SECTION 2.3:
BE CAREFUL WHEN USING DTS! IT'S BEST FOR "EASY" PROBLEMS!

You are expected to be able to find limits using rigorous algebra, not just through short cuts. It is easy to abuse the Dominant Term Substitution (DTS; this is my terminology, anyway) technique for finding limits of algebraic functions. You can use DTS if, at EVERY step in your solution, there is a clearly dominant term in the expression you're finding the limit of (or, if analyzing a fraction, if there is a clearly dominant term in the numerator and a clearly dominant term in the denominator).

For example, let's say we want $\lim_{x\to\infty}\left(x-\sqrt{x^2+x}\right)$. The answer is -1/2, not 0.

DTS can't be used here, because neither term (neither x nor $-\sqrt{x^2+x}$) is dominant; they are both "on the order of" x. A correct solution follows:

$$\lim_{x\to\infty}\left(x-\sqrt{x^2+x}\right)=\lim_{x\to\infty}\left(\frac{x-\sqrt{x^2+x}}{1}\cdot\frac{x+\sqrt{x^2+x}}{x+\sqrt{x^2+x}}\right)$$

$$=\lim_{x\to\infty}\frac{x^2-\left(x^2+x\right)}{x+\sqrt{x^2+x}}$$

$$=\lim_{x\to\infty}\frac{-x}{x+\sqrt{x^2+x}}$$

$$\left(\text{incorrectly applying DTS would give a '0' numerator}\right)$$

$$=\lim_{x\to\infty}\frac{\frac{-x}{x}}{\frac{x}{x}+\frac{\sqrt{x^2+x}}{x}}$$

$$=\lim_{x\to\infty}\frac{-1}{1+\sqrt{\frac{x^2+x}{x^2}}}\quad\left(\text{because }\sqrt{x^2}=\left|x\right|=x\text{ for }x>0\right)$$

$$=\lim_{x\to\infty}\frac{-1}{1+\sqrt{1+\frac{1}{x}}}$$

$$=-\frac{1}{2}$$