3.4: DERIVATIVES OF TRIG FUNCS.

(A) Prelims

You must make sure of something on your calc.

$\sin x$, etc.

must be $\boxed{RAD}$

Who said 360° in ○ Arbitrary! Babylonian? Was it ... "PAL"? Pts. in book

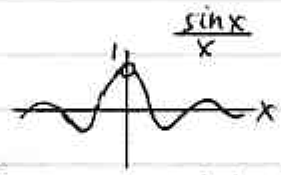
Need $\boxed{RAD}$ for →

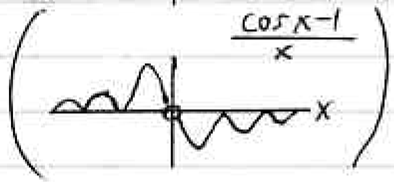
$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x\to 0} \frac{\cos x - 1}{x} = 0$$

Take these on faith.

(Book: $\lim_{\theta\to 0} \frac{1-\cos\theta}{\theta} = 0$.)

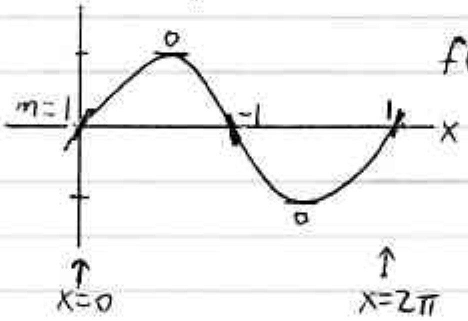




We know from (Squeeze Thm) (what?) that $\lim_{x\to\infty} \frac{\sin x}{x} = 0$

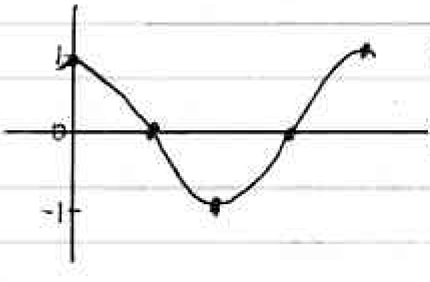
Derivs are slopes (of tan lines.) tangents cut thru concavity

Deriv. should be periodic.

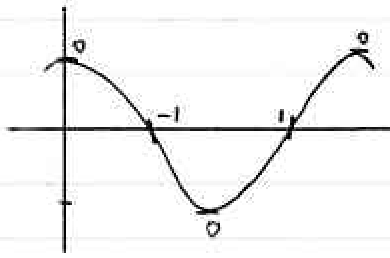


$f(x) = \sin x$:
period $= 2\pi$

What do you think? Periodic func.

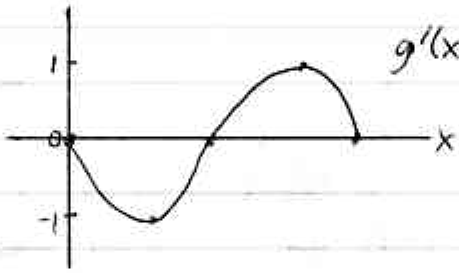


$f'(x) = \cos x$:
period $= 2\pi$



$g(x) = \cos x$

Deriv. is not $\sin x$, it's...



$g'(x) = -\sin x$

(B) Proofs (fair!)

(1) Prove $D_x(\sin x) = \cos x$

What's the lim def'n of f'?

or $\lim\limits_{h\to 0} \dfrac{\sin(x+h) - \sin(x-h)}{2h}$

$= \lim \dfrac{2\sin h \cos x}{2h}$ $\to 1$

don't need $\dfrac{\cos h - 1}{h} \to 0$!!

$$f(x) = \sin x$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\overbrace{\sin(x+h)}^{=\text{sum of mixed up products}} - \sin x}{h}$$

$$= \lim_{h\to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

collect $\sin x$ terms

$$= \lim_{h\to 0} \frac{(\sin x \cos h - \sin x) + \cos x \sin h}{h}$$

$$= \lim_{h\to 0} \frac{\overset{\text{Factored out}}{\sin x}(\cos h - 1) + \cos x \sin h}{h}$$

$$= \lim_{h\to 0} \left[\sin x \underbrace{\left(\frac{\cos h - 1}{h}\right)}_{0} + \cos x \underbrace{\left(\frac{\sin h}{h}\right)}_{1} \right]$$ Collect "h stuff."

$$= \cos x$$

② Prove $D_x(\cos x) = -\sin x$

$$f(x) = \cos x$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\overbrace{\cos(x+h)}^{=\text{cosines} - \text{sines}} - \cos x}{h}$$

$$= \lim_{h\to 0} \frac{\cos x \cos h - \sin x \sin h - \cos x}{h}$$ Collect cos x terms

$$= \lim_{h\to 0} \frac{(\cos x \cos h - \cos x) - \sin x \sin h}{h}$$

$$= \lim_{h\to 0} \frac{\overset{\text{Factored out}}{\cos x}(\cos h - 1) - \sin x \sin h}{h}$$

$$= \lim_{h\to 0} \left[\cos x \underbrace{\left(\frac{\cos h - 1}{h}\right)}_{0} - \sin x \underbrace{\left(\frac{\sin h}{h}\right)}_{1} \right]$$ Collect "h stuff."

$= -\sin x$

Keep 'em in suspense

③ Prove $D_x(\tan x) = \sec^2 x$

Use what?

$$D_x(\tan x) = D_x\left(\frac{\sin x}{\cos x}\right)$$ Quotient Rule

p.122

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} \leftarrow 1$$

$$= \sec^2 x \checkmark$$

④ Prove $D_x(\sec x) = \sec x \tan x$

$$D_x(\sec x) = D_x\left(\frac{1}{\cos x}\right)$$ Quotient or Reciprocal Rule: $\left(\frac{1}{g}\right)' = -\frac{g'}{g^2}$

$$= \cancel{-}\frac{\cancel{-}\sin x}{\cos^2 x}$$

p.122

$$= \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}$$

$$= \sec x \tan x \checkmark$$

In HW:

⑤ Prove $D_x(\cot x) = -\csc^2 x$

⑥ Prove $D_x(\csc x) = -\csc x \cot x$

Unless you want to keep reproving O↔A

© Memorize!

sin x and cos x are a pair of cofunctions.
tan x and cot x
sec x and csc x

sin, cos

Cover up 2nd column to review.

$D_x(\sin x) = \cos x$ $\quad\xrightarrow{\text{Cofunc.}}\quad$ $D_x(\cos x) = -\sin x$

$D_x(\tan x) = \sec^2 x$ $\quad\quad$ $D_x(\cot x) = -\csc^2 x$

$D_x(\sec x) = \sec x \tan x$ $\quad\quad$ $D_x(\csc x) = -\csc x \cot x$

$D_x(\cos x)$, too.

Memorize

"−", and
Replace each "piece" with its cofunction.

ⓓ Exs

Ex Find $D_x(x^2 \sec x + 3\cos x)$

$$= D_x(x^2 \sec x) + \underbrace{D_x(3\cos x)}_{3D_x(\cos x)}$$

^ copy
+ copy ^
(Product Rule)

$$= (2x)(\sec x) + (x^2)(\sec x \tan x) + 3(-\sin x)$$

$$= 2x\sec x + x^2 \sec x \tan x - 3\sin x$$

If your answer looks diff from back of book ⇒ not necess. wrong!

Trig Ids. → Other Forms?

What if $1-\sin^2 w$ Simp. 1st Rul second

Ex (#14) $R(w) = \dfrac{\cos w}{1-\sin w}$

Find $R'(w)$.

Quotient Rule

$$R'(w) = \frac{Lo\, D(Hi) - Hi\, D(Lo)}{(Lo)^2}$$

$$= \frac{(1-\sin w) D_w(\cos w) - (\cos w) D_w(1-\sin w)}{(1-\sin w)^2}$$

$$= \frac{(1-\sin w)(-\sin w) - \cos w(-\cos w)}{(1-\sin w)^2}$$

Are we done.

$$= \frac{-\sin w + \overbrace{\sin^2 w + \cos^2 w}^{=1}}{(1-\sin w)^2}$$

$$= \frac{1-\sin w}{(1-\sin w)^2}$$

$$= \frac{1}{1-\sin w}$$

Mart simp: They don't know Chain Rule yet!

If I graph f(x) do I get a nice smooth line? We have these domain issues lurking in the background

$D_x = 0$ on domain of f
What do we call discnts.? Up to 27 Removable

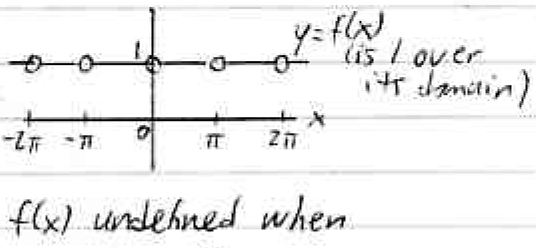
Ex Find $D_x(\underbrace{\sin^2 x \csc^2 x}_{f(x)})$

Simplify 1st!

$$= D_x\left(\cancel{\sin^2 x} \cdot \frac{1}{\cancel{\sin^2 x}}\right)$$

$$= D_x(1)$$

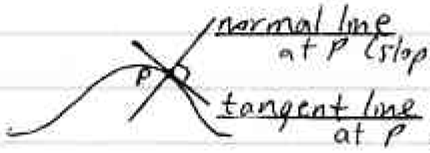
$$= 0$$



f(x) undefined when $\sin x = 0$ ($\Leftrightarrow \csc x$ und.)

(E) Tangent Lines

They tngt: $-\frac{1}{m}$



Ex $f(x) = 2\sin x - x$

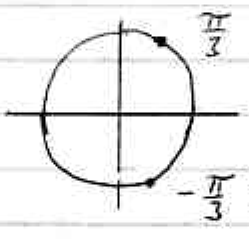
(a) Find the x-coords. of all pts. with a horizontal tangent line.

For what x is $f'(x) = 0$?

$$f(x) = 2\sin x - x$$

$$f'(x) = 2\cos x - 1 \overset{\text{set}}{=} 0$$

$$\cos x = \frac{1}{2}$$



must be in $\boxed{\text{RAD}}$

Can a (non-const.) poly have ∞ many horiz tan lines? NO

sin x, 2 sin x, ...

If x>0, or x↑, what does −x do to the graph? Guess!

sin x

2 sin x

−x depreciate

[−4π, 4π]

odd func.

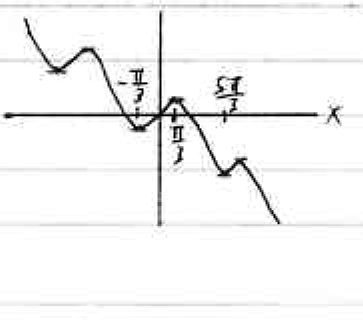
$x = \frac{\pi}{3} + 2\pi n$

$x = \frac{5\pi}{3} + 2\pi n$

n integer

or $x = \pm\frac{\pi}{3} + 2\pi n$, n int

Turns out



⑥ $f(x) = 2\sin x - x$

Note

$$f\left(\tfrac{\pi}{6}\right) = 2\sin\tfrac{\pi}{6} - \tfrac{\pi}{6}$$
$$= 2\left(\tfrac{1}{2}\right) - \tfrac{\pi}{6}$$
$$= 1 - \tfrac{\pi}{6}$$

Find eq. of tan line at $\left(\frac{\pi}{6}, 1-\frac{\pi}{6}\right)$

$$y - y_1 = m(x - x_1)$$

$$y - \left(1 - \tfrac{\pi}{6}\right) = \underset{f'\left(\frac{\pi}{6}\right)}{m}\left(x - \tfrac{\pi}{6}\right)$$

$$f'(x) = 2\cos x - 1$$
$$f'\left(\tfrac{\pi}{6}\right) = 2\cos\left(\tfrac{\pi}{6}\right) - 1$$
$$= 2\left(\tfrac{\sqrt{3}}{2}\right) - 1$$
$$= \sqrt{3} - 1$$

$$y - \left(1 - \tfrac{\pi}{6}\right) = \left(\sqrt{3} - 1\right)\left(x - \tfrac{\pi}{6}\right)$$

Blowing up previous graph
$\frac{\pi}{6} \approx .524$
$1 - \frac{\pi}{6} \approx .476$
$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Turns out