4.6: OPTIMIZATION

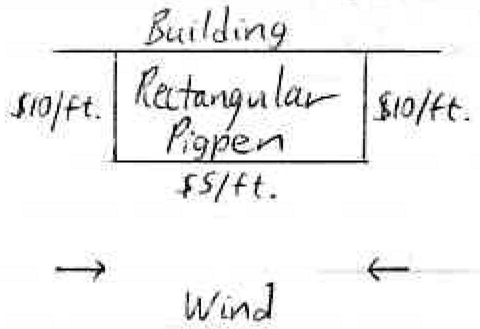
(Hand out 4.6 Handout)

Ex

So let me give you all \$300...

①,② You have \$300 to spend on fencing.

or river

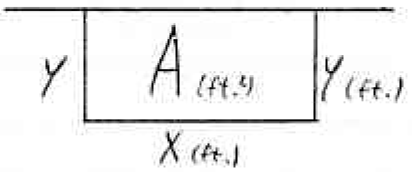


Usually, cost structure.

Wind tends to blow ll to this side of the bldg

Find the dimensions that will maximize the area (of the pigpen), and give that area.

②



③

Maximize A (area).

(PE) $A = xy$

④

Cost (\$)

Constraint

$$5x + 20y = 300$$

$$x + 4y = 60 \quad \text{(SE)}$$

(5)

(PE) $A = xy$

(SE) $x + 4y = 60$

$x = 60 - 4y$ (★)

(PE) → $A = (60-4y)y$

$A = \underbrace{60y - 4y^2}_{f(y)}$

Sometimes, an optional step but good habit.

(6) <u>Find Feasible Domain</u>

Need $x \geq 0$ and $y \geq 0$

(★) $60 - 4y \geq 0$

$60 \geq 4y$

$15 \geq y$

$y \leq 15$ → y $[0, 15]$

You could argue $x > 0, y > 0$. It's more convenient to have a closed interval. If $x = 0$ ~~or~~ $y = 0$ what's the area? | or —

<u>MAXIMIZE f</u>

<u>Find all critical #s (CNs) in $(0, 15)$</u>

Technically can't have CNs at endpts. EVT √s, anyway.

$f(y) = 60y - 4y^2$

$f'(y) = \underbrace{60 - 8y}_{\text{never DNE}} \overset{\text{set}}{=} 0$

$y = \left(\frac{15}{2} \text{ or}\right) 7.5$ ← CN

in $(0, 15)$ ✓

Verify that we have an A. Max. Pt. there (Don't appeal to "common sense".)

Method 1 (4.1) EVT applies

Sometimes it's better to keep the factored form.

y	$A = f(y) = (60-4y)y$
$a = 0$	0
7.5	225
$b = 15$	0

← A. Max. pt. on $[0,15]$

or $(0,15]$, if you wish/prefer

Method 2 (2nd DT)

$f'(7.5) = 0$, so can apply.

$f'(y) = 60 - 8y$

$f''(y) = -8$

$f''(7.5) = -8$ ⊖ ⇒ CD L. Max. Pt.

(on $[0,15]$,)
f cont.,
7.5 is the only CN
(no other opportunity for f to turn around)
⇒ also A. Max. Pt.

never DNE
CD everywhere ∩
121: Only CN in town. No chance to turn around again (Don't)

$f(7.5) = 225$

CD everywhere excludes

CD assumes cont.
f' ∃
(f' DNE ⇒ f'' DNE)

Method 3 (Where is f ↗, ↘?)

f': ⊕ on $(0, 7.5)$, ⊖ on $(7.5, 15)$ (y)

f: ↗ then ↘ → A. Max. Pt. at 7.5

ANSWERS

7.5 what?

$y = \left(\frac{15}{2} \text{ or}\right) 7.5$ ft.

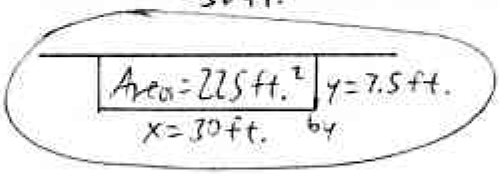
$A = 225$ ft.^{2}

Find x

PE $A = xy \Rightarrow 225 = x(7.5) \Rightarrow x = 30$ ft.

or ⓐ $x = 60 - 4y$
$= 60 - 4(7.5)$
$= 30$ ft.

Why so squished?
↑ 1 more expensive,
bldg. like matching funds
30 ft. by 7.5 ft.
Can do HW, but next Ex helps w/ 7, 23



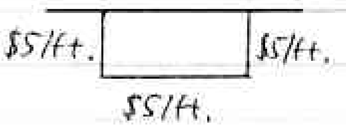
More basic/common version.

VARIATIONS

Instead of talking about total cost, we can talk about this partial perimeter "ice rule"

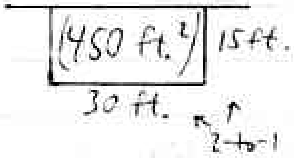
This 2-to-1 ratio works for similar probs.

(1)



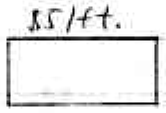
$300
(Partial P = 60 ft.)

Optimal



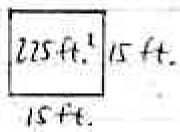
What shape is optimal? Given a fixed P, max A. You're going to prove this in #3x.

(2)



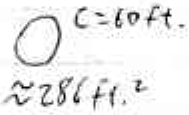
$300
P = 60 ft.

Optimal



Square!

If we go beyond rects., what is better than a square? (circle)

Better: 

Circle!

Can do this, but next Ex helps w/ 7.

≈ 2-car prob. 1.8

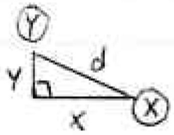
A → a
I use for endpt.
I'm me
You're you
What?
Opt. problem.

Ex ① At noon, Car X is 50 miles due east of Car Y.
Car X moves west at 30 mph.
Car Y moves north at 10 mph.
What is the closest the cars get?

I'll define t later.

② Noon

Later



How do we relate d, x, y?
Later, we'll differentiate.
Do you agree d^2 is min ⟺ d is?
As d ↑, d^2 ↑.

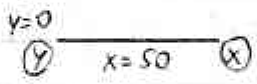
③ Minimize d.

(PE) $d^2 = x^2 + y^2$ ← Easier to min d^2 for now.

$d = \pm\sqrt{x^2+y^2}$

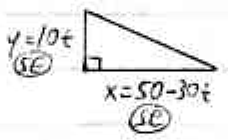
What haven't we used yet?
We need a new var.

④ Noon



t=0

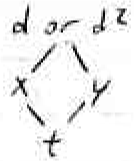
Let t = # hours after noon



t>0

shrinking at a rate of 30 mph
eventually, x goes negative.
d can't be neg, but x can.

⑤ (PE) $d^2 = x^2 + y^2$



(In Calc III, you draw bushy trees.)

Use (SE)s

$$d^2 = (50-30t)^2 + (10t)^2$$
$$= \underbrace{1000t^2 - 3000t + 2500}_{g(t)}$$

why g, not f? It's a reminder

⑥ <u>Feasible Domain</u>: $t \geq 0$
$[0, \infty)$

What must be true of t? What values make sense? Earth flat, Jason/robot driving, Sun never burns out.

<u>MINIMIZE g, then f</u> $\leftarrow \sqrt{g}$

<u>Find all critical #s in $(0,\infty)$</u> (CNs) - Don't use $-\frac{b}{2a}$ formula; use calculus

$$g'(t) = \underbrace{2000t - 3000}_{\text{never DNE}} \overset{\text{set}}{=} 0$$

$t = \frac{3}{2}$ or 1.5 in Dom of g ✓

<u>Note</u> Cars are closest at 1:30 p.m. (We think.)

<u>Verify we have an A. Min. there</u> (2 arguments you could use:)

$g''(t) = 2000 > 0 \quad \forall t$ in $(0,\infty)$

$\Rightarrow g$ ∪ CV on $(0,\infty)$

<u>or</u> $\Rightarrow g''(1.5) = 2000 > 0 \xRightarrow{\text{2nd DT}}$ L. Min. at 1.5

$g'(1.5) = 0$ needed for 2nd DT

g Cont. on $(0,\infty)$

g cont. on $(0,\infty)$
1.5 only CN there
"Only CN in town"

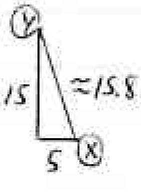
$g'' > 0$ ⇒ concave what? Only CN in town: 1.5! Of course, graph of g is a parabola opening up. Is 250 mi the ans.?

<u>ANSWER</u> $g(1.5) = 250$

$\Rightarrow f(1.5) = \sqrt{250}\ (= 5\sqrt{10}) \approx 15.8$ mi.

Easier for some people than $g \to f$. See if your ans. seems to work.

<u>Note</u> At 1:30 pm



Looks good! :)

(250?! No! <u>Triangle Inequality</u>: the sum of any two side lengths of a triangle must be greater than the third.) ▷ Detour