3.3: TECHNIQUES OF DIFFERENTIATION

Ⓐ Basic Rules

$h(x)$	$h'(x)$ (or $D_x h(x)$)	
c	0	
$mx+b$	m	
x^n	nx^{n-1}	(Power Rule)
$cf(x)$	$cf'(x)$	(Constant factors "pop out.")
$f(x)+g(x)$	$f'(x)+g'(x)$	(Can D_x term-by-term)
$-$	$-$	

Man overboard: $n-1$ laughing passengers left

new: D_x is a **linear operator** (like lim)

(where f, g are diff'e)

Ex ⓐ Find $D_x \underbrace{(-4x^3+6x-4)}_{f(x)}$

$$= D_x(-4x^3) + D_x(6x) - \cancel{D_x(4)}^{\,0} \quad \text{(can skip)}$$

$$= -12x^2 + 6 \leftarrow f'(x)$$

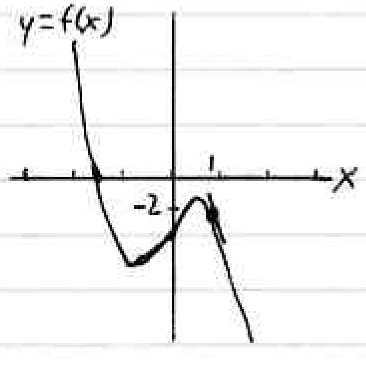
The constant term doesn't matter. Even if we have vertical shifts, the slope situation along the graph stays the same. The deriv. tells you about the slope situation along the graph.

same f' as in ⓐ

ⓑ $f(1) = -2$ ($\Rightarrow (1,-2)$ on graph of f)
Find an eq. of the tangent line at $(1,-2)$.

I'd have to be (kind of) a jerk to ask for a tan line at a pt. not on graph.

Turns out



I have a pt. on the tan line. What will give me the slope?

$$\text{Slope} = f'(1)$$
$$= \left[-12x^2+6\right]_{x=1} \text{ (evaluate at)}$$
$$= -12(1)^2+6$$
$$= -6$$

<u>Pt.-Slope form:</u>

$$y - y_1 = m(x - x_1)$$
$$y-(-2) = -6(x-1)$$
$$y+2 = -6(x-1)$$

or

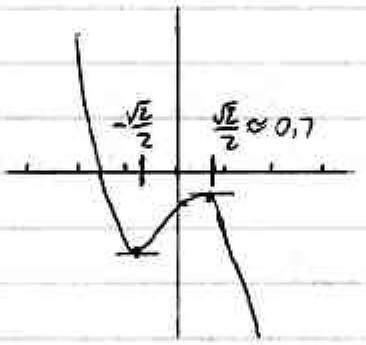
<u>Slope-Int. Form</u> $y = -6x+4$ ($y = mx+b$ Solve for b)

The graph of an eq. consists of all pts. whose coords satisfy the eq. $(1,-2)$ $m=-6$

ⓒ Solve $f'(x)=0$, or $D_x y = 0$

$$-12x^2+6=0$$
$$x^2 = \frac{1}{2}$$
$$x = \pm\sqrt{\frac{1}{2}} \quad \left(x = \pm\frac{\sqrt{1}}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}\right)$$
$$x = \pm\frac{\sqrt{2}}{2}$$

x can be what? Careful! Why do we care about when deriv=0? What kind of tan line?



(B) Product Rule

Is the deriv. of the sum = sum of derivs (if derivs exist?) yes.

$(fg)' = f'g'$ if f or $g = 0$; f and g are consts.

$$(fg)' \neq f'g' \text{ usually}$$

$$(fg)' = f'g + fg'$$

Could do using old methods: could distribute like crazy

Ex Find $D_x[(x^4+1)(x^2+4x-5)]$

Wag my finger

$$\begin{array}{ccc} & \wedge & \text{copy} \\ + & \text{copy} & \wedge \end{array} \Bigg\} \text{"Pointer Method"} \quad \wedge : D_x$$

$$= (4x^3)(x^2+4x-5) + (x^4+1)(2x+4)$$

OK to stop?

or

$$6x^5 + 20x^4 - 20x^3 + 2x + 4$$

Book does this – to me, it defeats the purpose if you want to use the product rule w/ polys.

Ex Find $D_x[(x+4)(x^2+2)(\sqrt[3]{x}-x)]$

or $(x^{1/3}-x)$

Grid: Slab of toxic waste along diagonal: D_x

$$\begin{array}{cccc} & \wedge & \text{copy} & \text{copy} \\ + & \text{copy} & \wedge & \text{copy} \\ + & \text{copy} & \text{copy} & \wedge \end{array} \Bigg\} \text{ For each term, } D_x \text{ one factor, copy the others.}$$

$$\begin{aligned} = &(1)(x^2+2)(\sqrt[3]{x}-x) \\ &+ (x+4)(2x)(\sqrt[3]{x}-x) \\ &+ (x+4)(x^2+2)\left(\tfrac{1}{3}x^{-\frac{2}{3}}-1\right) \end{aligned}$$

Normally, "the dreaded..."

Me Product + Quotient (Tricky) $h = \frac{f}{g}$, $gh = f$, $\Rightarrow h' = \ldots$ Analytic Fall 2007 p. 2

J-Lo (4th min. If err, off by a sign

How to Ace: HiD(Ho) -Chucky

© Quotient Rule

$$\left(\frac{f}{g}\right)' = \frac{\text{Lo D(Hi)} - \text{Hi D(Lo)}}{\text{the square of what's below } (\text{Lo})^2}$$

rhyme

$$= \frac{gf' - fg'}{g^2}$$

Hi $= f$
Lo $= g$

$(7x-3)(3x^2+1)^{-1}$ Need chain rule

Ex Find $D_x\left[\frac{7x-3}{3x^2+1}\right]$

$$\left(= \frac{\text{Lo D(Hi)} - \text{Hi D(Lo)}}{(\text{the square of what's below})} \leftarrow (\text{Lo})^2 \right)$$

$$= \frac{(3x^2+1)D_x(7x-3) - (7x-3)D_x(3x^2+1)}{(3x^2+1)^2}$$

$$= \frac{(3x^2+1)(7) - (7x-3)(6x)}{(3x^2+1)^2}$$

$\leftarrow$ Simplify (On test, maybe / maybe not)

$\vdots$

$$= \frac{-21x^2 + 18x + 7}{(3x^2+1)^2}$$

$f(x) = 1$

Don't bother? - quotient rule is suff. later - chain rule show this anyway. can't do $D_x(g^{-1})$ yet need Chain Rule!

Special Case $\left(\frac{1}{g}\right)' = -\frac{g'}{g^2}$ (Reciprocal Rule)