CALCULUS

CHAPTER 1, CHAPTER 2, SECTIONS 3.1-3.6

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COLOR CODING

WARNINGS are in red.
TIPS are in purple.

TECHNOLOGY USED

This work was produced on Macs with Microsoft Word, MathType, Mathematica (for most graphs) and Calculus WIZ, Adobe Acrobat, and Adobe Illustrator.

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Website: http://www.kkuniyuk.com
- You may download these and other course notes, exercises, and exams.

Feel free to send emails with suggestions, improvements, tricks, etc.

LICENSING

PARTIAL BIBLIOGRAPHY / SOURCES

Algebra: Blitzer, Lial, Tussy and Gustafson
Trigonometry: Lial
Precalculus: Axler, Larson, Stewart, Sullivan
Calculus: Larson, Stewart, Swokowski, Tan
Complex Variables: Churchill and Brown, Schaum's Outlines
Discrete Mathematics: Rosen
Online: *Britannica Online Encyclopedia*: http://www.britannica.com,
Wikipedia: http://www.wikipedia.org,
Wolfram MathWorld: http://mathworld.wolfram.com/
Other: *Harper Collins Dictionary of Mathematics*
People: Larry Foster, Laleh Howard, Terrie Teegarden, Tom Teegarden (especially for the Frame Method for graphing trigonometric functions), and many more.

TABLE OF CONTENTS

CHAPTER 1: REVIEW

CHAPTER 2: LIMITS AND CONTINUITY

CHAPTER 3: DERIVATIVES

See the website for more: http://www.kkuniyuk.com

ASSUMPTIONS and NOTATION

Unless otherwise specified, we assume that:

• f and g denote **functions**.

•• g sometimes denotes Earth's gravitational constant.

•• h may denote a function, or it may denote the "run" in some difference quotients in Chapter 3.

• a, b, c, k, and n denote **real constants** (or simply **real numbers**).

•• c sometimes denotes the speed of light in a vacuum.

•• d may denote a constant or a distance function.

•• e denotes a mathematical constant defined in Chapter 7. $e \approx 2.718$.

•• n might be restricted to be an integer $\left(n \in \mathbb{Z}\right)$.

• The **domain** of a function, which we will denote by $\text{Dom}\left(f\right)$ for a function f (though this is nonstandard), is its implied (or mathematical) domain.

•• This might not be the case in applied "word problems."

•• In single variable calculus (in which a function is of only one variable), we assume that the **domain** and the **range** of a function only consist of **real** numbers, as opposed to imaginary numbers. That is, $\text{Dom}\left(f\right) \subseteq \mathbb{R}$, and $\text{Range}\left(f\right) \subseteq \mathbb{R}$. ($\subseteq$ means "is a subset of.")

• **Graphs extend** beyond the scope of a figure in an expected manner, unless **endpoints** are clearly shown. Arrowheads may help to make this clearer.

• In single variable calculus, "real constants" are "real constant **scalars**," as opposed to vectors.

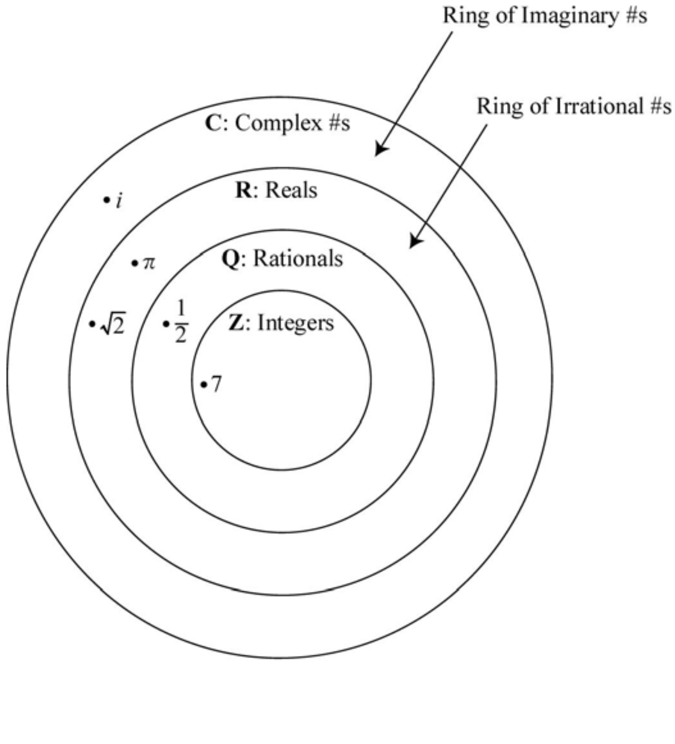
•• This will change in multivariable calculus and linear algebra.

MORE NOTATION

Sets of Numbers

Notation	*Meaning*	*Comments*
$\mathbb{Z}^+$, $\mathbf{Z}^+$	the set of positive integers	This is the set (collection) $\{1, 2, 3, ...\}$. “Zahlen” is a related German word. $\mathbb{Z}$ is in blackboard bold typeface; it is more commonly used than **Z**.
$\mathbb{Z}$, $\mathbf{Z}$	the set of integers	This set consists of the positive integers, the negative integers ($-1, -2, -3, \ldots$), and 0.
$\mathbb{Q}$, $\mathbf{Q}$	the set of rational numbers	This set includes the integers and numbers such as $\frac{1}{3}$, $-\frac{9}{4}$, 7.13, and $14.3\overline{587}$. $\mathbb{Q}$ comes from “Quotient.”
$\mathbb{R}$, $\mathbf{R}$	the set of real numbers	This set includes the rational numbers and irrational numbers such as $\sqrt{2}$, π, e, and 0.1010010001….
$\mathbb{C}$, $\mathbf{C}$	the set of complex numbers	This set includes the real numbers and imaginary numbers such as i and $2+3i$.

The Venn diagram below indicates the (proper) subset relations: $\mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$. For example, every integer is a rational number, so $\mathbb{Z} \subset \mathbb{Q}$. ($\subseteq$ permits equality.) Each disk is contained within each larger disk.



Set Notation

Notation	*Meaning*	*Comments*
$\in$	in, is in	This denotes set membership. Example: $7 \in \mathbb{Z}$.
$\notin$	not in, is not in	Example: $1.7 \notin \mathbb{Z}$.
$\ni$	such that	
$\mid$ or :	such that (in set-builder form)	Example: $\{x \in \mathbb{R} \mid x > 3\}$, or $\{x \in \mathbb{R} : x > 3\}$, is the set of all real numbers greater than 3.
$\forall$	for all, for any	This is called the universal quantifier.
$\exists$	there is, there exists	This is called the existential quantifier.
$\exists!$	there exists a unique, there is one and only one	This is called the unique quantifier. Example: $\exists! x \in \mathbb{R} \ni x = 3$, which states that there exists a unique real number equal to 3.
$\forall x \in \mathbb{R}$	for every real number (denoted by x)	More precisely: for any arbitrary element of the set of real numbers; this element will be denoted by x. Example: $\forall x \in \mathbb{R},\ x < x + 1$; that is, every real number is less than one added to itself.
$\forall x, y \in \mathbb{R}$	for every pair of real numbers (denoted by x and y)	More precise notation: $\forall (x, y) \in \mathbb{R}^2$.
$\varnothing$ or $\{\}$	empty set (or null set)	This is the set consisting of no elements. Example: The solution set of the equation $x = x + 1$ is $\varnothing$. The symbol $\varnothing$ is not to be confused with the Greek letter phi (ϕ).
$\cup$	set union	Example: If $f(x) = \csc x$, then $\text{Dom}(f) = (-\infty, -1] \cup [1, \infty)$. $\cup$ is used to indicate that one or more number(s) is/are being skipped over.
$\cap$	set intersection	Example: $[4, 6] \cap [5, 7] = [5, 6]$. Think: "overlap."
\ or $-$	set difference, set complement	Example: If $f(x) = \frac{1}{x}$, then $\text{Dom}(f) = \mathbb{R} \setminus \{0\}$, or $\mathbb{R} - \{0\}$.

Logical Operators

Notation	*Meaning*	*Comments*
$\vee$	or, disjunction	Example: If $f(x)=\csc x$, then $\operatorname{Dom}(f)=\{x\in\mathbb{R}\mid x\le -1 \ \vee \ x\ge 1\}$.
$\wedge$	and, conjunction	Example: If $f(x)=\frac{\sqrt{x-3}}{x-4}$, then $\operatorname{Dom}(f)=\{x\in\mathbb{R}\mid x\ge 3 \ \wedge \ x\ne 4\}$.
$\sim$ or $\neg$	not, negation	Example: The statement $\sim(x=3)$ is equivalent to the statement $x\ne 3$.
$\Rightarrow$	implies	Example: $x=2 \ \Rightarrow \ x^2=4$.
$\Leftrightarrow$	if and only if (iff)	Example: $x+1=3 \ \Leftrightarrow \ x=2$.

Greek Letters

The lowercase Greek letters below (especially θ) often denote **angle measures**.

Notation	*Name*	*Comments*
α	alpha	This is the first letter of the Greek alphabet.
β	beta	This is the second letter of the Greek alphabet.
γ	gamma	This is the third letter of the Greek alphabet.
θ	theta	This is frequently used to denote angle measures.
ϕ or φ	phi	This is not to be confused with $\varnothing$, which denotes the empty set (or null set). ϕ also denotes the golden ratio, $\frac{1+\sqrt{5}}{2}$, which is about 1.618. Tau (τ) is also used.

The lowercase Greek letters below often denote (perhaps infinitesimally) **small positive quantities** in calculus, particularly when defining **limits**.

Notation	*Name*	*Comments*
δ	delta	This is the fourth letter of the Greek alphabet.
ε	epsilon	This is the fifth letter of the Greek alphabet. This is not be confused with $\in$, which denotes set membership.

Some other Greek letters of interest:

Notation	*Name*	*Comments*
Δ	(uppercase) delta	This denotes "change in" or increment. Example: slope is often written as $\frac{\Delta y}{\Delta x}$. It also denotes the discriminant, $b^2 - 4ac$, from the Quadratic Formula.
κ	(lowercase) kappa	This denotes the curvature of a curve.
λ	(lowercase) lambda	This denotes an eigenvalue (in linear algebra), a Lagrange multiplier (in multivariable optimization), and a wavelength (in physics).
π	(lowercase) pi	This is a famous mathematical constant. It is the ratio of a circle's circumference to its diameter. $\pi \approx 3.14159$. It is irrational.
Π	(uppercase) pi	This is the product operator.
ρ	(lowercase) rho	This denotes mass density and also the distance between a point in 3-space and the origin (ρ is a spherical coordinate).
Σ	(uppercase) sigma	This is the summation operator.
τ	(lowercase) tau	This denotes the golden ratio, though phi (ϕ) is more commonly used.
ω	(lowercase) omega	This is the last letter of the Greek alphabet. It denotes angular velocity.
Ω	(uppercase) omega	This denotes ohm, a unit of electrical resistance.

More lowercase Greek letters:

zeta (ζ), eta (η), iota (ι), mu (μ), nu (ν), xi (ξ), omicron (o), sigma (σ), upsilon (υ), chi (χ), psi (ψ)

Geometry

Notation	*Meaning*	*Comments*
$\angle$	angle	
$\parallel$	is parallel to	
$\perp$	is perpendicular to, is orthogonal to, is normal to	

Vector Operators

Notation	*Meaning*	*Comments*
$\bullet$	dot product, Euclidean inner product	See Precalculus notes, Section 6.4.
$\times$	cross product, vector product	See Precalculus notes, Section 8.4.

Other Notations

Notation	*Meaning*	*Comments*
$\therefore$	therefore	This is placed before a concluding statement.
Q.E.D., or $\square$	end of proof	Q.E.D. stands for "quod erat demonstrandum," which is Latin for "which was to be demonstrated / proven / shown."
$\approx$, $\cong$	is approximately	
$\lfloor\ \rfloor$ or $[\![\ ]\!]$	floor, greatest integer	Think: "round down." Examples: $\lfloor 2.9 \rfloor = 2$, $\lfloor -2.9 \rfloor = -3$
∞	infinity	
min	minimum	The least of …
max	maximum	The greatest of …
$\text{Dom}(f)$	domain of a function f	The set of legal (real) input values for f
$\deg(f(x))$	degree of a polynomial $f(x)$	
$\circ$	composition of functions	Example: $(f \circ g)(x) = f(g(x))$.