So powerful
like Chain Rule for $\int$s

5.2: "u" SUBS (are to $\int$s what the Chain Rule is to D_x)

#8
$u = \sec x$ works $\frac{\sec^2 x}{2} + C$
#63: $\int \sin x \cos x \, dx$
shows trig equiv.

Ex $\int \tan x \sec^2 x \, dx$

Put in form $\int \underbrace{f(u)}_{\text{nice}} du$

$\tan x, \sec^2 x$
How are these related?

Let $u = \tan x$

Idea: Quotient of differentials $dx \to 0$ (like $h \to 0$ in limit defn of deriv.)

$\frac{du}{dx} = \sec^2 x$

technically inappropriate to say we're mult. by dx
"Differential form"

$\Rightarrow du = \sec^2 x \, dx$

$u = \sec x$ works & equivalent.

$\int \underbrace{\tan x}_{u} \underbrace{\sec^2 x \, dx}_{du}$

$= \int \underbrace{u}_{\text{Nice!}} du$

What? +C
Am I done?

$= \frac{u^2}{2} + C$

$= \frac{\tan^2 x}{2} + C$ Go back to x!
Can ✓ by D_x

Ex $\int x^2 (4x^3 - 5)^{10} dx$

You could've done in 5.1 - take you a day. Maybe Bin Thm, Pascal's Δ.

Let $u = 4x^3 - 5$ ← Why?
D_x in $\int$, except constant factor
u^{10} nicer
Common: u = "inside", exp., denom.
(Not always, though!)
(Do HW!!)

of a composite func.

$\rightarrow du = 12x^2 dx$

I can't just throw in a 12 arbitrarily. Const. factor pop out anyway. Like LTS. I guess mixing u, x OK, but awkward.

Method 1 (Compensation)

Balance

$$\frac{1}{12}\int \underbrace{12x^2 \overbrace{(4x^3-5)^{10}}^{u^{10}} dx}_{du}$$ Only pop out constants. NO! $\frac{1}{x}\int x \cdots$

$$= \frac{1}{12}\int u^{10}\,du$$
$$= \frac{1}{12}\left(\frac{u^{11}}{11}\right) + C$$ Don't need $\frac{C}{12}$
$$= \frac{u^{11}}{132} + C$$
$$= \frac{(4x^3-5)^{11}}{132} + C$$

Method 2 (Solve for whomever you have to kill)

$$\int \underbrace{x^2 \overbrace{(4x^3-5)^{10}}^{u^{10}} dx}_{\text{KILL!}}$$

$u = 4x^3 - 5$

$du = 12x^2\,dx$

$\Rightarrow \frac{1}{12}du = \underbrace{x^2\,dx}_{\text{KILL!}}$

Is $\frac{du}{12}$ OK?

$$= \int u^{10} \cdot \frac{1}{12}\,du$$
$$= \frac{1}{12}\int u^{10}\,du$$

etc. (see Method 1)

Your answer better be equiv. to prev. answer. C can absorb consts.

Ex $\int \sin(2x+1)\,dx$

$u = 2x+1$

$du = 2\,dx$

$$= \frac{1}{2}\int \sin(2x+1)\cdot 2\,dx$$
$$= \frac{1}{2}\int \sin u\,du$$
$$= \frac{1}{2}(-\cos u) + C$$
$$= -\frac{1}{2}\cos(2x+1) + C$$

Better?

$u = 2x+1$

$du = 2\,dx \Rightarrow dx = \frac{1}{2}du$

$$= \int \sin u \cdot \frac{1}{2}du$$
$$= \frac{1}{2}\int \sin u\,du$$
$$= \frac{1}{2}(-\cos u) + C$$
$$= -\frac{1}{2}\cos(2x+1) + C$$

sin and sec!
1st step - not calc.

Ex (#38) $\int \sin(2x)\sec^5(2x)\,dx$

$$= \int \frac{\sin(2x)}{\cos^5(2x)}\,dx$$

Could do u = 2x
→ would need 2 subs.
Try to let u =
biggest guy
whose deriv.
is in there.

$$u = \cos(2x)$$
$$du = -\sin(2x)\cdot 2\,dx$$
$$= -2\sin(2x)\,dx$$

$$= \left(-\frac{1}{2}\right)\int \frac{-2\sin(2x)}{\cos^5(2x)}\,dx$$

$$= -\frac{1}{2}\int \frac{du}{u^5}$$
$$= -\frac{1}{2}\int u^{-5}\,du$$
$$= -\frac{1}{2}\left[\frac{u^{-4}}{-4}\right]+C$$
$$= \frac{1}{8}\left(\frac{1}{u^4}\right)+C$$
$$= \frac{1}{8\cos^4(2x)}+C$$

$$\text{or} \quad \frac{1}{8}\sec^4(2x)+C$$

Book doesn't yet have this, but standard trick. Ch. 9

Ex $\int x\sqrt{x-1}\,dx$

$u = x-1 \Rightarrow x = u+1$ (Take our sub statement, solve for x.)

$du = dx$

du = 1 dx

What do we do w/ x?

Does ∫prod = prod ∫s? NO
Does ∫sum = sum ∫s? YES if ∃

$$= \int (u+1)\sqrt{u}\,du$$
$$= \int (u+1)u^{1/2}\,du \qquad \left(\neq \int(u+1)\,du\cdot\int u^{1/2}\,du\right)$$
$$= \int \left(u^{3/2}+u^{1/2}\right)+C \qquad \text{\& stick w/ algebra}$$

Don't skip.
Many blow
coeffs.

$$= \frac{u^{5/2}}{5/2}+\frac{u^{3/2}}{3/2}+C$$
$$= \frac{2}{5}(x-1)^{5/2}+\frac{2}{3}(x-1)^{3/2}+C$$

When you ÷ by a frac
...

Get practice on Hw!!