

CHAPTER 5: INTEGRALS

SECTION 5.1: ANTIDERIVATIVES / INDEFINITE INTEGRALS

- 1) Evaluate the following indefinite integrals. Assume that all integrands are defined [and continuous] “where we care.”

a) $\int \left(2x^3 - \frac{\sqrt[4]{x^3}}{5} - \frac{x^5}{2} + \frac{2}{3\sqrt{x}} - 3 \right) dx$

b) $\int y\sqrt{y} \, dy$

c) $\int \frac{3t^3 - 2t^2 + \sqrt{t}}{t} dt$

d) $\int (w+3)(w+4) dw$

e) $\int (3z)^4 dz$

f) $\int \frac{(t^2 + 3)^2}{t^6} dt$. Your final answer must not contain negative exponents.

g) $\int \frac{x^3 + 8}{x + 2} dx$

h) $\int (3\sin x + 5\cos x) dx$

i) $\int \frac{4}{\cos^2 \theta} d\theta$

j) $\int (\sec t)(\csc t)(\cot t) dt$

k) $\int \frac{\sin r}{\cos^2 r} dr$

l) $\int (\cot \theta)(1 + \cot^2 \theta)(\sin \theta) d\theta$

m) $\int \pi^2 dx$

n) $\int \sin^5 \pi \, dx$

o) $\int (a^{10} + abt) dt$, if a and b represent real constants

p) $\int D_x \left[\tan^5(x^4) \right] dx$

- 2) Evaluate $D_x \left(\int \sqrt{x^5 + x} \, dx \right)$.
- 3) For each part below, solve the differential equation subject to the given conditions.
- a) $f'(x) = 6x^2 + 2x - 1$ subject to: $f(-2) = 30$
 - b) $\frac{dy}{dx} = 2\sqrt{x}$ subject to: $y = 34$ if $x = 9$
 - c) $f''(x) = 3x + 2$ subject to: $f'(1) = \frac{1}{2}$ and $f(-1) = 7$
 - d) $\frac{d^2y}{dx^2} = 7\sin x + 2\cos x$ subject to: both ($y = 10$ and $y' = 4$) if $x = 0$
- 4) Assuming that a particle is moving on a coordinate line with acceleration $a(t) = 2 - 6t$ and with initial conditions $v(0) = -5$ and $s(0) = 4$, find the position function rule $s(t)$ for the particle. $s(t)$ is measured in feet, t in minutes, $v(t)$ in feet per minute, and $a(t)$ in feet per minute per minute.
- 5) On Earth, a projectile is fired vertically upward from ground level with a velocity of 1600 feet per second. Ignore air resistance. Use -32 feet per second per second as the Earth's [signed] gravitational constant of acceleration.
- a) Find the projectile's height $s(t)$ above ground t seconds after it is fired.
(The formula will be relevant until the time the projectile hits the ground.)
 - b) Find the projectile's maximum height.
- 6) A brick is thrown directly downward from a height of 96 feet with an initial speed of 16 feet per second. Use -32 feet per second per second as the Earth's [signed] gravitational constant of acceleration.
- a) Find the brick's height above ground t seconds after it is thrown.
(Your formula will be relevant up until the time at which the brick hits the ground.)
 - b) Find how long it takes for the brick to hit the ground (starting from the time the brick is thrown).
 - c) Find the velocity at which the brick hits the ground.
- 7) A country has natural gas reserves of 250 billion cubic feet. If the gas is consumed at the rate of $3 + 0.02t$ billion cubic feet per year (where $t = 0$ corresponds to now, and t is measured in years), how long will it be before the reserves are depleted? Round off your answer to four significant digits.

SECTION 5.2: u SUBSTITUTIONS

- 1) Evaluate the following indefinite integrals. In this section, assume that all integrands are defined [and continuous] “where we care.”

a) $\int 3x(x^2 + 5)^3 dx$

b) $\int \sqrt{4p - 5} dp$

c) $\int \alpha^4 \sin(\pi - 2\alpha^5) d\alpha$

d) $\int \frac{2r^3 + r}{\sqrt{r^4 + r^2}} dr$

e) $\int (\sin x)(\sec^5 x) dx$

f) $\int [\sin(3\theta) + \theta] d\theta$

g) $\int \frac{\sec^2(\sqrt{x})}{(\sqrt{x}) \tan^3(\sqrt{x})} dx$

h) $\int \frac{1}{x^2 + 2x + 1} dx$

i) $\int \frac{\sec \beta \tan \beta}{4 + 12 \sec \beta + 9 \sec^2 \beta} d\beta$

j) $\int \frac{x}{\sqrt{x+1}} dx$

k) $\int \sin^2 x dx$

- 2) Use “Guess-and-Check” to evaluate the following indefinite integrals.

a) $\int \cos(4x) dx$; b) $\int \sin(3\theta) d\theta$; c) $\int \sec^2\left(\frac{\theta}{2}\right) d\theta$; d) $\int \sec(2\alpha) \tan(2\alpha) d\alpha$;

e) $\int \csc(7\alpha) \cot(7\alpha) d\alpha$

- 3) We will evaluate $\int \frac{(\sqrt{x} - 1)^2}{\sqrt{x}} dx$ in two different ways.

a) Evaluate $\int \frac{(\sqrt{x} - 1)^2}{\sqrt{x}} dx$ **without** using a substitution.

b) Evaluate $\int \frac{(\sqrt{x} - 1)^2}{\sqrt{x}} dx$ by using a substitution.

- c) Show that your answers to a) and b) are equivalent. (This is mostly algebra.)

ADDITIONAL PROBLEMS (#4-6)

4) Evaluate $\int \left(2 + \frac{1}{\sqrt{z}}\right)^4 \cdot \frac{1}{z\sqrt{z}} dz$.

5) Evaluate $\int (u^2 + 1)\sqrt{u - 2} du$. Note: Do not use u as your substitution variable.

6) Evaluate $\int \frac{x^2 + 2x}{x^2 + 2x + 1} dx$.

SECTIONS 5.3/5.4: AREA and DEFINITE INTEGRALS

- 1) Assume that $\int_1^4 \sqrt{x} \, dx = \frac{14}{3}$. Evaluate the following integrals based on this information.

a) $\int_1^4 \sqrt{t} \, dt$

b) $\int_4^4 \sqrt{x} \, dx + \int_4^1 \sqrt{x} \, dx$

- 2) Evaluate the following definite integrals using areas.

a) $\int_{-1}^5 6 \, dx$

b) $\int_{-3}^2 (2x + 6) \, dx$

c) $\int_0^3 |x - 1| \, dx$. We will revisit this problem in Section 5.6.

d) $\int_0^3 \sqrt{9 - x^2} \, dx$

e) $\int_{-2}^2 \left(3 + \sqrt{4 - x^2} \right) dx$

- 3) We will approximate $\int_1^5 (2x + 3) \, dx$ using Riemann approximations and the partition P of the interval $[1, 5]$, where P is given by: $\{1, 3, 4, 5\}$.

- a) Use a Left-Hand Riemann Approximation.
- b) Use a Right-Hand Riemann Approximation.
- c) Use a Midpoint Riemann Approximation.
- d) Use an area argument to find the exact value of the integral.

- 4) Approximate $\int_0^6 \left(8 - \frac{1}{2}x^2 \right) dx$ by using a Midpoint Riemann Approximation and a regular partition of the interval $[0, 6]$ into six subintervals.

SECTION 5.5: PROPERTIES OF DEFINITE INTEGRALS

- 1) Assuming that $\int_1^4 x^2 dx = 21$ and $\int_1^4 x dx = \frac{15}{2}$, evaluate the following definite integrals.

a) $\int_1^4 (3x^2 + 5) dx$

b) $\int_4^1 (2 - 9t - 4t^2) dt$

- 2) True or False: $\int_0^{4\pi} (1 + \sin x) dx \geq 0$. Explain.

- 3) True or False: $\int_{3\pi}^{5\pi} (-\cos^2 x) dx \leq 0$. Explain.

- 4) Express as one integral: $\int_5^1 f(x) dx + \int_{-3}^5 f(x) dx$.

Assume that f is everywhere continuous on \mathbb{R} .

- 5) Express as one integral: $\int_c^{c+h} g(t) dt - \int_c^h g(t) dt$.

Assume that g is everywhere continuous on \mathbb{R} . c and h represent real constants.

- 6) Let $f(x) = x^2 + 1$. Assume that $\int_{-2}^1 f(x) dx = 6$; you will be able to verify this after you study Section 5.6.

- a) Find the average value of f on $[-2, 1]$.

- b) Find a number z that satisfies the conclusion of the Mean Value Theorem (MVT) for Integrals, where the interval of interest is $[-2, 1]$.

- 7) Let $g(x) = 3\sqrt{x+1}$. Assume that $\int_{-1}^8 g(x) dx = 54$; you will be able to verify this after you study Section 5.6.

- a) Find the average value of g on $[-1, 8]$.

- b) Find a number z that satisfies the conclusion of the Mean Value Theorem (MVT) for Integrals, where the interval of interest is $[-1, 8]$. (Consider g instead of f .)

SECTION 5.6: FUNDAMENTAL THEOREM OF CALCULUS (FTC)

1) Evaluate the following definite integrals.

a) $\int_{-2}^1 (x^2 + 1) dx$. We saw this in Section 5.5, Exercise 6. Yes, the answer is 6.

b) $\int_4^9 dt$

c) $\int_9^{16} \frac{r+2}{\sqrt{r}} dr$

d) $\int_{-8}^8 (\sqrt[3]{x^2} - 2) dx$. Hint: Do you see how you can reduce your work?

e) $\int_4^2 \frac{t^2 - 1}{t - 1} dt$

f) $\int_5^5 \sin^4(\theta^2) d\theta$.

Hint: If you're spending too much time on this, you're doing it wrong!

g) $\int_0^\pi 5 \sin x dx$

h) $\int_0^{\pi/4} (\sec t)(\sec t + \tan t) dt$

i) $\int_0^2 f(x) dx$, where $f(x) = \begin{cases} x, & \text{if } 0 \leq x < 1 \\ x^2, & \text{if } 1 \leq x \leq 2 \end{cases}$

2) Evaluate the following definite integrals by using u substitutions (or maybe "Guess-and-Check" if the u sub is linear).

a) $\int_{-2}^3 x^2 (x^3 + 1)^3 dx$

b) $\int_1^4 \sqrt{5-x} dx$

c) $\int_0^1 \frac{1}{(3-2x)^2} dx$

d) $\int_1^4 \frac{1}{\sqrt{x}(\sqrt{x}+1)^3} dx$

e) $\int_{\pi/2}^\pi \cos\left(\frac{\theta}{3}\right) d\theta$.

In e), give an exact answer and also give an approximate answer rounded off to three significant digits.

f) $\int_{\pi/4}^{\pi/3} [4 \sin(2x) + 6 \cos(3x)] dx$

In f), give an exact answer and also give an approximate answer rounded off to two significant digits.

g) $\int_0^{\pi/3} \frac{\sin x}{\cos^2 x} dx$

3) We will evaluate $\int_{-2}^{-1} (8 - 5x)^2 dx$ in two different ways.

a) First expand $(8 - 5x)^2$, and then integrate as usual.

b) Use a u substitution.

4) In Sections 5.3/5.4, Exercise 2c, you evaluated $\int_0^3 |x - 1| dx$ using areas. Now, use the definition of absolute value and evaluate this integral using the FTC.

5) Evaluate $\int_{-\pi}^{\pi} (x + \sin x) dx$ by finding an antiderivative and applying the FTC as usual. Why does your answer make sense?

6) Let $f(x) = \frac{x}{\sqrt{x^2 + 9}}$.

a) Find the average value of f on $[0, 4]$.

b) Find a number z that satisfies the conclusion of the Mean Value Theorem (MVT) for Integrals, where the interval of interest is $[0, 4]$.

7) Find $D_x \left(\int_{\pi/4}^{\pi} \sin^{10} x dx \right)$ without performing the integration.

8) Find $D_x \left(\int_{\pi/4}^x \sin^{10} t dt \right)$ without performing the integration.

SECTION 5.7: NUMERICAL APPROXIMATION OF DEFINITE INTEGRALS

- 1) We will find approximations for $\int_1^3 \frac{1}{1+x^2} dx$ by using a regular partition of the interval $[1, 3]$ into $n = 4$ subintervals. Round off intermediate results to five decimal places, and round off final answers to four decimal places.

a) Use the Trapezoidal Rule.

b) Use Simpson's Rule.

Note: The exact value of the integral is $\tan^{-1}(3) - \frac{\pi}{4} \approx 0.4636$. You will learn how to work out this integral exactly in Chapter 8.

- 2) We will find approximations for $\int_0^{\pi} \sqrt{\sin x} dx$ by using a regular partition of the interval $[0, \pi]$ into $n = 6$ subintervals. Round off intermediate results to five decimal places, and round off final answers to four decimal places.

a) Use the Trapezoidal Rule.

b) Use Simpson's Rule.

Note for Kuniyuki's class: If related questions are placed on exams, then you will be given a formula for the Trapezoidal Rule and/or Simpson's Rule, as appropriate.