4.8: NEWTON'S METHOD

Ⓐ Review

Solve an eq.
Can - RHS

Find a sol'n of $f(x)=0$ (f x-ints, f' crit. #s, f'' PIN #s)
i.e., Find a zero (or "root") of f.

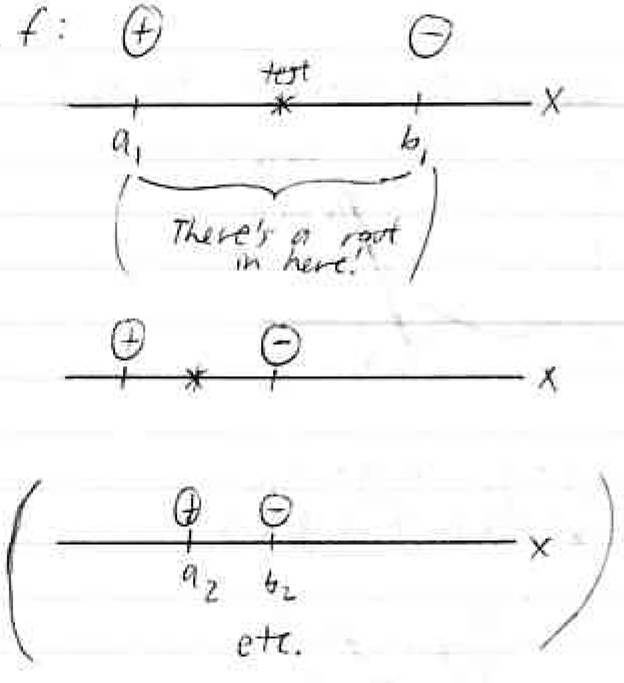
10^{th}-degree; factoring, rational zero test ($\pm\frac{p}{q}$), etc. fail?
Trig? $\sqrt{\ }$?

Approximate a root!

2.5: Bisection Method (based on IVT)

When are we guaranteed to have a root inside here?
seeds a_1, b_1

cont. f:



rounding error
IT uh ray tiv
secant method

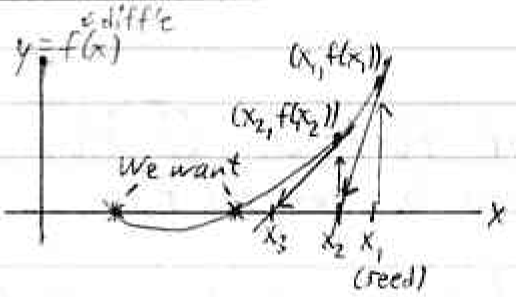
Iterative

Form or known really by Simpson! (CMC³-S '03)

Really Simpson's?

(B) Newton's Method: Idea

We want the 0s of the f so we want the what on the graph? (x-ints.) We just need one.



(Ski down tangent line to new approx.)

This graph might be invisible to us. Ideally, we could ski down / x-int. Instead, we'll ski down? Tan line approx. the graph near this pt.

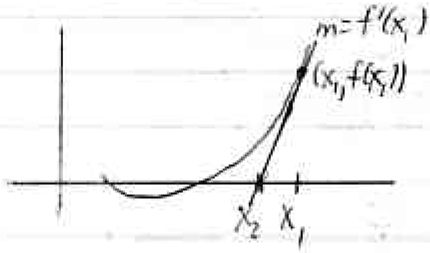
Seed (initial guess) = x_1 (maybe based on a graph on a graphing calc.)

Let x_2 = x-int. of tangent line at $(x_1, f(x_1))$

x_3 = " " $(x_2, f(x_2))$

etc.

(C) How do we get from x_1 to x_2?



This is the same x_1

$$y - y_1 = m(x - x_1)$$

$$y - f(x_1) = [f'(x_1)](x - x_1)$$

When we find x-ints

Plug in $y=0$

Solve for x

$$x_1 - \frac{f(x_1)}{f'(x_1)} = x$$

← if $\neq 0$

$$\boxed{x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}}$$ (prev. approx. − ratio of func. value to deriv. value there)

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$ ← provided $\neq 0$ $n = 1, 2, 3, \ldots$

Stuck ⇒ New seed!

(D) Ex

what does this have to do w/ finding 0s? roots

Can use differentials but one step.

Let's say you have a cheapy calc.

Approx. $\sqrt{5}$ to 4 dec. places.

i.e., Approx. the "+" root of $f(x) = x^2 - 5$.

$f'(x) = 2x$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{x_n^2 - 5}{2x_n}$$

$$= \frac{2x_n^2 - x_n^2 + 5}{2x_n}$$

$$= \frac{x_n^2 + 5}{2x_n}$$

$\sqrt{5}$ has to be between what and what?

Try to avoid rounding off

Seed: $x_1 = 2$

$$x_2 = \frac{(2)^2 + 5}{2(2)} = 2.2500$$

$$x_3 = \frac{(2.25)^2 + 5}{2(2.25)} \approx 2.2361$$

$$x_4 \overset{\text{turns out}}{\approx} 2.2361$$

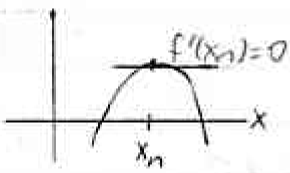
same, so STOP!

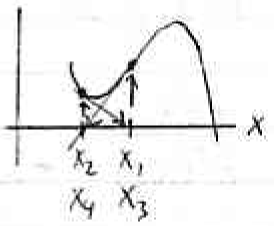
(*If* the method works, when you get "close" to a root, the # of correct decimal places doubles w/ each iteration.

Quadratic convergence)

When Animals Attack

Ⓔ When Newton Fails!





Why does the method fail? ①
Stuck in a valley ②
$x^{\frac{1}{3}}$: diverges ③ $\forall x_1 \neq 0$
Hit tan line intervals – approx. often not good far away
Larson 218:
Suff. for conv.:
$\left|\frac{f(x)f''(x)}{[f'(x)]^2}\right| < 1$
on an open interval containing the root

(Why are we talking about this now?
Applic. of derivs. and $f'(x)=0$ – find critical #s
$f''(x)=0$ – find PINs)