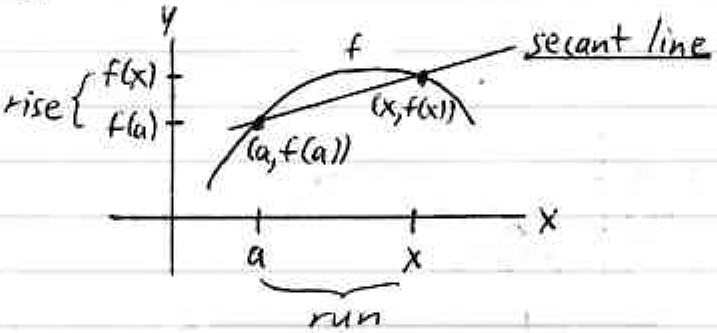
3.1: TANGENT LINES and RATES OF CHANGE

(A) Tangent Lines

familiar?
Diff of outputs over diff of inputs
secare = to cut
$y=f(x)$, $x=a$
what are the coords?

if I draw this line, what is $f(x)-$

$$\frac{f(x)-f(a)}{x-a} \qquad \text{Difference quotient}$$



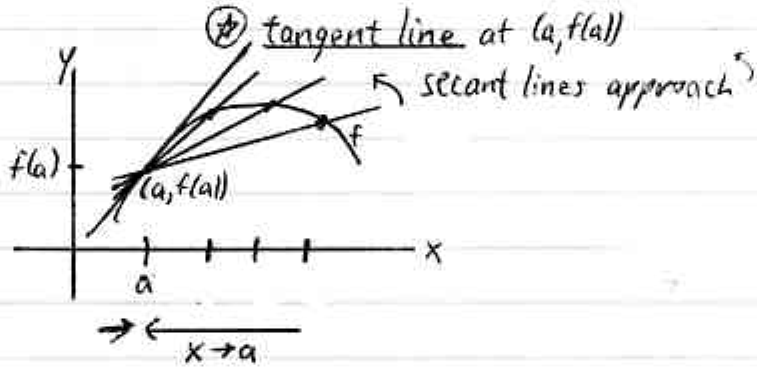
$= \dfrac{\text{rise}}{\text{run}}$

$=$ slope of secant line

Algebra → 2 pts.
Calculus! → slope!

"limit line"

What happens as $x \to a$?



If we repeat this process on the left, we get the same line.

The slope of ✪ is

$$\boxed{m_a = \lim_{x \to a} \overbrace{\frac{f(x)-f(a)}{x-a}}^{\text{slopes of secant lines}} \text{ if it exists}}$$

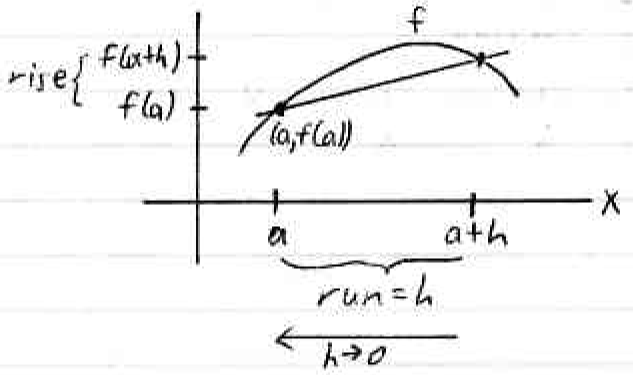
(Otherwise, undefined.

Derivatives are slopes.

$m_a = f'(a)$, the derivative of f at a. (3.2)

<u>Alternative approach</u> (More common)

Let $x = a+h$



The world revolves around a.

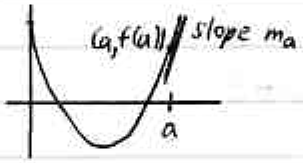
$$\frac{\text{rise}}{\text{run}}$$

$$m_a = \lim_{h\to 0} \overbrace{\frac{f(a+h)-f(a)}{h}}^{} \quad \text{if it exists}$$

Take lim as ?→? Then, "h→0"

<u>Ex</u> $f(x) = x^2 - 5x + 4$, Find m_a.

x-ints 1, 4
y-int 4
V $(2\tfrac{1}{2}, -2\tfrac{1}{4})$



$\frac{1}{3}$ - $\frac{1}{2}$ have seen
a: fixed arbit. constant
In 3.2, $a \leftarrow x$

$$m_a = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{[(a+h)^2 - 5(a+h) + 4] - [a^2 - 5a + 4]}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{a^2} + 2ah + h^2 \cancel{- 5a} - 5h \cancel{+ 4} \cancel{- a^2} \cancel{+ 5a} \cancel{- 4}}{h}$$

$$= \lim_{h \to 0} \frac{2ah + h^2 - 5h}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2a + h - 5)}{\cancel{h}}$$

cancel $(h \neq 0)$

$$= \lim_{h \to 0} (2a + \underset{\downarrow \atop 0}{h} - 5)$$

$$= 2a + 0 - 5$$

$$= 2a - 5$$ Slope formula

Handy formula for slope.

(What is the slope of the tangent line at the point w/x-coord a? $2a - 5$.
10? 15.
100? 195.)

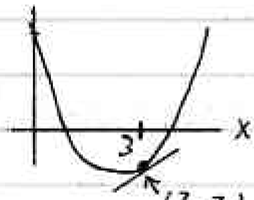
a: specific value for x

Case $a=3$ $(x=3)$

$$f(x)=x^2-5x+4$$
$$f(3)=(3)^2-5(3)+4$$
$$=-2$$

why do I care if I want the tan line at 3?

I'm not going to trick you w/pt. not on graph.



Find an eq. of the tangent line at (3,-2).

Slope: $m_a=2a-5$

$m=m_3=2(3)-5=1$

Point-slope form

$$y-y_1=m(x-x_1)$$
$$y-(-2)=(1)(x-3)$$
$$y+2=x-3$$

$\Rightarrow$ Slope-int. form

$$y=x-5$$

Another approach:

We need to know m, b.
we already know

Model
force things through to find unknown parameter

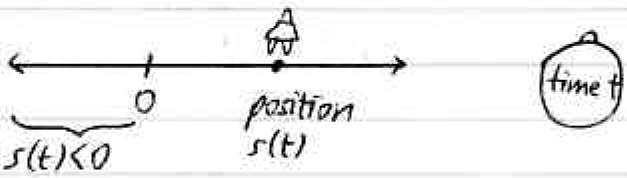
$$y = mx + b$$
$$(-2) = (1)(3) + b$$
$$b = -5$$
$$y = (1)x + (-5)$$
$$y = x - 5$$

(The graph of an eq. consists of all pts. whose coords. satisfy the eq.)

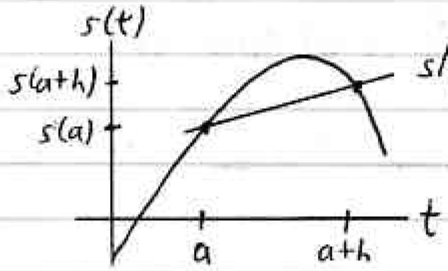
Ⓑ Rectilinear Motion

- along a line

My idea of a car
tank!
stellar
Dave Letterman



Be precise!



slope = average velocity on $[a, a+h]$

$= \frac{\text{change in } s}{\text{change in } t}$

Why?

Idea: $d = r\,t$
distance rate time

$$r = \frac{d}{t}$$

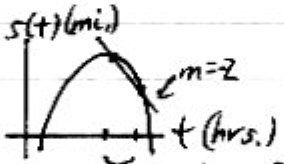
Similarly,

$$\text{avg. vel.} = \frac{\text{change in } s}{\text{change in } t}$$

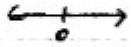
$$= \frac{s(a+h)-s(a)}{h}$$

$$= \text{slope of secant line}$$
(can be <0)

Ex



avg. vel. $= -2 \frac{\text{mi.}}{\text{hr.}}$
on this interval



(If car ← and →
[avg. vel.] ≠ avg. speed
avg. speed] = 2 here
bec. 1 direction)

Which way

[instantaneous] velocity at time a

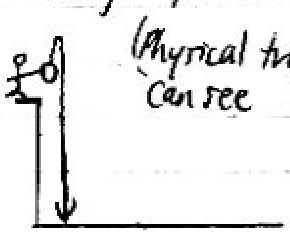
$$= \lim_{h \to 0} \frac{s(a+h)-s(a)}{h} \quad \text{if it exists}$$

Projectile

Balloon
If you're mean,
a cannonball
textbook

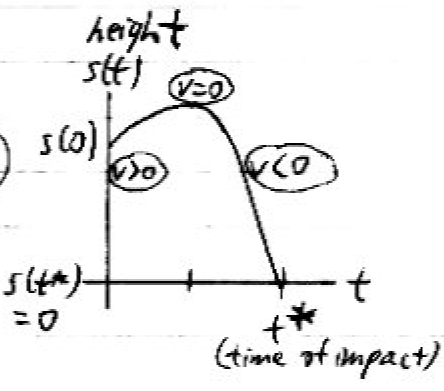
We'll be solving
these later on
↓ is vel + or (−)?
−3 mph
−4 mph hurts more
when hits first, but...
Up to 15

straight up/down



(Physical trajectory)
can see

VS.



Write during break!

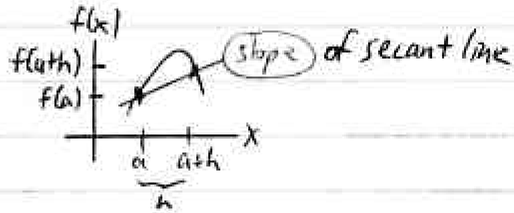
© Rates of Change

don't say distance bec. we have the sign issue.

Ex Velocity = rate of change of position with respect to (wrt) time.

① Average rate of change of $y=f(x)$ wrt x on $[a, a+h]$

$$= \frac{f(a+h)-f(a)}{h}$$

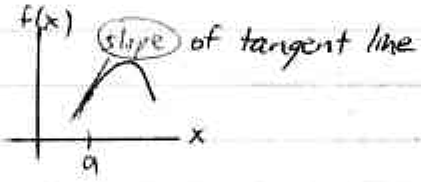


② (Instantaneous) rate at a

One modification term

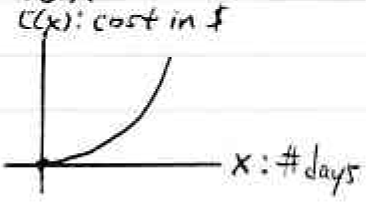
$$= \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$$

$$= f'(a)$$



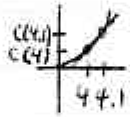
Ex $C(x) = 10x^2$

evil CEO draining co. like a vampire works 24 hrs. a day



0231 FBI monitoring equipment

Avg. rate of change of C wrt x on $[4, 4.1]$



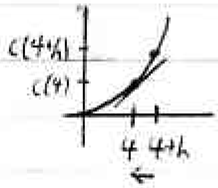
$$= \frac{C(4.1)-C(4)}{4.1-4}$$

$$= \frac{10(4.1)^2-10(4)^2}{0.1}$$

$$= 81 \text{ \$/day}$$

Inst. rate of change of C wrt x at 4

$$= \lim_{h \to 0} \frac{C(4+h) - C(4)}{h}$$



$$= \lim_{h \to 0} \frac{10(4+h)^2 - 10(4)^2}{h}$$

$$= \lim_{h \to 0} \frac{10(16+8h+h^2) - 160}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{160} + 80h + 10h^2 - \cancel{160}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(80+10h)}{\cancel{h}}$$

$$= \lim_{h \to 0} (80 + 10h)$$

$$= 80 \text{ \$/day}$$

Close to what we had before. $h = 0.1$ unless $C(x) = 1000x^{1M}$ avg. rate of change $\approx$ inst.

If table of values, a way to approx. deriv (use small h)