Most Valuable Thm? Rarely "see", but used in proofs, FTC! e.g., developing arc length Larson (ES: some think most imp. thm.

4.2: MEAN VALUE THEOREM (MVT)

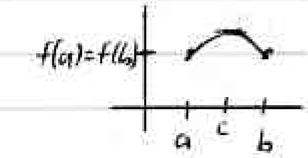
pron. rawls

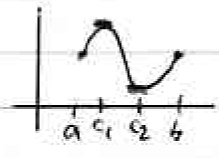
(A) Special Case: Rolle's Thm.

Assuming f is cont., What goes up.. must come down.

When is f *guaranteed* to have a critical # (CN)?

Look at all these CNs! (shake cable) (I've looked at your tests... Hope I don't hurt myself.)





My cable!

Our CNs can't have f' be DNE. ←

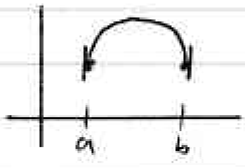
Thm If (1) f is cont. on $[a,b]$,
(2) f is diff'e on (a,b), and
(3) $f(a)=f(b)$
} hypotheses

Why not $[a,b]$? (a,b) more powerful. At a, b, only one sided derivs.

Then, $f'(c)=0$ for some c in (a,b). ← conclusion
(Maybe more than one!)
(i.e., ∃ horiz. tangent line somewhere in there)

Note Why not "diff'e on $[a,b]$"?

would exclude this Ex. (Do you remember a bounded curve where we had vertical tan lines at the ends?)



Rolle OK!
(We want Rolle's Thm. to be powerful enough to apply to these cases, too.)

If we have DNE somewhere, FINE! (1) ensures that that # is in Dom(f)

(Note 2 If (2) fails, we get critical #s (CNs) where f' is DNE. ~~Y~~)

If (2) fails, we get crit #s where f' DNE ~~Y~~

Ex Consider $f(x)=x^4-18x^2$ on $[-4,4]$

(1),(2) $f(x)$ = poly. → f is con't $[-4,4]$ + diff'e $(-4,4)$ (everywhere)

(3) $f(-4)=(-4)^4-18(-4)^2=-32$
$f(4)=(4)^4-18(4)^2=-32$ ⇐ =
} or f is even (-4, 0, 4 sym.)

So, Rolle's Thm. applies.

Find all #s "c" in $(-4,4)$ such that $f'(c)=0$.

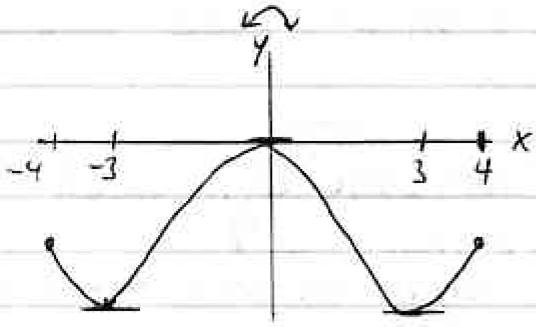
Back of book uses c

$$f(x) = x^4 - 18x^2$$
$$f'(x) = 4x^3 - 36x$$
$$= 4x(x^2-9)$$
$$= 4x(x+3)(x-3) \overset{\text{set}}{=} 0$$

$0, -3, 3$ ← all in $(-4,4)$ ✓

Turns out

f even, diff'e → We have to have either a turnaround pt. or a flat graph at 0. corresp. to a constant func. around there



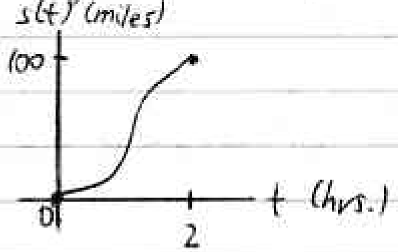
f even

Could do 3, 5, 7, not 1
[illegible]

(B) MVT in General

Christine
It drives itself
What can you say about the vel.?
No Back to the Future stunts
whoosh → disappears → continuity

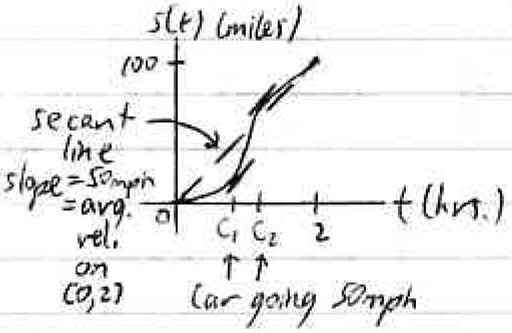
Ex A car goes 100 miles in 2 hours. → (In one direction)



s is cont. on $[0,2]$
diff'e on $(0,2)$

What's the slope of secant line?
What do you think the MVT allows us to conclude? If the avg...
Speedometer must hit 50.
It tell scenario "go 100 mi. in 2 hrs. You die if 50mph, die if don't make it on time" Goodbye...
At how many times is

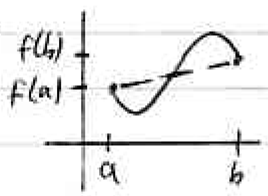
Average (mean) velocity on $[0,2]$ is 50 mph.
MVT ⇒ The car must go 50mph at some instant in $(0,2)$.



Where is tangent line ∥ secant line?
parallel to
(same slope)

$f'(c_1) = 50$ mph
c_2

(MVT)
Thm



If f is
① cont. on $[a,b]$, and
② diff'e on (a,b)
} (Often satisfied!)

Then $\exists$ c in (a,b) such that...

familiar

$$f'(c) = \underbrace{\text{slope of secant line}}_{\dfrac{f(b)-f(a)}{b-a}}$$

Bec. looks like 1-liners!

$$\text{or} \quad f(b)-f(a) = [f'(c)][b-a]$$

<u>Special Case</u>: If $f(b) = f(a)$, $\exists c$ in (a,b):

$$f'(c) = \frac{\overset{0}{f(b)-f(a)}}{\underset{\neq 0}{b-a}} = 0$$

Rolle's Thm is used in proof

[illegible] perspective

$\sim \to \sim$ [illegible] a function

Rolle  (Again, we look for where tangent line $\parallel$ secant line.)
horiz.

<u>Ex</u> Consider $f(x) = x^4 - x$ on $[2,5]$

f is $\underset{[2,5]}{\text{cont.}}$, $\underset{(2,5)}{\text{diff'e}}$ $\Rightarrow$ MVT applies.

Find a "c" in $(2,5)$ such that ...

$$f(b)-f(a) = [f'(c)][b-a]$$
$$f(5)-f(2) = [f'(c)]\underbrace{[5-2]}_{=3}$$
$$f(5) = (5)^4 - (5) = 620$$
$$f(2) = (2)^4 - (2) = 14$$
$$620 - 14 = [f'(c)][3]$$
$$\underline{f'(c) = 202}$$

$$f'(c) = \frac{f(b)-f(a)}{b-a} = \frac{f(5)-f(2)}{5-2} = \frac{620-14}{3} = 202$$

For what c is $f'(c)=202$?

$$f(x)=x^4-x$$
$$f'(x)=4x^3-1 \overset{\text{set}}{=} 202$$

$$x^3=\frac{203}{4}$$

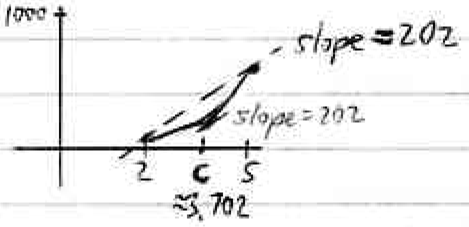
$203 \div 4 =$
$\boxed{A}$ .333.. $\boxed{=}$
$1 \boxed{a^{b}/c} 3$

$$x \text{ or } c = \sqrt[3]{\frac{203}{4}} \approx 3.702 \text{ in } (2,5) \checkmark$$

Turns out

Ain't no inflection on this interval

Draw tan line 1st

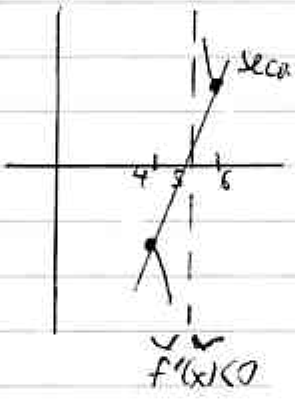


Ex Consider $f(x)=\frac{1}{x-5}$ on $[4,6]$

MVT might not apply but concl. may work out.

f is discont. at $5 \Rightarrow$ MVT does not apply

Are there any pts in here w/a ∥ tangent line?



$$f(x)=(x-5)^{-1}$$
$$f'(x)=-(x-5)^{-2}(1)$$
$$=-\frac{1}{(x-5)^2}$$
$$<0 \quad \forall x \neq 5$$

∥ tangent line is possible, but not guaranteed!

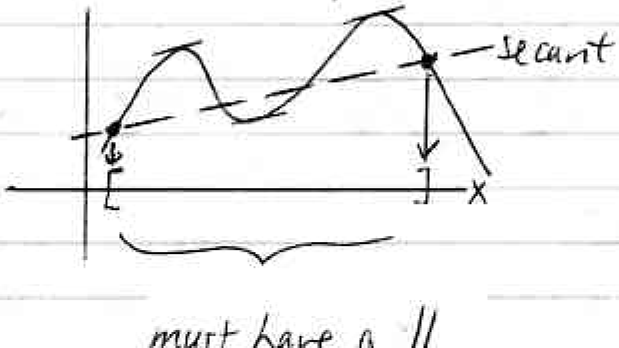
That's not the neat part.

Corollary of MVT (Neat!)

(implies cont. everywhere)

If f is diff'e everywhere, pick any 2 distinct points on graph

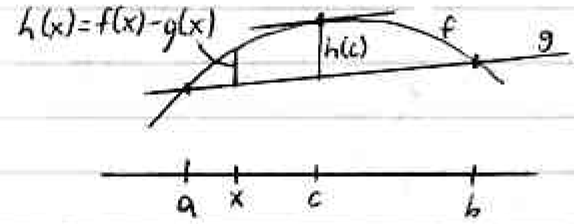
what's all this junk about closed intervals? Cooked up?



Here's your closed interval.

must have a $\parallel$ tangent line in here! (MVT applies)

MVT Proof Idea



$h(a)=0$
$h(b)=0$
h cont. on $[a,b]$
diff'e on (a,b)
(bec. f, g are)

$\Rightarrow$ Apply Rolle's Thm. to h on $[a,b]$

$\Rightarrow \exists c$ in (a,b): $h'(c)=0$

i.e., $f'(c)-g'(c)=0$

$f'(c)=g'(c)$

slope of secant line

Note: c is where there's max. dist. bet f, g (in the leaf)