4.4/4.5: GRAPHING EXS

(A) Checklist

Old Issues

where do we literally have a ?

what property do we have? Same if f rat'l, alg. Discrepancy

what kind of discont.? Rat'l helps w/graphing

A graph can cross a HA.

VA: $\exists c$ s.t. What if

① Find Dom(f)
(Maybe find Range(f) later.) } Same if f algebraic (rational, plus roots OK)

② Where is f cont.?
(VAs? Holes? What kind of discont.?)

③ VAs? HAs?

$\left(\lim_{x \to c^- \text{ or } c^+} |f(x)| = \infty\right)$ Investigate $\lim_{x \to \infty, -\infty} f(x)$. Do we get real #s?

④ x-ints? y-int?

Solve $f(x) = 0$ or y (Can be hard)

$f(0)$ (Usually easy)

⑤ Symmetry?

f even   f odd

New Issues

Find $f'(x)$

① Find critical #s (CN_s) "c"

f'(c) ⇒ can't ONE apply 2nd DT when was 2nd DT inconclusive?

③ Where is f ↗, ↓?

④ Use 1st DT to classify pts. at CN_s.

(Also) Look for V, V.

Find $f''(x)$

2nd DT? Need $f'(c)=0$ (horiz. tan line)

② Classify pts. at CN_s L. Max/Min pts.? (Use 1st DT if you have to graph f, anyway.)

If f'(c) DNE, If $f''(c)=0$ or DNE, If f'' hard

⑤ Find PINs.

⑥ Where is f graph CU, CD? IPs?

⑦ Graph Range!

(B) Ex 5

Don't skimp, need big picture. Precalc

Ex Graph $f(x) = 4x^5 - 25x^4$

Old

① $\text{Dom}(f) = \mathbb{R}$
② f cont. on $\mathbb{R}$
③ No VAs, HAs

} $f(x)$ = nonconstant poly.

We can't skip the old stuff

Constant → HA

④ Find x-ints (if feasible)

Set $4x^5 - 25x^4 = 0$

$x^4(4x - 25) = 0$

$x = 0$ $\quad$ $x = \frac{25}{4} = 6.25$

(0,0)

y-int $= f(0) = 0$

⑤ f not even, odd

New

$f(x) = 4x^5 - 25x^4$
$f'(x) = 20x^4 - 100x^3$
$f''(x) = 80x^3 - 300x^2$

prior → nice derivs not even vertical tangent lines!

① Find critical #s (CNs)

$f'(x) = \underbrace{20x^4 - 100x^3}_{\text{never ONE factor!}} \overset{\text{set}}{=} 0$

$20x^3(x - 5) = 0$

(Do not $\div x^3$. Just factor the left side; you want a re-expression of $f'(x)$. Don't even $\div 20$!!)

0 $\quad$ 5 in $\text{Dom}(f)$ ✓

CNs

(Since we want to graph f, anyway, we can skip ②.)

② <u>Use 2nd DT to classify pts. at CNs.</u> (or 1st DT ③)
(Fair for a "shorter" question.)

Factoring a little weird

$f''(x) = 80x^3 - 300x^2$

Jack squat

✓ $f'(0)=0$, but $f''(0) = 0 \Rightarrow$ Use 1st DT for $x=0$.
Inconclusive!

✓ $f'(5)=0$, $f''(5) = 2500$

Sign of f'' ⇒ Think concavity!

⊕ ⇒ (CU)

f ∪

L. Min. pt. at $(5, f(5))$ $(5, -3125)$

③ <u>Where is f ↗, ↘?</u>

④ <u>Use 1st DT</u> (to classify pts. at CNs)

Need 1st DT for x=0.

Break at any disconts., CNs.

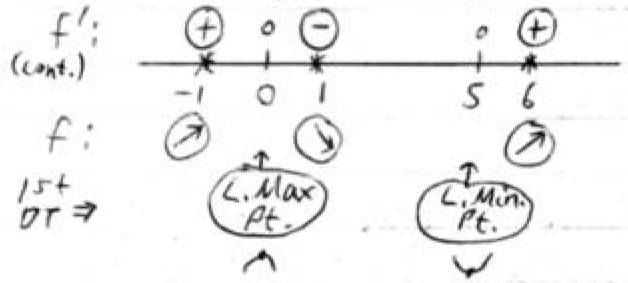
Factored form better for sign analysis.

Assume cont.

f' ⊕ ⊖ ; $f''(c)$ not positive

See f vs. f'

$f'=0$, $f'<0$, f' DNE



$f'(x) = (20)(x^3)(x-5)$
$f'(-1) = (+)(-)(-) = ⊕$
$f'(1) = (+)(+)(-) = ⊖$
$f'(6) = (+)(+)(+) = ⊕$

$f'(-1)=120$
$f'(1)=-80$

$(0, f(0))$ → $(0,0)$ also y-int.

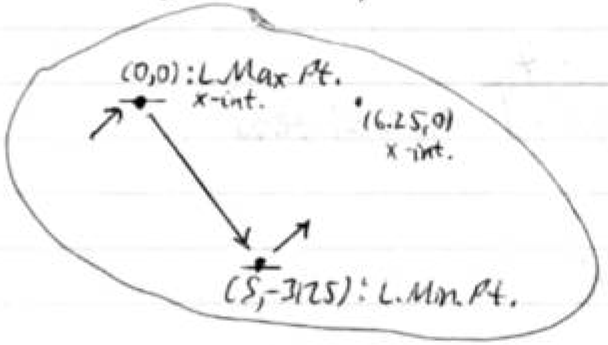
$(5, f(5))$ → $(5, -3125)$

⇐ (Make sure you plug into $f(x)$, not $f'(x)$ or $f''(x)$.)

If f defined, discont. at c ⇒ $f'(c)$ DNE, anyway, c a CN bec. in Dom(f)

f ↗ on $(-\infty, 0] \cup [5, \infty)$
f ↘ on $[0, 5]$

Skeleton graph (optional, but helpful):



(If $f'(0)$ were DNE, don't put horiz. tan line —.)

⑤ Find PINs

$$f''(x) = \underbrace{80x^3 - 300x^2}_{\text{never DNE}} \overset{\text{set}}{=} 0$$

$$20x^2(4x-15) = 0$$

$x = 0 \quad x = \frac{15}{4}$ in Dom(f) ✓ (PINs)

(0 happens to be both CN, PIN.)

Can look → at our guess graph?

⑥ Where CU, CD? IPs?

Break at any disconts., PINs.

f'' (cont.): ⊖ on $(-\infty, 0)$ [test -1], ⊖ on $(0, \frac{15}{4})$ [test 1], ⊕ on $(\frac{15}{4}, \infty)$ [test 5] ← knew from ②

$\frac{15}{4} = 3.75$

f: CD, CD, CU

no IP! (at 0)

IP at $\left(\frac{15}{4}, f\left(\frac{15}{4}\right)\right)$

$(3.75, \approx -1977.54)$

$f''(x) = 20x^2(4x-15)$

$f''(-1) = (+)(+)(-) = ⊖$

$f''(1) = (+)(+)(-) = ⊖$

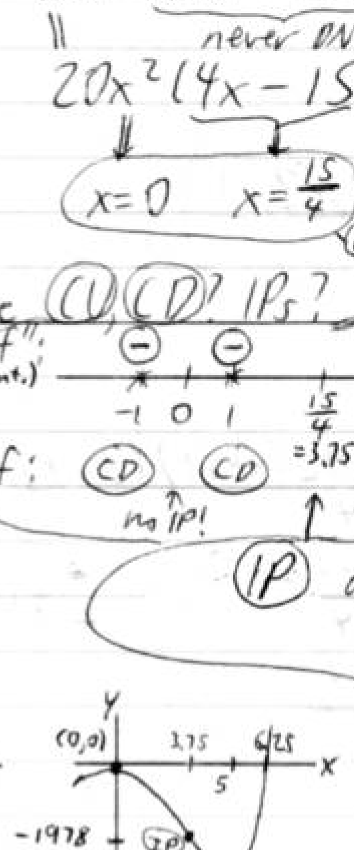
$f''(5) = (+)(+)(+) = ⊕$

$\frac{15}{4} = 3\frac{3}{4}$

If f defined, discont. at $c \Rightarrow c$ CN anyway, but not IP. corner

CD CU possible for LMax but for poly f' DNE? Do 4.4

⑦ Graph



Note: Range = $\mathbb{R}$ $(-\infty, \infty)$

Equidist. from 5: 3.75, 6.25

Unlike Microsoft Windows, these work.

(Windows! Between, beyond CNs, IPs, disconts.)

Ex (4.5 #22)

Old

Sketch $f(x) = \dfrac{-4}{x^2+1}$ ← no real zeros

① $\text{Dom}(f) = \mathbb{R}$

② f cont. on $\mathbb{R}$

nice rat'l func. no weird behavior

③ No VAs

HA: $y = 0$ ($f(x)$ = proper rational)

f always (-)

④ No x-ints ($f(x)$ never 0)

y-int $= f(0) = -4$

⑤ f is even ← y-axis

New

$$f(x) = \frac{-4}{x^2+1}$$

$$f'(x) = \frac{(x^2+1)(0) - (-4)(2x)}{(x^2+1)^2} \quad ((x^2+1)(0) \to 0)$$

$$= \frac{8x}{(x^2+1)^2}$$

Skip in class

$$f''(x) = \frac{(x^2+1)^2(8) - 8x[2(x^2+1)(2x)]}{(x^2+1)^4}$$

$$= \frac{8(x^2+1)^2 - 32x^2(x^2+1)}{(x^2+1)^4} \quad \leftarrow \div (x^2+1) \neq 0$$

$$= \frac{8(x^2+1) - 32x^2}{(x^2+1)^3}$$

$$= \frac{-24x^2 + 8}{(x^2+1)^3}$$

If just sticking to local ext. 1st DT
Granted, didn't have to simplify

$$f''(x) = \frac{-8(3x^2-1)}{(x^2+1)^3}$$

① <u>Find CNs</u>

$$f'(x) = \frac{8x}{(x^2+1)^2} \overset{\text{set}}{=} 0$$

$(x^2+1)^2$ never DNE (x^2+1 never 0)

Don't mess w/
When does a fraction = 0?
Num = 0, Den ≠ 0

$8x = 0$

$x = 0$ (CN) in Dom(f) ✓

② <u>2^{nd} DT</u> (can skip)

$f'(0) = 0$, so we can apply 2^{nd} DT.

$f''(0) = 8$ ⊕

(CU) f

L. Min. pt. at $(0, f(0))$
$(0, -4)$ — also y-int.

③ <u>Where is f ↗, ↘?</u>

④ <u>1^{st} DT</u>

f′: ⊖ (at −1), 0 (at 0), ⊕ (at 1) — x

f: ↘ (−1), ↑ L. Min pt. (0), ↗ (1)

1^{st} DT ⇒ L. Min pt. $(0, f(0))$ $(0,-4)$

$$f'(x) = \frac{(8)(x)}{(x^2+1)^2}$$

$$f'(-1) = \frac{(+)(-)}{(+)} = \ominus$$

$$f'(1) = \frac{(+)(+)}{(+)} = \oplus$$

f ↘ on $(-\infty, 0]$
f ↗ on $[0, \infty)$

↘ . ↗
$(0,-4)$

(5) Find PINs

$$f''(x) = \underbrace{\frac{-8(3x^2-1)}{(x^2+1)^3}}_{\text{never DNE}} \overset{\text{set}}{=} 0$$

$$3x^2 - 1 = 0$$

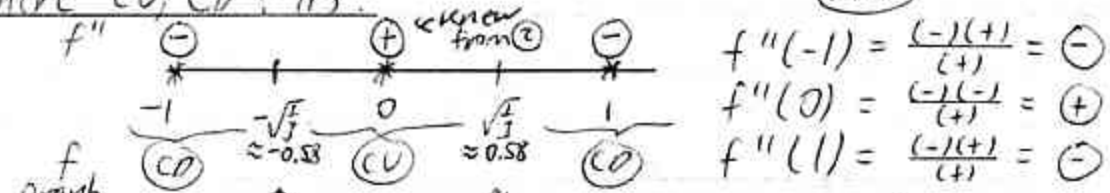
$$x^2 = \frac{1}{3} \quad \text{or } \pm\frac{\sqrt{3}}{3}$$

$$x = \pm\sqrt{\frac{1}{3}} \approx \pm 0.58 \text{ in Dom}(f) \checkmark$$

PINs

(6) Where CU, CD? IPs?

Graph sym?



$$f''(-1) = \frac{(-)(+)}{(+)} = \ominus$$

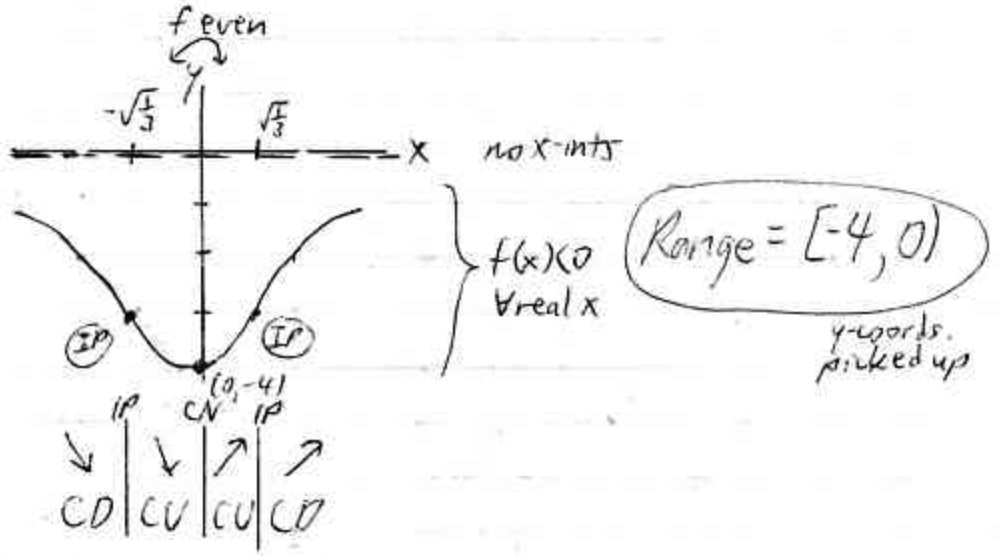
$$f''(0) = \frac{(-)(-)}{(+)} = \oplus$$

$$f''(1) = \frac{(-)(+)}{(+)} = \ominus$$

IPs at $\left(-\sqrt{\frac{1}{3}}, f\left(-\sqrt{\frac{1}{3}}\right)\right), \left(\sqrt{\frac{1}{3}}, f\left(\sqrt{\frac{1}{3}}\right)\right)$

$\left(-\sqrt{\frac{1}{3}}, -3\right), \left(\sqrt{\frac{1}{3}}, -3\right)$

(7) Graph



f always ↓ 0

If f even, cont. at 0, 0 a CN ⇒ either L.Max/Min. pt. there. Windows

$f(x) < 0$ $\forall$ real x

Range = $[-4, 0)$

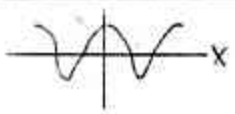
y-coords. picked up

$D_x(\cos x) = -\sin x$

Y corner, not cusp

$\lim_{x \to \pm\infty}$

Ex $\cos x$



$|\cos x|$