4.4: f''

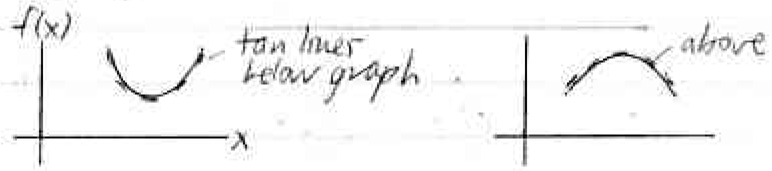
SAT analogies

(A) f is to f' as f' is to f''

f ↗ if f' (+)
f' ↗ if f'' (+)

What does f' tell me about the graph of f? What about f''? (locally)

(A) Concavity



f graph is

concave up (CU) ∪ ∪ ∪   concave down (CD)

On which graph are the slopes ↗?

what does that tell me about f'? (can't write: conjuring)

not ⟺
If f' ↗ maybe f'' DNE

(A) If f'' (+) ⇒ f' ↗ ⇒ f graph (CU) (key)

If f'' (−) ⇒ f' ↘ ⇒ f graph (CD) (key)

"Go together" word play

How to Ace ui

Up like a cup

Down like a frown

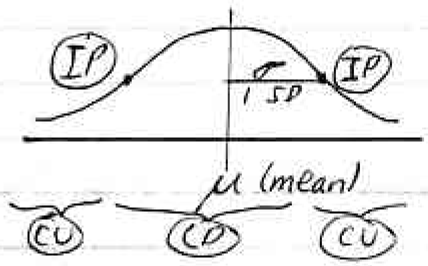
Ⓑ Inflection Pts. (IPs)

where concavity changes (CU ⇄ CD
f cont.

Exs

Berkeley upper div bio 2000 students

Bell curve (Normal)

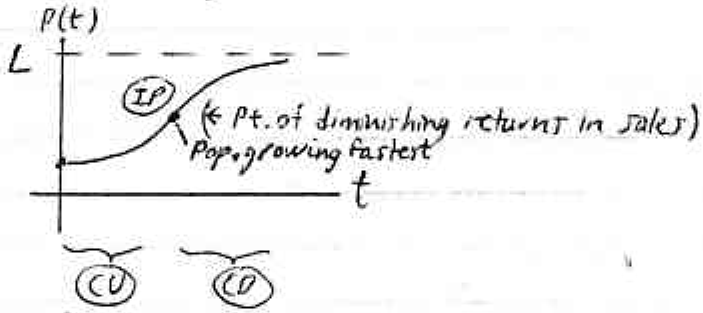


≈68% of data w/in 1 SD
≈95% 2
≈99.7% 3

Malthus
Pop. exp. ↗
Food supply linearly ↗
carrying capacity
1960s [illegible]
L=10B

Don't get bang for buck you used to

Pop. growth (Logistic Model)



f' ↗   f' ↘
"India"   "China"
f ↗ at ↗ rate   f ↗ at ↓ rate

(2nd deriv.: rate of change of rate of change of f)

Extra Credit if you give me your ATM PIN #

A CN is a #
in Dom(f) where
f' = 0 or DNE

A possible inflection #
PIN
f''
PINs
IPs

Special Pts. for f, f'

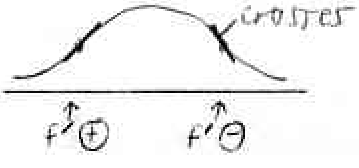
CNs are the ~~only~~ places where
(L.Max/Min. Pts.) can appear.

f' can be anything at an IP.

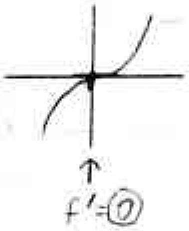
Tan line crosses bec. f'' + 0 − / − 0 + above → below / below → above

Skim

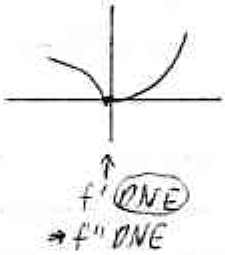
Exs



$f(x)=x^3$



f can't be DNE at IP, though



(No tan line at 0)

from 4.3 ©

4.3 Ex $f(x) = 2x^3 - 9x^2 - 24x$

What does f'' tell me?

ⓐ Where is the f graph (CU), (CD)?
ⓑ Find any IPs.

just need f' to find f''

$$f'(x) = 6x^2 - 18x - 24$$
$$f''(x) = 12x - 18 \overset{\text{set}}{=} 0 \quad (f'' \text{ never DNE})$$
$$x = \tfrac{3}{2}$$

only PIN in Dom(f) ✓

f'': (−) at 0, (0) at $\frac{3}{2}$, (+) at 2 on the x line

Test
$$f''(0) = 12(0) - 18 = -18 \quad (-)$$
$$f''(2) = 12(2) - 18 = 6 \quad (+)$$

f'': (−) (0) (+) on the x line, with $\frac{3}{2}$ marked

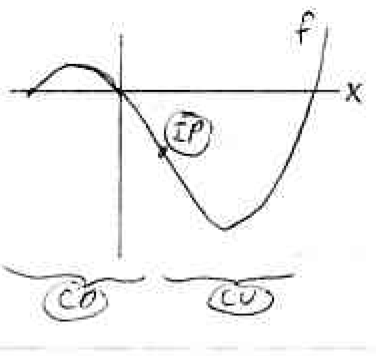
f graph: (CD) $\xrightarrow{\text{change}}$ (CU)

$(-\infty, \frac{3}{2})$ (IP) $(\frac{3}{2}, \infty)$ } answer to ⓐ

IP at $(\frac{3}{2}, f(\frac{3}{2}))$
$(\frac{3}{2}, -49.5)$ } ⓑ

If I tell you here $f' = 0$ and f graph CU, do you believe

Before, we saw a No Man's Land
Can better inter/extrapolate graph



(c) 2^{nd} DT

Alternative to 1^{st} DT for classifying pts. at CNs.

((2^{nd} DT may be easier to use, but it may not apply or work sometimes.) when 1^{st} DT does

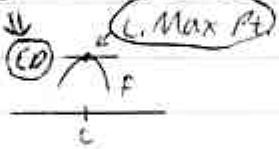
Assume $\underbrace{f'(c)=0}_{\text{horiz. tan line}}$ (if DNE $\Rightarrow$ Use 1^{st} DT)

2nd DT may be easier to use, no need to do a sign chart to find graph, but not as applicable. If $f'(c)=0$ ⇒ f defined at c, c in Dom(f), c is CN

1^{st} DT: Use ↗, ↘
2^{nd} DT: Use concavity + ∃ horiz. tan line

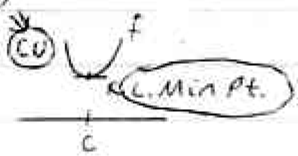
What's $f''(c)$?

(1) If $\ominus$



(Wordplay fails. Think: Concavity.)

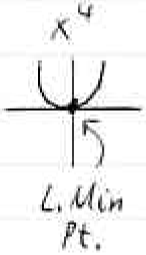
(2) If $\oplus$

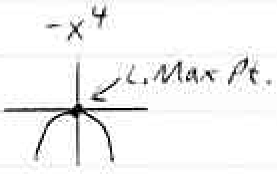


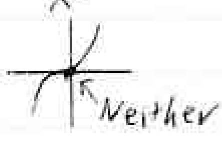
(3) If $\odot$ or DNE ⇒
2^{nd} DT is useless!
Use 1^{st} DT.

Much like Spears.

Exs ($f'(0)=0$, $f''(0)=0$)







Anything goes!

Anything goes!

If odd func cont. at 0, has crit. pt. there → neither unless ?

e.g.,

If even, L. Max or Min pt.