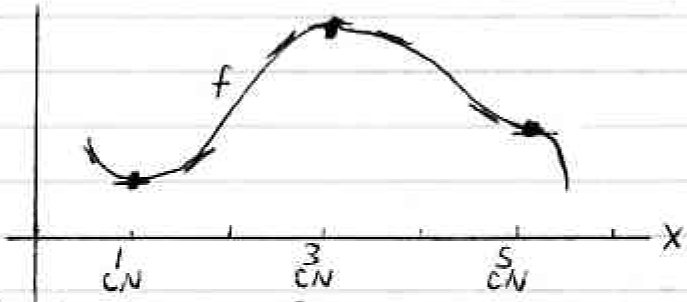
4.3: THE FIRST DERIVATIVE TEST (1st DT)

Ⓐ Where is f Increasing (↗), Decreasing (↘)?

Ex



Gotta be defined on (1, 3)
No

This breaks it decreasing

$f'(x)$: − 0 + 0 − 0 − (Slopes of tangent lines)

f can ↗ anyway; maybe

Look → f′ ove

Point on graph when describing f ↗.

$f'(x) > 0$ (+) on (1, 3) (interval), so f ↗ on (1, 3) or [1, 3] (f cont.) (Pts. to the right are higher than pts. to the left.)

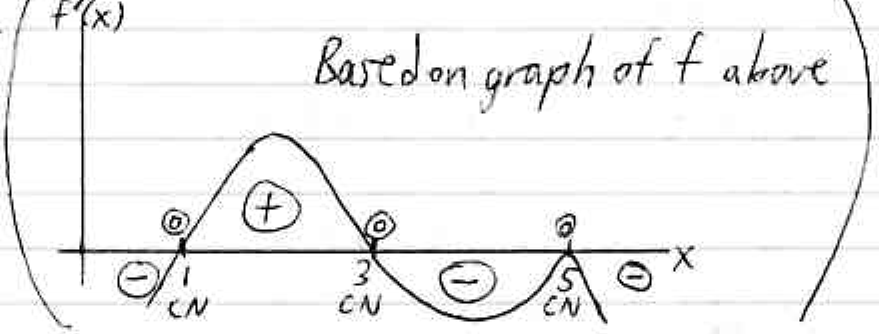
What do we mean by "increasing"?
(In [1, 3], if $d > c$, then $f(d) > f(c)$.)

$f'(x) < 0$ (−) on (3, 5), so f ↘ on (3, 5)

"Test Value" Idea

Based on f ↗

Note ($f'(x)$ Based on graph of f above)



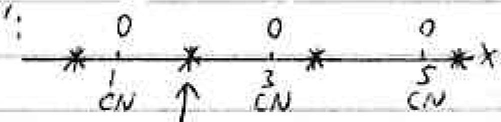
Gelbaum 36
f′ ∃ but discont. at 0
$f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$
We sort of reverse IVT.
IVT ⇒ if f′ cont., change sign bet. a, b, then 0 in between.

The CNs (and where f′ discont.) are the only places where f′ can change sign.

Does it have to change sign? Look at 5.

Let's say I didn't have any graphs, but I knew the 0s are at 1,3,5.
Book calls f'(k) the test value

f':



To completely analyze the sign of f', we just have to sample the sign at these 4 places.
∞ problem → finite problem

The sign of f' here is the same throughout $(1,3)$.
Can test $x = 2, 1.1, 2.5, \ldots$

What's the graph of that? Don't use that! (Precalc knowledge. Use calc, where possible.)

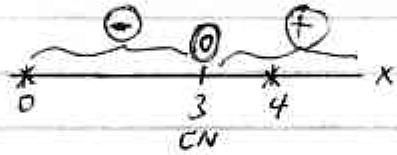
<u>Ex</u> $f(x) = x^2 - 6x + 8$

$f'(x) = \underbrace{2x - 6}_{\text{never DNE}} \overset{\text{set}}{=} 0$

$x = 3$

can also pick 2

f'



Book calls −6, 2 test values

$f'(0) = 2(0) - 6 = -6$ ⊖
$f'(4) = 2(4) - 6 = 2$ ⊕

<u>Graph f</u>

What is the easiest pt. to find on the graph?

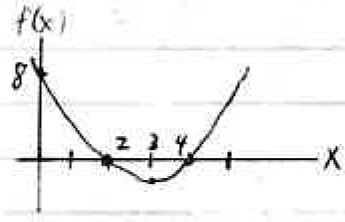
$y\text{-int.} = f(0)$
$= 8$

Find x-ints: $f(x) = x^2 - 6x + 8 \overset{\text{set}}{=} 0$
$(x-4)(x-2) = 0$
$x = 2, 4$

y-int = 8, but we don't show that

Vertex: (3, −1)

What kind of pt. do we have here?



f' ⊖ ⓪ ⊕
$\Rightarrow f$ ↘ ↗

(B) 1st DT

to classify pts. at CNs as L. Max. Pts., L. Min. Pts., or Neither

Assume f cont. at c, a CN (NO)

Same order as in book

Don't say if local max pt. → ⊕ → ⊖

Counterex.

(1)

L. Max Pt.

If f': ⊕ 0 ⊖ then

c

f: ↗ ↘

L. Max Pt.

⊕ DNE ⊖

c

(Doesn't matter if $f'(c) = 0$ or DNE)

(2)

L. Min Pt.

If f': ⊖ ⊕ then

c

f: ↘ ↗

When do we have Neither?

(3)

Neither

If f': ⊕ ⊕ then

c

f ↗ ↗

Neither

If f': ⊖ ⊖ then

c

f ↘ ↘

Ⓒ Exs

Ex $f(x) = 2x^3 - 9x^2 - 24x$

What can f' tell you? What secrets?

ⓐ Find the intervals on which $f \nearrow$ or $\searrow$.
ⓑ Find the local extrema.

$$f'(x) = 6x^2 - 18x - 24 \stackrel{\text{set}}{=} 0 \quad (f' \text{ never DNE})$$

or $\quad 6(x^2 - 3x - 4) = 0$

(Avoid ÷6; you won't have f'(x) anymore.) $\quad 6(x-4)(x+1) = 0$

$$\underbrace{x = 4, -1}_{\text{CNs}} \quad \text{in Dom}(f) \checkmark$$

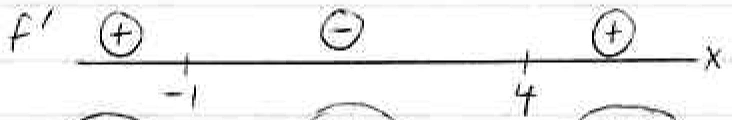
What might be convenient?

If 2 terms "+" / "−", do I know sign of sum? No!

$f'(x) = (6)(x-4)(x+1)$ ← Factored form, better for sign analysis.

$f'(-2) = (6)(-2-4)(-2+1)$ ← Can skip.
$= (+)(-)(-)$
$= \oplus$

$f'(0) = (+)(-)(+) = \ominus$
$f'(5) = (+)(+)(+) = \oplus$



$f \nearrow$ on $(-\infty, -1]$ $\qquad$ $f \searrow$ on $[-1, 4]$ $\qquad$ $f \nearrow$ on $[4, \infty)$

Apply 1st DT: (f cont.) $\qquad$ L. Max $\frown$ $\qquad$ L. Min $\smile$

L. Max Pt.

Be careful! Don't plug into f': what get? If you get 0, be suspicious.

$$(-1, \underbrace{f(-1)}) \quad \leftarrow \text{not } f'$$

$$f(-1) = 2(-1)^3 - 9(-1)^2 - 24(-1)$$
$$= 13$$

NOT f'!!

$(-1, 13)$

L. Min Pt.

$$(4, \underbrace{f(4)})$$

$$f(4) = 2(4)^3 - 9(4)^2 - 24(4)$$
$$= -112$$

$(4, -112)$

© Sketch the graph of f.

Find x-ints (if you can) — I'll tell you if I want the x-ints.

$$f(x) = 2x^3 - 9x^2 - 24x \overset{\text{set}}{=} 0$$
$$x(2x^2 - 9x - 24) = 0$$

$x = 0$ QF: $x = \frac{9 \pm \sqrt{273}}{4}$

$x \approx -1.9, 6.4$

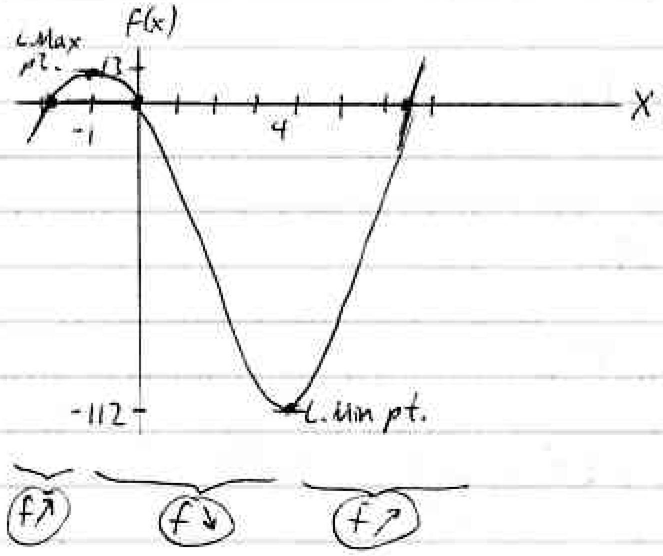
Sol'ns of $ax^2 + bx + c = 0$ ($a \neq 0$) are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Factoring works nicely if $b^2 - 4ac$ (discriminant) is a nice square.

y-int. $= f(0) = 0$

Using basic observations regarding f, f'.

No man's land ☺ Later, we'll find out more about graphs, we'll better interpolate, extrapolate.



What does cos x look like? What does +x do? Upward drift! Do we still get hills, valleys?

Ex Consider $f(x) = \cos x + x$ on $[-2\pi, 2\pi]$.

Find the local extrema.

$f'(x) = -\sin x + 1 \overset{set}{=} 0$ ($f'(x)$ never DNE)

$1 = \sin x$

$x = \frac{\pi}{2} + 2\pi n$, n int.

$x = \frac{\pi}{2}, -\frac{3\pi}{2}$ in $[-2\pi, 2\pi]$

Could do $\pm 2\pi$

$-\sin(-\frac{2\pi}{4})$
$= \sin(\frac{2\pi}{4})$ +
$= -\frac{\sqrt{2}}{2}$

f': + 0 + 0 + $\Rightarrow$ No local ext.

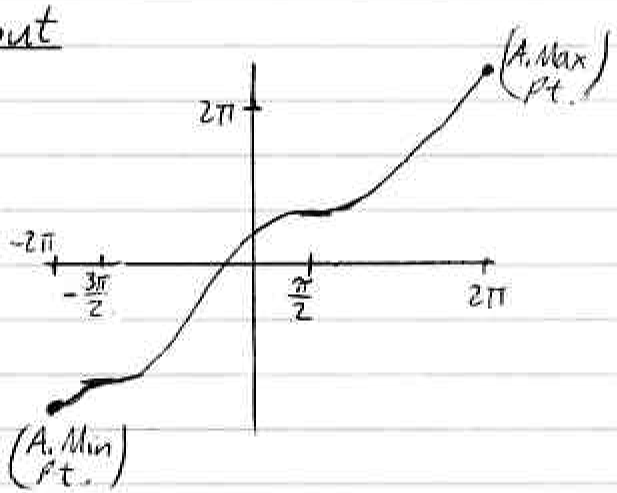
-2π, $-\frac{3\pi}{2}$ CN, $\frac{\pi}{2}$ CN, 2π

$f'(x) = 1 - \underbrace{\sin x}_{\text{in } [-1,1]}$ never $-$, only 0 at CNs

(Don't have to do test values!)

By EVT...
$(2\pi, 1+2\pi)$
$(-2\pi, 1-2\pi)$
$\exists$ A Max, Min.

Turns out



(No local ext.)

$f \nearrow$ on $[-2\pi, 2\pi]$, even though $f' = 0$ sometimes

You'll get a hor. tan line every 2π units if you extend the graph