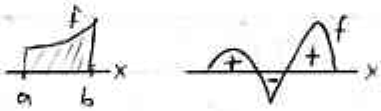
5.5: PROPERTIES OF DEFINITE $\int$s

Ⓐ Review

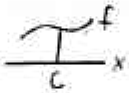
cont. on $[a,b]$ $\Rightarrow$ int. there

Assume integrability.

$\int_a^b f(x)\,dx$



① $\int_c^c (\quad) = 0$



$\int_a^b + \int_b^a = \int_a^a = 0$

② $\int_b^a (\quad) = -\int_a^b (\quad)$

SKIP; see back

Ⓑ Integrand-Splitting Rules

If I'm $\int$ something complicated, I may be able to break it down.

like for $\int$, D_x

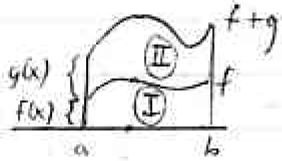
③ $\int_a^b c f(x)\,dx = c \int_a^b f(x)\,dx$

$f \geq 0$

Shorthand

④ $\int_a^b (f+g)\,dx = \underbrace{\int_a^b f\,dx}_{\mathrm{I}} + \underbrace{\int_a^b g\,dx}_{\mathrm{II}}$

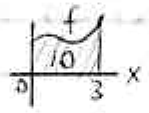
$f, g \geq 0$



⑤ $\quad - \qquad\qquad - $

(B) Integrand-Splitting (Linearity)

Ex Given: $\int_0^3 f(x)\,dx = 10$



Then, $\int_0^3 [4f(x)+1]\,dx$

*linear operator, like $\int$, D_x, lim

When I graph "1", I'm finding the area of a...

$$\left(= \int_0^3 4f(x)\,dx + \int_0^3 1\,dx\right)$$

$$= \underbrace{4\int_0^3 f(x)\,dx}_{=10} + \underbrace{\int_0^3 1\,dx}$$

Up to 9

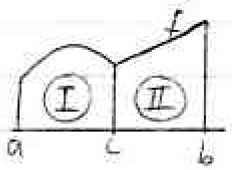
$$= 4(10) + 3$$

$$= 43$$

We can split the integrand, like for $\int$s; what can we also split for definite $\int$s?

(C) Interval-Splitting

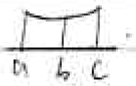
(maybe f is piecewise-defined)



$$\int_a^b f(x)\,dx = \underbrace{\int_a^{c} f(x)\,dx}_{\text{I}} + \underbrace{\int_{c}^b f(x)\,dx}_{\text{II}}$$

Even if c is outside the interval.

Even if



Can "cancel" $\int^x \int_x$ or $\int_x \int^x$

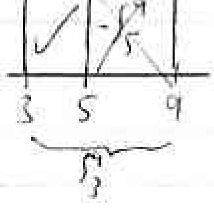
more intuitive
How can rewrite $\int_9^5$?

Ex
Simplify

$$\int_9^5 f(x)\,dx + \int_3^9 f(x)\,dx = \int_3^{9} \quad + \int_{9}^5$$

$$= \int_3^5 f(x)\,dx$$

You don't need to know what happens from 5 to 9, as long as you have integrability.



Name from Larson 278

(D) Inequality - Preserving Rules

Throughout $[a,b]$,

(i) If $f(x) \geq 0$,
then $\int_a^b f(x)\,dx \geq 0$.

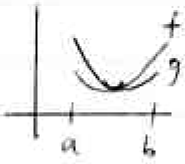
nonnegativity preserved

Up to 21
from (7):
$\int [f-g] \geq 0$

True even if f and/or g falls below 0
if one or both graphs fall below x-axis.
(i) Comparing f to 0
(ii) g

See m151! Comparison Idea

(ii) If $f(x) \geq g(x)$,
then $\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$.



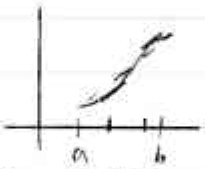
dominance preserved

(E) MVT for Definite $\int$s, f_{av}

In words do you know 'em?

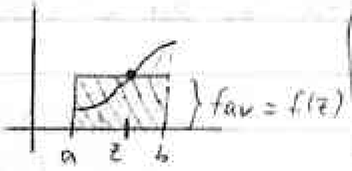
How do you avg. so many func. values?
Range game from Pierce is right.
Fixed width
Raising the bar until area of rect = area of piano
balance
cut-outs hole

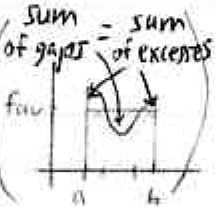
MVT for D_x



(somewhere in here, we get the avg. rate of change)

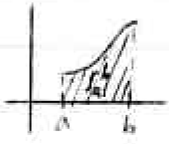
Now,





(somewhere in here, we get the avg. func. value)

If f is cont. on $[a,b]$, then $\exists z$ in (a,b) such that



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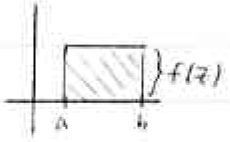


equiv.; if $a=b$, (a,a) is silly.

$$\int_a^b f(x)\,dx = f(z)(b-a) \quad \leftarrow \text{MVT concl. in book}$$

i.e.,

$$\underbrace{f(z) = \frac{\int_a^b f(x)\,dx}{b-a}}_{= f_{av}} \quad \begin{matrix} \leftarrow \text{"Sum"} \\ \leftarrow \text{"Input Size"} \end{matrix}$$

(like in "discrete averaging")

$= f_{av}$, the average value of f on $[a,b]$

Ex (#26)

Given: $\int_{-1}^{3} \underbrace{(3x^2 - 2x + 3)}_{f(x)}\, dx = 32$

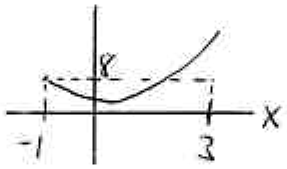
(b) Find f_{av} on $[-1,3]$.

Do 1st

$$f_{av} = \frac{\int_{-1}^{3} f(x)\,dx}{3-(-1)}$$

$$= \frac{32}{4}$$

$$= \boxed{8}$$



What's Part (a)? where is f_{av} taken on?

(a) (Find all z such that $f(z) = f_{av}$.)

$$f(z) = 8$$
$$3z^2 - 2z + 3 = 8$$
$$3z^2 - 2z - 5 = 0$$
$$\underbrace{(3z-5)}_{\Downarrow}\underbrace{(z+1)}_{\Downarrow} = 0$$

$\boxed{z = \frac{5}{3}}$ $\quad \cancel{z = -1}$ not in $(-1,3)$

How do you solve? There's no "8-factor property."

QF if desperate. Either way, $\ldots = 0$