Miscarceptions See AMATYC Fall 2002 p.25

CH 2: LIMITS

2.1: INTRO

Write → graph in during break

(A) Easy Exs

Ex If $f(x) = 3x^2 + x - 1$, find $\lim_{x \to 1} f(x)$

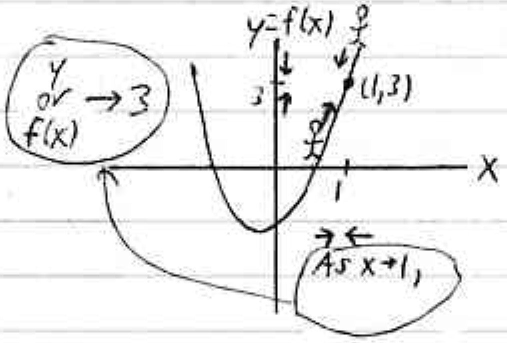
(What # does $f(x)$ approach as x approaches 1?)

Sol $f(x)$ = a poly.
Plug in $x = 1$ and evaluate!

$$\lim_{x\to 1} f(x) = f(1) = 3(1)^2 + (1) - 1 = 3$$

Note 1

Lovers running towards each other.



Note 2 Write: $\lim_{x\to 1} f(x) = 3$, or

$f(x) \to 3$ as $x \to 1$

$\underline{\text{Ex}}$ Find $\displaystyle\lim_{x \to 3} \underbrace{\left(\frac{2x+1}{x-2}\right)}_{f(x)}$

This isn't a poly func.

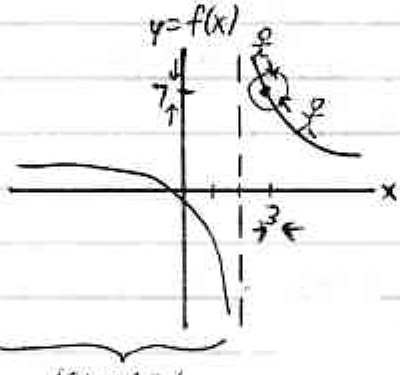
rational func. $\left(\frac{\text{poly}}{\text{poly}_{\neq 0}}\right)$ 3 is in domain ✓

$= f(3)$

$= \dfrac{2(3)+1}{(3)-2}$

$= 7$

$\underline{\text{Note}}$



limits are "LOCAL"

once you get beyond the asym.

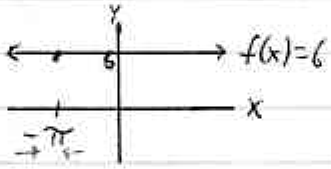
officially, cont at 3 is a consequence of lim = f(3)

We only care what happens "immediately around" x=3. LOCALLY!

How do you graph 6 as a func.

Up to ?

$\underline{\text{Ex}}$ Find $\displaystyle\lim_{x \to -\pi} 6 = 6$



Ⓑ Harder Exs

Is the limit f(3)?

Ex Find $\lim_{x\to 3} \underbrace{\dfrac{x^2-9}{x-3}}_{f(x)}$

This is a problem...

$f(3)$ is undefined!
Simplify 1st.

$$= \lim_{x\to 3} \frac{(x+3)\cancel{(x-3)}}{\cancel{x-3}}$$

If x=3, this is garbage. Enron stock

cancel as long as $x \neq 3$

$$= \lim_{x\to 3} (x+3)$$

Now plug in $x=3$

$$= 3+3$$

$$= 6$$

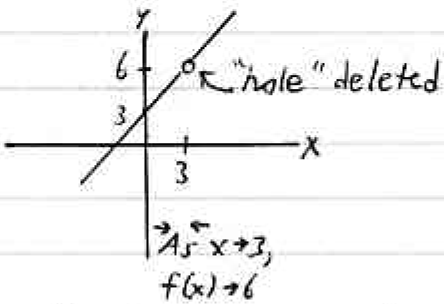
what is going on?

Note $\dfrac{x^2-9}{x-3} = x+3$ if $x \neq 3$

There's one diff

Our lovers approach this point, but they can't quite meet. We can be mean to them.

We'll come back to $3^{\pm}$ later.



x	f(x)	
2.9	5.9	As $x\to 3^-$
2.99	5.99	
3	und. OK!	6
3.01	6.01	
3.1	6.1	As $x\to 3^+$

Up to 2.1
How to Ace
Fan, guy's note

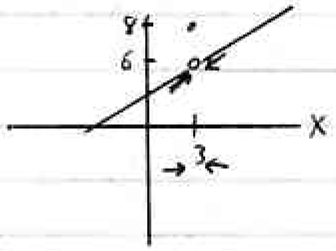
We technically don't care what happens at $x=3$!!

There's nothing inherently wrong w/ this func. No one said rule had to be a 1-liner. Very common in limit probs.

<u>Ex</u> If $f(x) = \begin{cases} \frac{x^2-9}{x-3} & \text{if } x \neq 3 \\ 8 & \text{if } x = 3 \end{cases}$

<u>"piecewise-defined" function</u>

find $\lim_{x\to 3} f(x) = \boxed{6}$, even though $f(3)=8$!!



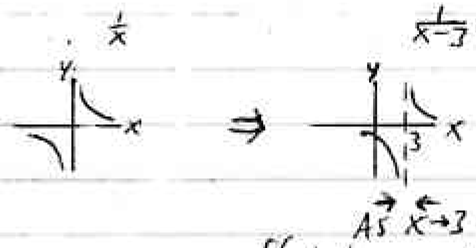
what is f(3)? what is the value of the func at 3? What is the lim? f(3) helped when we were dealing w/ polys, $\lim_{x\to3}$. Technically, it's not supposed to matter.

(Here, $f(3)$ is irrelevant!)

<u>Ex</u> Find $\lim_{x\to 3} \underbrace{\frac{x-3}{x^2-6x+9}}_{f(x)}$

$= \lim_{x\to 3} \frac{x-3}{(x-3)^2}$

$= \lim_{x\to 3} \frac{1}{x-3}$ can cancel if $x \neq 3$



f(x) does not approach a single real #

£ goes to hell He's Ken Lay

Can up to 23

This limit does not exist (DNE)

© One-Sided Limits

Left-hand limit

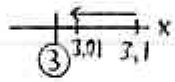
$$\lim_{x \to a^-} f(x)$$

as x approaches a from the left (lower #s)

Right (hand limit)

$$\lim_{x \to a^+} f(x)$$

as x " " right (higher #s)



Two-sided limit

$\lim_{x \to a} f(x)$ exists $\xleftrightarrow{\text{if and only if}}$ these both exist (as real #s) and are =

In particular

Can write $\longleftrightarrow$ $\lim_{x \to a^-}$ " "

$$\boxed{\lim_{x \to a} f(x) = L \iff \lim_{x \to a^-} f(x) = L \text{ and } \lim_{x \to a^+} f(x) = L}$$

a real #

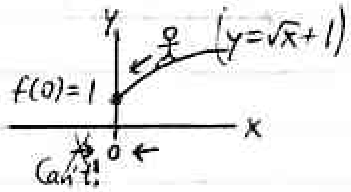
Multivar Calc.

Ex Let $f(x) = \sqrt{x} + 1$

What values for x will give me a nice real # here?

$$\text{Domain} = \{x \mid x \geq 0\} = [0, \infty)$$

What y coord. are we approaching?



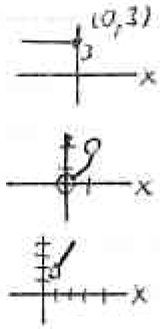
$$\lim_{x \to 0^+} f(x) = 1$$

$$\lim_{x \to 0^-} f(x) \quad \text{DNE}$$

$$\lim_{x \to 0} f(x) \quad \text{DNE}$$

Ex

$$\text{Let } f(x) = \begin{cases} 3 & \text{if } x \leq 0 \\ 2x^2 & \text{if } 0 < x < 1 \\ 2x & \text{if } x > 1 \end{cases}$$



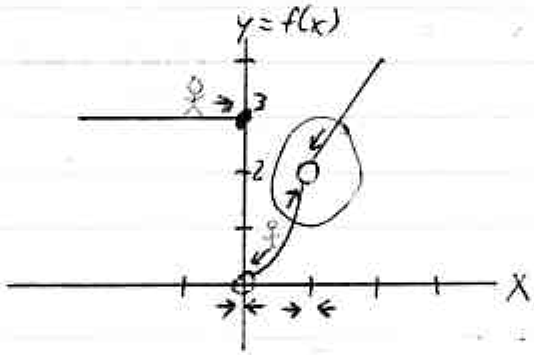
(0,3) included

Endpoints (0,0) and (1,2) excluded.

(Endpoint (1,2) still excluded.)

In this set, we just miss 0 and 1, but we can get the endpts of this piece of parabola when we plug in $x=0$ and $x=1$

If $x=1$ were included the func value would be 2. Superimpose Ignore stubborn guy 3 all his life.



$$\lim_{x \to 1^-} f(x) = 2$$

$$\lim_{x \to 1^+} f(x) = 2$$

$$\lim_{x \to 1} f(x) = 2$$

same real #!

$$\lim_{x \to 0^-} f(x) = 3$$

$$\lim_{x \to 0^+} f(x) = 0$$

$$\lim_{x \to 0} f(x) \quad \text{DNE}$$

$\neq$