{2x(1+x)}$

b) $2x\arcsin(3x)+\dfrac{3x^2}{\sqrt{1-9x^2}}$, or $\dfrac{\sqrt{1-9x^2}\left[2x\sqrt{1-9x^2}\arcsin(3x)+3x^2\right]}{1-9x^2}$

c) $-\dfrac{1}{x\sqrt{1-(\ln x)^2}}$

d) $5\left[e^t+\operatorname{arcsec}(t^4)\right]^4\left[e^t+\dfrac{4}{t\left(\sqrt{t^8-1}\right)}\right]$, or $\dfrac{5\left[e^t+\operatorname{arcsec}(t^4)\right]^4\left(te^t\sqrt{t^8-1}+4\right)}{t\left(t^8-1\right)}$

e) $\dfrac{1}{(1+x^2)\sqrt{1-(\arctan x)^2}}$

2) a) and b) 1.

In a), use inverse properties. Observe that $\arcsin(\sin x)=x$ if $-\dfrac{\pi}{2}<x<\dfrac{\pi}{2}$.

Note: There are other real values of x for which $\arcsin(\sin x)\neq x$;

for example, consider $x=\dfrac{5\pi}{6}$.

In b), observe that $\sqrt{1-\sin^2 x}=\sqrt{\cos^2 x}=\left|\cos x\right|=\cos x$, because $-\dfrac{\pi}{2}<x<\dfrac{\pi}{2}$.

3) No. The "-1" superscript is an exponent (indicating reciprocal or multiplicative inverse) in $(\sin x)^{-1}$, but it indicates an inverse function in $\sin^{-1} x$.

$D_x\left[(\sin x)^{-1}\right] = D_x\left[\frac{1}{\sin x}\right] = D_x\left[\csc x\right] = -\csc x \cot x$.

This is not equivalent to $D_x\left(\sin^{-1} x\right)$, which is $\frac{1}{\sqrt{1-x^2}}$.

4)

a) $5\arcsin t + C$, or $5\sin^{-1} t + C$

b) $\arcsin\left(\frac{x}{4}\right) + C$, or $\sin^{-1}\left(\frac{x}{4}\right) + C$

c) $-\sqrt{16-x^2} + C$, or $C - \sqrt{16-x^2}$. Hint: Use a *u*-sub instead of the techniques of this chapter.

d) $\frac{1}{5}\arctan\left(\frac{x}{5}\right) + C$, or $\frac{1}{5}\tan^{-1}\left(\frac{x}{5}\right) + C$

e) $\frac{1}{2}\operatorname{arcsec}\left(\frac{x}{2}\right) + C$, or $\frac{1}{2}\sec^{-1}\left(\frac{x}{2}\right) + C$

f) $\arcsin\left(e^x\right) + C$, or $\sin^{-1}\left(e^x\right) + C$

g) $\frac{3}{20}\arctan\left(\frac{x^2}{10}\right) + C$, or $\frac{3}{20}\tan^{-1}\left(\frac{x^2}{10}\right) + C$

h) $\frac{(\arctan x)^2}{2} + C$, or $\frac{\left(\tan^{-1} x\right)^2}{2} + C$

i) $\frac{1}{12}\operatorname{arcsec}\left(\frac{x^4}{3}\right) + C$, or $\frac{1}{12}\sec^{-1}\left(\frac{x^4}{3}\right) + C$.

Hint: If $u = x^4$, then $du = 4x^3\,dx$. Multiply the numerator and the denominator by $4x^3$.

5) $\frac{\pi}{12}$

SECTION 8.3: HYPERBOLIC FUNCTIONS

1) $\sinh(1) \approx 1.175$, $\cosh(1) \approx 1.543$, $\tanh(1) \approx 0.7616$

2) Prove that $D_x\left(\dfrac{e^x - e^{-x}}{2}\right) = \dfrac{e^x + e^{-x}}{2}$.

3) Prove that $\left(\dfrac{e^x + e^{-x}}{2}\right)^2 - \left(\dfrac{e^x - e^{-x}}{2}\right)^2 = 1$.

4)

a) $3\cosh(3x)$

b) $3\sinh(3x)$

c) $4e^x \operatorname{sech}^2(e^x)$

d) $\ln(\operatorname{sech} x) - x\tanh x$

e) $6x - 2^{\operatorname{csch} x}(\ln 2)\operatorname{csch} x \coth x$

f) $-4\sec t \tan t \coth^3(\sec t)\operatorname{csch}^2(\sec t)$

g) $\dfrac{(1+x^2)\arctan x \sinh x - \cosh x}{(1+x^2)(\arctan x)^2}$. Hint: $\dfrac{\arctan x \sinh x - \dfrac{\cosh x}{1+x^2}}{(\arctan x)^2}$ simplifies to this.

5)

a) $\dfrac{1}{3}\sinh(3x) + C$

b) $\dfrac{1}{3}\cosh(3x) + C$

c) $\dfrac{7}{8}\tanh(4x^2 - 1) + C$

d) $-2\operatorname{sech}(\sqrt{x}) + C$

e) $-\dfrac{1}{2}\coth^2(e^t) + C$, or $C - \dfrac{\coth^2(e^t)}{2}$, or $C - \dfrac{\operatorname{csch}^2(e^t)}{2}$