SECTION 4.3: THE FIRST DERIVATIVE TEST ("1st DT")

LEARNING OBJECTIVES

- Construct sign charts of $f'$ to determine intervals of increase or decrease for $f$.
- Use the First Derivative Test ("1st DT") to classify points at critical numbers (CNs) as L.Max.Pts., L.Min.Pts., or neither.
- Analyze $f$ and $f'$ to graph, say, $y = f(x)$.

PART A: INTERVALS OF INCREASE OR DECREASE FOR A FUNCTION

Example 1 (Interpreting the Sign of the First Derivative)

Below is the graph of $y = f(x)$ for a function $f$, where:

- $\text{Dom}(f) = \mathbb{R}$.
- $f$ has only three critical numbers (CNs): 1, 3, and 5.
- $f'(1) = 0$, $f'(3)$ does not exist (DNE), and $f'(5) = 0$.

What is the sign of $f'(0)$? … of $f'(2)$?

\[ \begin{align*}
\text{y} & \quad \text{1} \\
\text{1} & \quad \text{3} \quad \text{3} \quad \text{5} \quad \text{CN} \quad \text{CN} \quad \text{CN}
\end{align*} \]
A derivative is a slope of a tangent line. Judging from the red tangent lines in the preceding figure,

- \( f'(0) < 0 \) (negative, “–”). This is because the corresponding tangent line falls. As a result, \( f \) decreases on a neighborhood of \( x = 0 \).

  • We may informally say that \( f \) decreases “at” 0, but we’re not supposed to. The “decreasing property” requires at least two distinct points on the graph to compare; the one on the right must always be higher than the one on the left. We will discuss \( f \) decreasing on intervals.

- \( f'(2) > 0 \) (positive, “+”). This is because the corresponding tangent line rises. As a result, \( f \) increases on a neighborhood of \( x = 2 \).

Interpreting the Sign of the First Derivative

<table>
<thead>
<tr>
<th>Condition</th>
<th>Conclusion</th>
</tr>
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<tbody>
<tr>
<td>( f'(c) &gt; 0 )</td>
<td>( f ) increases on ( x = c ).</td>
</tr>
<tr>
<td>( f'(c) &lt; 0 )</td>
<td>( f ) decreases on ( x = c ).</td>
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</tbody>
</table>

Example 2 (False Convereses of the Derivative-Sign Interpretation Theorems)

If we lose differentiability, the converses are false. In the counterexample below, \( f \) increases on a neighborhood of \( x = 1 \), yet \( f''(1) \) does not exist (DNE); it is not positive. Here is the graph of \( y = f(x) \):

In fact, we can say that \( f \) increases on the interval \([0, 2]\).
The First Derivative Test ("1st DT")

4.3.3

Intervals of Increase or Decrease

Consider points on the graph of \( y = f(x) \) "on" an \( x \)-interval.

**\( f \) increases** on an interval ⇔
1) \( f \) is defined throughout the interval, and
2) "on" that interval, for any two distinct points on the graph, the point on the right is **higher** than the point on the left.
• We call such an interval an **interval of increase**.

**\( f \) decreases** on an interval ⇔
1) \( f \) is defined throughout the interval, and
2) "on" that interval, for any two distinct points on the graph, the point on the right is **lower** than the point on the left.
• We call such an interval an **interval of decrease**.

• More precisely, a function \( f \) increases on an interval ⇔
1) \( f \) is defined throughout the interval, and
2) for any pair of values \( x_1 \) and \( x_2 \) in the interval such that \( x_1 < x_2 \) (equivalently, \( x_2 > x_1 \)), \( f(x_2) > f(x_1) \).

• More precisely, a function \( f \) decreases on an interval ⇔
1) \( f \) is defined throughout the interval, and
2) for any pair of values \( x_1 \) and \( x_2 \) in the interval such that \( x_1 < x_2 \) (equivalently, \( x_2 > x_1 \)), \( f(x_2) < f(x_1) \).

The First Derivative and Intervals of Increase or Decrease

(Assume \( a < b \).)

If \( f'(x) > 0 \)(positive, "+" ) on \( (a, b) \), then \( f \) increases on \( (a, b) \).
• Furthermore, if \( f \) is continuous on \( [a, b] \), then we can say that \( f \) increases on \( [a, b] \).

If \( f'(x) < 0 \)(negative, "-" ) on \( (a, b) \), then \( f \) decreases on \( (a, b) \).
• Furthermore, if \( f \) is continuous on \( [a, b] \), then we can say that \( f \) decreases on \( [a, b] \).
Other forms for intervals are: \((-\infty, \infty)\), \((-\infty, b)\), \((-\infty, b]\), \((a, \infty)\), \([a, \infty)\), \([a, b)\), and \((a, b]\). We can say a function increases or decreases on these kinds of intervals, as well. For example, if \(f'(x) > 0\) (positive, “+”) on \((a, b)\) and \(f\) is left-continuous at \(x = b\), then \(f\) increases on \((a, b]\).

If \(f''(x) > 0\) (positive, “+”) on \((a, b)\) and \(f\) is right-continuous at \(x = a\), then \(f\) increases on \([a, b)\). Think: “Pushing inside-out.”

**Example 3 (Sign Charts of the First Derivative; Intervals of Increase or Decrease; Revisiting Example 1)**

Consider the function \(f\) from Example 1. Here is the graph of \(y = f(x)\):

- Construct a sign chart for \(f'\).
- What are “the” intervals of increase or decrease for \(f\)? (It is assumed that we provide the most “ambitious” possible answers. It is true that \(f\) increases on \([1.9, 2.1]\), say, but that’s not very ambitious!)

**Solution**

Remember that derivatives are slopes of tangent lines.

We place “fenceposts” at the three CNs to divide the real number line into four “windows,” roughly corresponding to intervals.
“First window”
\( f'(x) < 0 \) (negative, “−”) on \((-\infty, 1)\), so \( f \) decreases on \((-\infty, 1]\).
Furthermore, \( f \) is left-continuous at \( x = 1 \), so \( f \) decreases on \((-\infty, 1]\).

“Second window”
\( f'(x) > 0 \) (positive, “+”) on \((1, 3)\), so \( f \) increases on \((1, 3)\).
Furthermore, \( f \) is continuous on \([1, 3]\), so \( f \) increases on \([1, 3]\).

• What about 1, itself? As mentioned in Example 1, we’re not supposed to talk about \( f \) increasing or decreasing at a particular \( x \)-value such as 1. In Example 1, we informally said that \( f \) decreased “at” 0, because \( f \) decreased on some neighborhood of \( x = 0 \). Now in Example 3, we cannot say that \( f \) increases on (throughout) a neighborhood of \( x = 1 \), nor can we say that it decreases on such a neighborhood. Even informally, we cannot say that \( f \) is increasing or decreasing “at” 1.

“Third window”
\( f'(x) < 0 \) (negative, “−”) on \((3, 5)\), so \( f \) decreases on \((3, 5)\).
Furthermore, \( f \) is continuous on \([3, 5]\), so \( f \) decreases on \([3, 5]\).

“Fourth window”
\( f'(x) < 0 \) (negative, “−”) on \((5, \infty)\), so \( f \) decreases on \((5, \infty)\).
Furthermore, \( f \) is right-continuous at \( x = 5 \), so \( f \) decreases on \([5, \infty)\).

• Combining overlapping intervals. \( f \) decreases on \([3, 5]\) and also on \([5, \infty)\), and the intervals overlap (even if it’s just at one number, such as 5 here). Therefore, \( f \) decreases on their union, \([3, \infty]\).

• Beware of combining non-overlapping intervals. It may be misleading to say that \( f \) decreases on the union \((-\infty, 1] \cup [3, \infty)\). That might imply, for instance, that the point at \( x = 3 \) is lower than the point at \( x = 1 \), which is not the case. Also, the function \( g \), graphed below as \( y = g(x) \), does not increase on \([1, 3]\), even though \( g \) increases on both \([1, 2]\) and \((2, 3]\).
“The” interval of increase for $f$ is $[1, 3]$.

“The” intervals of decrease for $f$ are $(-\infty, 1]$ and $[3, \infty)$. §

PART B: TEST VALUE METHOD FOR SIGN CHARTS OF THE FIRST DERIVATIVE

In Example 3, we used the graph of $y = f(x)$ to construct a sign chart for $f'$. Usually, we do the reverse; we construct a sign chart for $f'$ to help us graph $y = f(x)$. 

\text{“Fencepost” Theorem for Sign Charts of the First Derivative}

For a function $f$, the only places (“fenceposts”) on the real number line where $f'$ can change sign are:

- at CNs of $f$,
- where $f$ is undefined, and
- anywhere else $f'$ is discontinuous.

(We won’t fret about this last case.) That is, where $f'$ is 0 or discontinuous (particularly “DNE”).

- All such “fenceposts” must be indicated on a sign chart for $f'$.
- A “large fencepost” may indicate something like a “large gap” in $\text{Dom}(f)$.
- See Footnote 1 for a proof.

\textit{Example 4 ("Fencepost” Theorem for Sign Charts of the First Derivative; Revisiting Examples 1 and 3)}

Take the sign chart for $f'$ from Example 3.

\begin{center}
\begin{tabular}{cccc}
\hline
$x$ & - & + & - & - \\
\hline
$1$ & CN & $3$ & CN & $5$ & CN \\
\hline
\end{tabular}
\end{center}

• WARNING 1: The sign of $f'(x)$ might not change at a “fencepost.”

Here, the sign of $f'(x)$ does not change at $x = 5$, a CN. §
**Example 5 ("Fencepost" Theorem for Sign Charts of the First Derivative)**

Let \( f(x) = \frac{1}{x^2} \). Here is the graph of \( y = f(x) \):

![Graph of \( y = \frac{1}{x^2} \)](image)

Even though \( f \) has no CNs (Check this!), we do need a “fencepost” at \( x = 0 \), since \( f \) is undefined there. In fact, \( f' \) does change sign there as we cross the vertical asymptote (VA) on the graph.

- **WARNING 2:** Don’t forget to put “fenceposts” where \( f \) is undefined.

The “Fencepost” Theorem motivates the following method for constructing such sign charts.

**Test Value Method for Sign Charts of the First Derivative**

- For a function \( f \), assume the sign chart for \( f' \) has a finite number of “fenceposts” that divide the real number line into “windows,” each corresponding to an open interval when the “fenceposts” are removed.

- Within each interval, pick a test value \( c \); let’s say \( x = c \).

- The sign of \( f'(c) \) will be the sign of \( f'(x) \) for all \( x \)-values throughout that interval.

- Some sources call \( f'(c) \) the “test value,” since it is a “value” of the derivative function.
Example 6 (Test Value Method for Sign Charts of the First Derivative; Revisiting Examples 1, 3, and 4)

Take our function \( f \) from Example 1. Let’s say we only know that \( \text{Dom}(f) = \mathbb{R} \) and the only “fenceposts” for the sign chart of \( f' \) are at 1, 3, and 5.

- The three “fenceposts” determine four “windows” corresponding to four open \( x \)-intervals: \((-\infty, 1), (1, 3), (3, 5), \) and \((5, \infty)\).

- Pick a test value in each of the intervals. Let’s pick 0, 2, 4, and 6.

- At each test value \( c \), evaluate \( f'(c) \). The sign of \( f'(c) \) is shared by the other values of \( f'(x) \) throughout the \( x \)-interval.

  **TIP 1:** We only care about signs. We usually don’t care about the value of \( f'(c) \) if we know its sign. Sometimes, an extreme test value like 1,000,000 is preferable if the sign of \( f'(1,000,000) \) is obvious.

- Let’s say \( f'(0) = -2 < 0 \). Then, \( f'(x) < 0 \) on \((-\infty, 1)\).
- Let’s say \( f'(2) = 2 > 0 \). Then, \( f'(x) > 0 \) on \((1, 3)\).
- Let’s say \( f'(4) = -2 < 0 \). Then, \( f'(x) < 0 \) on \((3, 5)\).
- Let’s say \( f'(6) = -2 < 0 \). Then, \( f'(x) < 0 \) on \((5, \infty)\).

\[
\begin{array}{c|cccc|c}
& 0 & 1 & 2 & 3 & 4 & 5 & 6 \\
\hline
f' & - & + & - & - & & & \\
\end{array}
\]

- Cleaning up:

\[
\begin{array}{c|cccc|c}
& 1 & 3 & 5 \\
\hline
f' & - & + & - & - & \\
\end{array}
\]

If we know that \( f \) is continuous at the “fenceposts,” then we can say that:

- The interval of increase for \( f \) is \([1, 3]\).
- The intervals of decrease for \( f \) are \((-\infty, 1) \) and \([3, \infty)\).
PART C: FIRST DERIVATIVE TEST ("1st DT")

The purpose of the First Derivative Test ("1st DT") is to classify a CN of a function as a L.Max., a L.Min., or "Neither."

Example 7 (Motivating the First Derivative Test; Revisiting Examples 1, 3, 4, and 6)

Consider the function $f$ from Example 1. Here is the graph of $y = f(x)$:

$f$ is continuous at the CNs. How can we use the sign chart of $f'$ to classify a CN of $f$ as a L.Max., a L.Min., or "Neither"?

§ Solution

• 1 is a CN of $f$, and $f$ is continuous at $x = 1$. $f'$ changes from negative to positive at $x = 1$. By the 1st DT, $x = 1$ is a L.Min.

• 3 is a CN of $f$, and $f$ is continuous at $x = 3$. $f'$ changes from positive to negative at $x = 3$. By the 1st DT, $x = 3$ is a L.Max.

• 5 is a CN of $f$, and $f$ is continuous at $x = 5$. $f'$ stays negative “around” $x = 5$. By the 1st DT, $x = 5$ is neither a L.Min. nor a L.Max.

(See Footnote 2 for more precision.) §
First Derivative Test ("1st DT")

Assume a function \( f \) is continuous at a CN \( c \).
Consider the graph of \( y = f(x) \).

- If \( f' \) changes from negative to positive at \( x = c \), then \( x = c \) is a L.Min.
- If \( f' \) changes from positive to negative at \( x = c \), then \( x = c \) is a L.Max.
- If \( f' \) stays negative or stays positive “around” \( x = c \), then \( x = c \) is **neither** a L.Min. nor a L.Max.

**WARNING 3:** Only for CNs. The 1st DT is only applied to CNs. Local extrema can only occur at CNs.

**TIP 2:** Little pictures such as \( \cup \) and \( \cap \) can help. We use here the set union and intersection symbols for convenience; pretend they don’t fail the Vertical Line Test (VLT).

PART D: USING THE FIRST DERIVATIVE TO GRAPH A FUNCTION

*Example 8 (Using the First Derivative to Graph a Function)*

Let \( f(x) = 2x^3 - 9x^2 - 24x \).

a) What are “the” intervals of increase or decrease for \( f \)?

b) Find any L.Max./Min. Pts. on the graph of \( y = f(x) \).

c) Sketch the graph of \( y = f(x) \).

§ Solution

- \( \text{Dom}(f) = \mathbb{R} \).
- \( f'(x) = 6x^2 - 18x - 24 = 6(x^2 - 3x - 4) = 6(x+1)(x-4) \).

**TIP 3:** Factoring helps with sign analyses. The sign of a product can be determined by the signs of its factors. On the other hand, the sign of a sum is often not determined by the signs of its terms.

**TIP 4:** Other techniques. If it is not easy to factor \( f'(x) \), then other techniques such as the Quadratic Formula ("QF") can help with finding zeros of \( f'(x) \). The Test Value Method can help with sign analyses.
• \( f \) and \( f' \) are everywhere continuous on \( \mathbb{R} \). This means that the only “fenceposts” on the sign chart of \( f' \) where \( f' \) can change sign are at the CNs of \( f \).

• Find the CNs of \( f \). \( f' \) is never undefined (“DNE”). Solve:

\[
f'(x) = 0 \\
6(x + 1)(x - 4) = 0
\]

**WARNING 4: Avoid division.** Try not to divide both sides by 6 here. Let’s not be confused about what \( f'(x) \) really is; it is precisely \( 6(x + 1)(x - 4) \). Even more dangerously, dividing by a negative number could well lead to a faulty sign analysis.

The CNs are \(-1\) and \(4\), which are in \( \text{Dom}(f) \).

• Construct the sign chart for \( f' \).

Test Value Approach

\[
\begin{array}{cccc}
& + & 0 & - & 0 & + \\
-2 & 0 & -1 & 4 & 5 & x \\
\end{array}
\]

\[
f'(x) = (6)(x + 1)(x - 4)
\]

This column doesn’t have to be written.

\[
f'(-2) = (6)(-2 + 1)(-2 - 4) = (+)(-)(-) = + \\
f'(0) = (6)(0 + 1)(0 - 4) = (+)(+)(-) = - \\
f'(5) = (6)(5 + 1)(5 - 4) = (+)(+)(+) = +
\]

(TIP 1). We only care about the sign of \( f' \) at each test \( x \)-value. Extreme test values may be easier to analyze. For example, it may be easier to see that \( f'(1,000,000) > 0 \) than \( f'(5) > 0 \).

Also, \( f'(x) = 6x^2 - 18x - 24 \Rightarrow f'(0) = 6(0)^2 - 18(0) - 24 = -24 \), which is negative.
Sketch the graph of $y = f'(x)$, where

\[ f'(x) = 6x^2 - 18x - 24 = 6(x+1)(x-4). \]

This is an upward-opening parabola along which the $y$-values, here the values of $f'(x)$, stay positive in the leftmost and rightmost windows. The parabola sags beneath the $x$-axis in the middle window, indicating a negative sign for values of $f'(x)$ there.

- Also, the multiplicities of $-1$ and $4$ as zeros of $f'(x)$ are both 1, which is odd. (The exponents on the $(x+1)$ and $(x-4)$ factors are both 1.) This confirms that the parabola “cuts through” the $x$-axis at both “fenceposts,” and the sign of $f'(x)$ alternates from window to window.

- Clean up the sign chart for $f'$, find “the” intervals of increase or decrease for $f$, and apply the 1st DT to classify the CNs.

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>$(-1, f(-1))$</td>
<td>$(4, f(4))$</td>
</tr>
<tr>
<td>$(-1, 13)$</td>
<td>$(4, -112)$</td>
</tr>
</tbody>
</table>
• WARNING 5: Use \( f(x) \), not \( f'(x) \), to find \( y \)-coordinates of L.Max./Min. Pts. Be suspicious if you obtain 0 or “DNE,” since \( f'(x) = 0 \) or “DNE” at CNs. Also, \( y \) cannot be “DNE” for a L.Max./Min. Pt.

• TIP 5: Remainder Theorem. The Remainder Theorem from the Precalculus notes on Synthetic Division (Section 2.3) can be used to efficiently evaluate polynomial functions. Here, we evaluate \( f(4) \):

\[
\begin{array}{ccccccc}
2 & -9 & -24 & 0 \\
2 & -1 & -28 & -112 \\
\end{array}
\]

Answer to a). By the sign chart for \( f' \) and continuity of \( f \) at the CNs, “the” intervals of increase for \( f \) are \((-\infty, -1]\) and \([4, \infty)\).

“The” interval of decrease is \([-1, 4]\).

Answer to b). The sole L.Max.Pt. is at \((-1, 13)\). The sole L.Min.Pt. is at \((4, -112)\).

• Find the \( y \)-intercept of the graph of \( y = f(x) \), if any.

Set \( x = 0 \) and evaluate: \( f(0) = 0 \). The \( y \)-intercept is at \((0, 0)\).

• Find the \( x \)-intercept(s) of the graph of \( y = f(x) \), if any.

Set \( y = 0 \) and solve the equation \( f(x) = 0 \) for real \( x \)-values.

\[
f(x) = 0 \\
2x^3 - 9x^2 - 24x = 0 \\
x(2x^2 - 9x - 24) = 0 \\
x = 0 \text{ or } 2x^2 - 9x - 24 = 0 \\
x = 0 \text{ or (by the QF:) } x \approx -1.9 \text{ or } x \approx 6.4
\]

The \( x \)-intercepts are at \((0, 0)\), about \((-1.9, 0)\), and about \((6.4, 0)\).

• WARNING 6: \( x \)-intercepts are sometimes difficult (or impossible) to find by hand.
• $f$ is neither even nor odd.

• $f$ is a nonconstant, nonlinear polynomial function, so the graph of $y = f(x)$ has no asymptotes (vertical, horizontal, or slant), holes, corners, or cusps. The leading term is $2x^3$, which has odd degree and a positive leading coefficient, so the graph resembles a “rising snake” in the long run.

• Sketch the graph of $y = f(x)$. Answer to c).

![Graph of $f(x)$]

(Axes are scaled differently)

§

Example 9 (Using the First Derivative to Graph a Function)

Let $g(\theta) = \cos(\theta) + \theta$, where $\text{Dom}(g) = [-2\pi, 2\pi]$.

Sketch the graph of $y = g(\theta)$. Do not worry about \theta-intercepts.

§ Solution

• $g'(\theta) = -\sin(\theta) + 1 = 1 - \sin(\theta)$.

• $g$ and $g'$ are continuous on $[-2\pi, 2\pi]$. This means that the only “fenceposts” on the sign chart of $g'$ where $g'$ can change sign are at the CNs of $g$ in $(-2\pi, 2\pi)$. 
• Find the CNs of \( g \) in \((-2\pi, 2\pi)\). \( g' \) is never undefined ("DNE") in \((-2\pi, 2\pi)\). Solve:

\[
\begin{array}{c|c}
  g'(\theta) & 0 \\
  1 - \sin(\theta) & 0 \\
  \sin(\theta) & 1 \\
\end{array}
\]

The CNs in \((-2\pi, 2\pi)\) are \( \frac{\pi}{2} \) and \( -\frac{3\pi}{2} \).

• Construct the sign chart for \( g' \).

The methods in Example 8 work, but we can observe that \( g'(\theta) = 1 - \sin(\theta) > 0 \) in all three windows.

•• \( \sin(\theta) \) can never be greater than 1, so \( 1 - \sin(\theta) \) can never be negative:

\[
-1 \leq \sin(\theta) \leq 1 \quad \Rightarrow \quad 1 \geq -\sin(\theta) \geq -1 \quad \Rightarrow \quad 2 \geq 1 - \sin(\theta) \geq 0
\]

•• \( 1 - \sin(\theta) \) is equal to 0 in the \( \theta \)-interval \((-2\pi, 2\pi)\) only at the CNs \( \frac{\pi}{2} \) and \( -\frac{3\pi}{2} \).

By the sign chart for \( g' \) and continuity of \( g \) on \([-2\pi, 2\pi]\), \( g \) increases on \([-2\pi, 2\pi]\). \( g \) has no local extrema in \((-2\pi, 2\pi)\).
• Find the **y-intercept** of the graph of \( y = g(\theta) \), if any.

Set \( \theta = 0 \) and **evaluate**: \( g(0) = 1 \). The y-intercept is at \( (0,1) \).

• \( g \) is neither even nor odd.

• The graph of \( y = g(\theta) \) has no holes, corners, or cusps. No graph of finite length can have asymptotes.

• Sketch the graph of \( y = g(\theta) \).

\[ g \text{ is continuous on } [-2\pi, 2\pi], \text{ so the EVT applies.} \]

The A.Min.Pt. is at: \( (-2\pi, g(-2\pi)) = (-2\pi, 1-2\pi) \).

The A.Max.Pt. is at: \( (2\pi, g(2\pi)) = (2\pi, 1+2\pi) \).

• In \( g(\theta) = \cos(\theta) + \theta \), the “\( +\theta \)” term appears to produce an upward drift in the graph, even “ironing out the bumps.”
FOOTNOTES

1. **Proof of the “Fencepost” Theorem.** Let’s say we have identified all “fenceposts” for a function \( f \) on the real number line. Consider any open interval \((a, b)\) containing no “fenceposts,” where \( a < b \). The lack of “fenceposts” means that \( f’(x) \) is real and nonzero, \( \forall x \in (a, b) \), and \( f’ \) is continuous on \((a, b)\). We will use proof by contradiction to show that \( f’ \) cannot change sign in \((a, b)\). Assume there exist \( x_1 \) and \( x_2 \) in \((a, b)\), where \( f’(x_1) \) and \( f’(x_2) \) have different signs and \( x_1 < x_2 \). Since \( f’ \) is continuous on \((a, b)\), it is also continuous on \([x_1, x_2]\). By the Intermediate Value Theorem (IVT), applied to \( f’ \), \( f’(c) = 0 \) for some \( c \in [x_1, x_2] \). That contradicts our assumption that \((a, b)\) has no “fenceposts,” since \( c \) would then be a “fencepost” in \([x_1, x_2]\) and therefore in \((a, b)\). Therefore, there cannot exist \( x_1 \) and \( x_2 \) in \((a, b)\), where \( f’(x_1) \) and \( f’(x_2) \) have different signs. \( f’ \) cannot change sign in \((a, b)\). Q.E.D.

2. **The sign of \( f’ \) around a CN - precise statements.** In Example 7, it is more precise to say that \( f’ \) is negative on a left-neighborhood of \( x = 1 \) and positive on a right-neighborhood of \( x = 1 \). \( f’ \) is positive on a left-neighborhood of \( x = 3 \) and negative on a right-neighborhood of \( x = 3 \). \( f’ \) is negative on a punctured (or deleted) neighborhood of \( x = 5 \).

3. **Converses of the statements in the First Derivative Test are false.** Point \( T \) in Section 4.1, Example 1 provides a counterexample.