CHAPTER 8: INVERSE TRIGONOMETRIC and HYPERBOLIC FUNCTIONS

SECTION 8.1: INVERSE TRIGONOMETRIC FUNCTIONS

1) Evaluate the following. If an expression is not defined as a real number, write "undefined."

   a) $\sin^{-1}\left(\frac{1}{2}\right)$, also known as $\arcsin\left(\frac{1}{2}\right)$

   b) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$, also known as $\arcsin\left(-\frac{\sqrt{3}}{2}\right)$

   c) $\sin^{-1}\left(\frac{\pi}{2}\right)$, also known as $\arcsin\left(\frac{\pi}{2}\right)$

   d) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$, also known as $\arccos\left(-\frac{\sqrt{2}}{2}\right)$

   e) $\tan^{-1}(-1)$, also known as $\arctan(-1)$

   f) $\sin^{-1}\left[\sin\left(\frac{\pi}{5}\right)\right]$, also known as $\arcsin\left[\sin\left(\frac{\pi}{5}\right)\right]$

   g) $\cos^{-1}\left[\cos\left(\frac{7\pi}{6}\right)\right]$, also known as $\arccos\left[\cos\left(\frac{7\pi}{6}\right)\right]$

2) Rewrite the following as algebraic expressions in x. Assume $x > 0$.

   a) $\tan\left[\sin^{-1}\left(\frac{x}{5}\right)\right]$, also known as $\tan\left[\arcsin\left(\frac{x}{5}\right)\right]$

   b) $\cos\left[\tan^{-1} x\right]$, also known as $\cos\left[\arctan x\right]$

KNOW THE GRAPHS, DOMAINS, AND RANGES OF THE THREE KEY INVERSE TRIGONOMETRIC FUNCTIONS!

SECTION 8.2: CALCULUS and INVERSE TRIGONOMETRIC FUNCTIONS

1) Find the following derivatives. Simplify where appropriate.
You do not have to simplify radicals or rationalize denominators.

   a) Let $f(x) = \arctan\left(\sqrt{x}\right)$, also written as $\tan^{-1}\left(\sqrt{x}\right)$. Find $f'(x)$.

   b) Find $\frac{d}{dx}\left[x^2 \arcsin(3x)\right]$, also written as $\frac{d}{dx}\left[x^2 \sin^{-1}(3x)\right]$.

   c) Find $D_x\left[\arccos(\ln x)\right]$, also written as $D_x\left[\cos^{-1}(\ln x)\right]$.

   d) Let $g(t) = \left[e^t + \operatorname{arcsec}\left(t^4\right)\right]^5$, also written as $g(t) = \left[e^t + \sec^{-1}\left(t^4\right)\right]^5$.
   Find $g'(t)$. Assume that the range of the arcsec function is
   given by $\left[0, \frac{\pi}{2}\right) \cup \left[\pi, \frac{3\pi}{2}\right)$, as we do in the notes. Bear in mind that the range is defined differently in other sources.

   e) Let $y = \arcsin(\arctan x)$, also written as $\sin^{-1}\left(\tan^{-1} x\right)$. Find $\frac{dy}{dx}$.

2) We will find $D_x\left[\arcsin(\sin x)\right]$, also written as $D_x\left[\sin^{-1}(\sin x)\right]$, in two different ways. Assume $-\frac{\pi}{2} < x < \frac{\pi}{2}$; the importance of this assumption is part of the point of this problem.

   a) Simplify first before finding the derivative.

   b) Do not simplify first before finding the derivative.

3) Yes or No: Is $D_x\left[(\sin x)^{-1}\right]$ equivalent to $D_x\left(\sin^{-1} x\right)$?

4) Evaluate the following indefinite integrals. You may use C, D, etc. as representing arbitrary constants.

• Assume that the range of the inverse secant (arcsecant) function is given by $\left[0, \frac{\pi}{2}\right) \cup \left[\pi, \frac{3\pi}{2}\right)$, as we do in the notes. Bear in mind that the range is defined differently in other sources.

a) $\int \frac{5}{\sqrt{1-t^2}}\, dt$

b) $\int \frac{1}{\sqrt{16-x^2}}\, dx$

c) $\int \frac{x}{\sqrt{16-x^2}}\, dx$

d) $\int \frac{dx}{25+x^2}$

e) $\int \frac{1}{x\sqrt{x^2-4}}\, dx$

f) $\int \frac{e^x}{\sqrt{1-e^{2x}}}\, dx$

g) $\int \frac{3x}{100+x^4}\, dx$

h) $\int \frac{\arctan x}{1+x^2}\, dx$

i) $\int \frac{1}{x\sqrt{x^8-9}}\, dx$. Hint: After deciding on a u-sub, multiply the numerator and the denominator by the same expression.

5) Evaluate the definite integral $\int_0^3 \frac{1}{x^2+9}\, dx$. Observe that we can find the **exact** value of this integral; in Chapter 5, we would have numerically approximated it using Riemann sums, the Trapezoidal Rule, or Simpson's Rule.

SECTION 8.3: HYPERBOLIC FUNCTIONS

1) Evaluate $\sinh(1)$, $\cosh(1)$, and $\tanh(1)$. Round off to four significant digits.

2) Prove that $D_x(\sinh x) = \cosh x$.

3) Prove that $\cosh^2 x - \sinh^2 x = 1$.

4) Find the following derivatives. Simplify where appropriate.

   a) $D_x\left[\sinh(3x)\right]$

   b) $D_x\left[\cosh(3x)\right]$

   c) $D_x\left[4\tanh\left(e^x\right) - 1\right]$

   d) $D_x\left[x\ln(\operatorname{sech} x)\right]$

   e) $D_x\left[3x^2 + 2^{\operatorname{csch} x}\right]$

   f) $\dfrac{d}{dt}\left(\left[\coth(\sec t)\right]^4\right)$

   g) $D_x\left(\dfrac{\cosh x}{\arctan x}\right)$

5) Evaluate the following indefinite integrals.

   a) $\int \cosh(3x)\,dx$. Try using Guess-and-Check here.

   b) $\int \sinh(3x)\,dx$. Try using Guess-and-Check here.

   c) $\displaystyle\int \frac{7x}{\cosh^2\left(4x^2 - 1\right)}\,dx$

   d) $\displaystyle\int \frac{\operatorname{sech}\left(\sqrt{x}\right)\tanh\left(\sqrt{x}\right)}{\sqrt{x}}\,dx$

   e) $\int e^t \coth\left(e^t\right)\operatorname{csch}^2\left(e^t\right)\,dt$