# CHAPTER 3 REVIEW

## SECTION 3.1: INTRODUCTION

Secant lines (related to the graph of $y = f(x)$)

slope = average rate of change of $y$ with respect to $x$ on $[a, a+h]$ or $[a, b]$

$$= \frac{f(a+h) - f(a)}{h} \quad \text{or} \quad \frac{f(b) - f(a)}{b - a}$$

If $h \approx 0$, this $\approx f'(a)$, if it exists.

Tangent lines

slope = instantaneous rate of change of $y$ with respect to $x$ at $a$

$= \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$ if it exists. Use algebra to work through this!

$= f'(a)$ (You may use our shortcuts unless you're told to use the limit definition.)

Slopes and equations of tangent lines

Rectilinear motion

Velocity = [Instantaneous] rate of change of position with respect to time.
$v(t) = s'(t)$

## SECTION 3.2: $f'(x)$

Limit definition of the derivative

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ if it exists. Use algebra to work through this!

Key cases when the derivative is "DNE"

Where there is a…

($m_L$ = left-hand derivative; $m_R$ = right-hand derivative)

1) Discontinuity
2) Corner — $m_L \neq m_R$ (Maybe one is $\pm\infty$, but not both.)
3) Vertical tangent line — $m_L$ is $\infty$ or $-\infty$, and $m_R$ is $\infty$ or $-\infty$.
4) Cusp — $m_L$ and $m_R$: one is $\infty$; the other is $-\infty$. This is a special case of 3).

## SECTIONS 3.2 and 3.3: SHORTCUTS FOR DIFFERENTIATION

$c, m, b,$ and $n$ are constants; $f$ and $g$ are functions of $x$.

| | **Derivative with respect to *x*** | **Comments** |
|---|---|---|
| $c$ | $0$ | |
| $mx+b$ | $m$ | |
| $x^n$ | $nx^{n-1}$ | Power Rule |
| $cf$ | $cf'$ | Constant factors "pop out" |
| $f+g$ | $f'+g'$ | Sum Rule |
| $f-g$ | $f'-g'$ | Difference Rule |
| $fg$ | $f'g+fg'$ | Product Rule ("Pointer Method") |
| $\frac{f}{g}$ | $\frac{gf'-fg'}{g^2}$ | Quotient Rule; $\frac{\text{LoD(Hi)}-\text{HiD(Lo)}}{\text{the square of what's below}}$ |

Solve $f'(x)=0$. (Where are tangent lines horizontal?)
Slopes and equations of tangent lines and normal lines

## SECTION 3.4: TRIG

RADIAN mode!!!

Prove $D_x(\sin x)=\cos x$, and $D_x(\cos x)=-\sin x$ using all of these:

1) The limit definition of the derivative

2) $\lim_{h\to 0}\frac{\sin h}{h}=1$

3) $\lim_{h\to 0}\frac{\cos h-1}{h}=0$

Derive these using $D_x(\sin x)=\cos x$, $D_x(\cos x)=-\sin x$, the Quotient Rule, and/or the Reciprocal Rule:

$D_x(\tan x)=\sec^2 x$ $\qquad$ $D_x(\cot x)=-\csc^2 x$

$D_x(\sec x)=\sec x\tan x$ $\qquad$ $D_x(\csc x)=-\csc x\cot x$

Use the table above (in 3.2/3.3) and trig identities/formulas.

Solve $f'(x)=0$ by solving a trig equation. (Where are tangent lines horizontal?)
Slopes and equations of tangent lines and normal lines

## SECTION 3.5: DIFFERENTIALS and LINEAR APPROXIMATIONS

$$f\Bigg(\underbrace{x + \overbrace{\Delta x}^{=dx}}_{=\text{new } x}\Bigg) \approx \underbrace{f(x)}_{\substack{\text{Known} \\ \text{exactly}}} + \underbrace{dy}_{\substack{= f'(x)dx \\ = (\text{slope})(\text{run}), \\ \text{where run } = \\ \text{new } x \text{ - old } x \\ = \text{change in } y \text{ along} \\ \text{tangent line at} \\ (x, f(x)); \text{" rise"}}}$$

## SECTION 3.6: CHAIN RULE

Let's say $u$ is a function of $x$.

$$\frac{dy}{dx} = \frac{dy}{du}\underbrace{\frac{du}{dx}}_{\text{tail}}$$

Generalized Power Rule

$$D_x\, u^n = \left(nu^{n-1}\right)\underbrace{\left(D_x u\right)}_{\text{tail}}$$

Generalized Trig Rules

$$D_x(\sin u) = (\cos u)\underbrace{\left(D_x u\right)}_{\text{tail}}$$

$$D_x(\sec u) = \left(\sec u \tan u\right)\underbrace{\left(D_x u\right)}_{\text{tail}}, \text{ etc.}$$

Think "Big Picture": Overall, do I have a power, or a trig function?
Use techniques and deal with issues discussed on the previous page.

## SECTION 3.7: IMPLICIT DIFFERENTIATION

Implicitly differentiate functions
Given an equation, find $y'$.
Slopes and equations of tangent lines and normal lines

## SECTION 3.8: RELATED RATES

1) Read the problem.
2) Define variables, maybe through a general diagram (nothing specific to the instant of interest).
3) Identify what we are GIVEN throughout the process and what we have to FIND at the instant of interest.
4) Set up the key relevant formula.
5) Implicitly differentiate both sides of the formula in 4).
6) Plug in (substitute) values at the instant of interest.
   Diagrams, the Pythagorean Theorem, and/or other formulas may help.
7) Solve for the desired rate (don't forget units).
   Remember issues of unit compatibility and DEG vs. RAD.
8) State your conclusion using proper English.
9) Does your answer make sense?