CHAPTER 1 AND 2: REVIEW OUTLINE

CHAPTER 1: REVIEW

(Topic 1) Functions

Types of Functions
Domain and Range
Representing Subsets of Real Numbers
  Set-Builder Form, Interval Form, Graphical Form
Graphs of Functions and the Vertical Line Test (VLT)
Even and Odd Functions; Symmetry
Combining Functions: Arithmetic Operations and Compositions

(Topics 2 and 3) Trig Functions

Angles and Evaluating Trig Functions
  “THE” Table of Special Trig Values
  Quadrants and Signs (ASTC)
  Quadrantal Angles, Coterminal Angles, Reference Angles
Right Triangle (SOH-CAH-TOA) and Unit Circle Approaches
Identities (Fundamental IDs and Advanced IDs)
Domains, Ranges, and Graphs of the Six Basic Trig Functions
Simplifying Trig Expressions and Verifying IDs
Solving Trig Equations
CHAPTER 2: LIMITS AND CONTINUITY

(2.1) Intro

Evaluating Limits at a Point
  Limit Theorems for Rational Functions
Limits are Local
One- and Two-Sided Limits
  Analyzing Piecewise-Defined Functions
  Drawing and Analyzing Graphs
“Ignore a” Theorems
When a Limit Does Not Exist (DNE)

(2.2) Properties of Limits

Algebra of Limits, Linearity of Limits
Limit of a Root: Discussion
Limit Theorem for Algebraic Functions

(2.3) Limits and Infinity I

Notation for Nonexistent Limits
Horizontal Asymptotes (HAs)
“Long-Run” Limits \( \lim_{x \to \infty} f(x) \), \( \lim_{x \to -\infty} f(x) \)
When a Limit can be Described as \( \infty \) or \( -\infty \) (“Special Cases of DNE”)
Limit Forms
  Signs and Rescaling
“Long-Run” Limit Rules for \( \frac{c}{x^k} \)
“Long-Run” Limits for Polynomial Functions and Dominance
“Long-Run” Limits for Rational Functions
  “Twin Limits” Property
  Technique for Evaluating Limits
    Dividing Numerator and Denominator by the Highest Power of \( x \) (for example) in the Denominator
Short Cut: Dominant Term Substitution
Short Cuts Based on Degrees of Numerator, Denominator
Long Division and Slant Asymptotes
“Long-Run” Limits for Algebraic Functions and Dominance
Word Problems
(2.4) Limits and Infinity II

Vertical Asymptotes (VAs)
Infinite Limits at a Point
Limit Forms
When a Limit can be Described as $\infty$ or $-\infty$ ("Special Cases of DNE")
Factoring, Sign Analysis can help

(2.5) The Indeterminate Forms $0/0$ and $\infty/\infty$

Techniques for the $0/0$ Form:
- Factoring and Canceling / Dividing
- Rationalizing a Numerator or Denominator
- Working with Compound Fractions
- L'Hôpital's Rule (to be seen later)

VAs and Holes

Techniques for the $\infty/\infty$ Form:
(See Section 2.3.)

(2.6) The Squeeze (Sandwich) Theorem

Using the Theorem to Prove a Limit Statement at a Point
Using the (Modified) Theorem to Prove a "Long-Run" Limit Statement

(2.7) Precise Definitions of Limits

Precise $\varepsilon$-$\delta$ Definition of a Limit at a Point: \[ \lim_{x \to a} f(x) = L \]
One-Sided Variations: \[ \lim_{x \to a^+} f(x) = L \quad \text{and} \quad \lim_{x \to a^-} f(x) = L \]
Proving a Limit Statement Involving a Constant or Linear $f$

Precise Definitions of "Long-Run" Limits
\[ \lim_{x \to \infty} f(x) = L \quad \text{and} \quad \lim_{x \to -\infty} f(x) = L \]

Precise Definitions of Infinite Limits at a Point
\[ \lim_{x \to a} f(x) = \infty \quad \text{and} \quad \lim_{x \to a} f(x) = -\infty \]
(2.8) Continuity

Definition of Continuity …
   1) at a Point
   2) on an Open Interval
   3) on a Closed Interval

Classifying Discontinuities
   Removable
   Jump
   Infinite

One-Sided Continuity

Continuity Theorems
   Algebra of Continuity
   Continuity of Rational Functions
   Continuity of Composite Functions
   Continuity of Basic Trig Functions
   Intermediate Value Theorem (IVT) and
       The Bisection Method for Approximating Zeros of a Function

Where is a Function Continuous / Discontinuous?