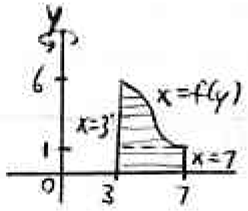
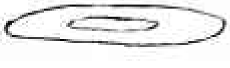
6.3: MORE VOLUMES OF SOLIDS OF REVOL.: CYLINDRICAL SHELL METHOD

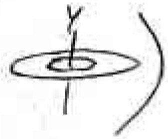
Ⓐ Review Washer Method



$$V = \int_0^1 \left(\pi \underbrace{r_{out}}_{(7)}{}^2 - \pi \underbrace{r_{in}}_{(3)}{}^2\right) dy$$

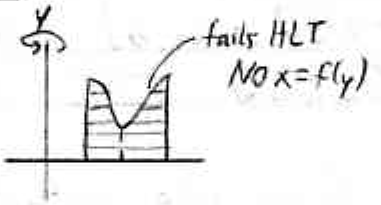


$$+ \int_1^6 \left(\pi \underbrace{r_{out}}_{[f(y)]}{}^2 - \pi \underbrace{r_{in}}_{[3]}{}^2\right) dy$$

(We're integrating areas of washers. )

Note

What do you get? Target



Washer method fails unless we split.

Ⓑ Intro Cyl. Shells

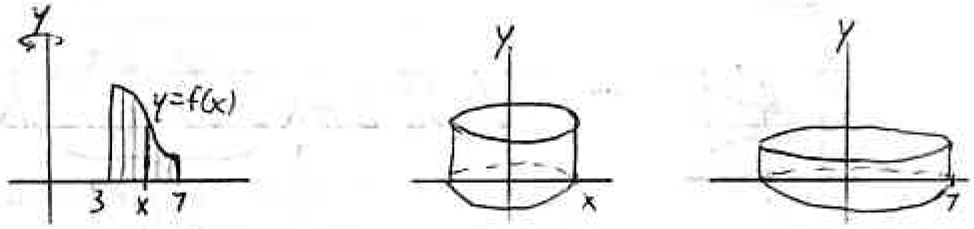
What if we take vertical line segments (scan lines) (|| to axis of revol.)

How would the cyls differ

ripples of cylinders

manipulative

label of can



We'll integrate the [lateral] surface areas of cylinders. Ⓐ

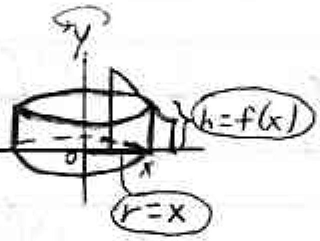
" Rippling out "

Can

☆

$\rightarrow$ $\boxed{A = 2\pi r h}$ h

$2\pi r$



$\boxed{A = 2\pi x f(x)}$

$$\boxed{V = \int_a^b 2\pi x f(x)\,dx}$$

no square height ≥ 0 vs. Disk Method

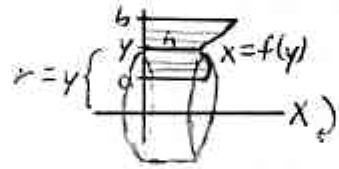
if f cont. $\Rightarrow$ ensures integrability

$f(x) \geq 0$ on $[a, b]$ $\Rightarrow$ height ≥ 0

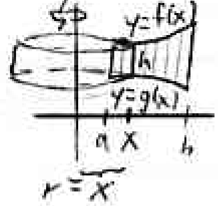
$0 \leq a < b$  $\Rightarrow$ radius ≥ 0

C. Variations

Key: Draw the picture! Memorization shaky...

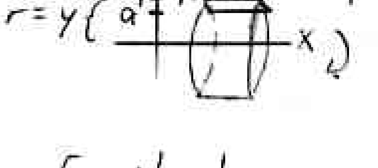


$$V = \int_a^b 2\pi y\, f(y)\,dy$$



$f - g \geq 0$

$$V = \int_a^b 2\pi \underset{\text{radius}}{x} \underbrace{\overbrace{[f(x) - g(x)]}^{\text{top} - \text{bot}}}_{\text{height } (h, \text{ or } h(x)) \text{ at } x} dx$$



$$V = \int_a^b 2\pi \underset{\text{radius}}{y} \underbrace{\overbrace{[f(y) - g(y)]}^{\text{right} - \text{left}}}_{\text{"height" } (h, \text{ or } h(y)) \text{ at } y} dy$$

Don't need $f, g \geq 0$. Need $f \geq g$.

For the linear measures radius and height, it's "top-bottom or right-left".
(later: Weird axis) (here)

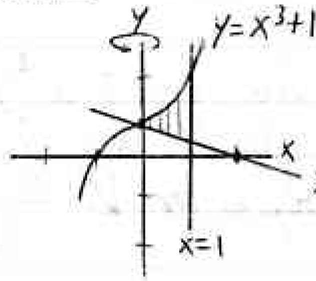
① Exs

Ex (#10) R is bounded by $y=x^3+1$, $x+2y=2$, $x=1$.
(R is revolved about y-axis.) $\circlearrowleft y$
Find V.

Sketch R

Intercept Method

vertical, || y-axis $\underbrace{\ }_{x=0}$



Washer Method

Split  $\uparrow$ scan (dy)

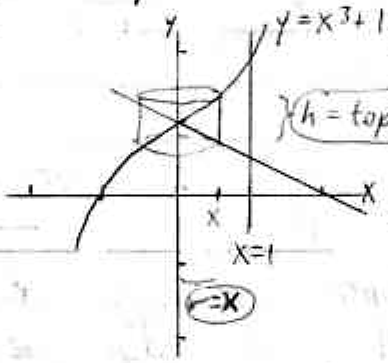
we scan R vertically.

Cyl Shells


scan → (dx)

Solve the eqs. for y in terms of x.

Scanner moves left-to-rt over R



$x+2y=2$
$2y=2-x$
$y=1-\frac{1}{2}x$ (bottom)

$$V = \int_0^1 \underbrace{2\pi x}_{\text{x-values}} \big[\underbrace{(x^3+1)}_{\text{top}} - \underbrace{(1-\tfrac{1}{2}x)}_{\text{bottom}}\big]\, dx$$

need

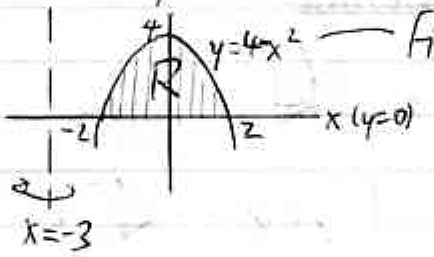
$$= \frac{11\pi}{15}$$

Ex (Weird axis)

(#20b) R is bounded by $y = 4-x^2$, $y=0$.
R is revolved about $x=-3$.
Set up the integral for V. (So we get a ready-made Ch5 problem, in principle.)

I'd rather scan R horizontally.

Sketch R



Find x-ints:
$0 = 4-x^2$
$x^2 = 4$
$x = \pm 2$

Scan → (dx)

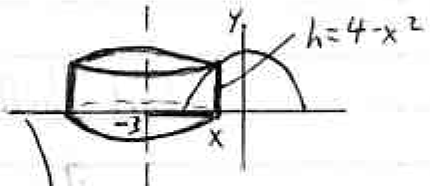
cyls of same height, diff radii

Disk

mean!

scanner moves left-to-rt over R

If $x=-3$, $x>0$, $3+x$



r = right − left
$= x-(-3)$
$= x+3$

(Note: Symmetry about y-axis not helpful: cyls have diff. radii.)

$$\boxed{V = \int_{-2}^{2} 2\pi \underbrace{(x+3)}_{\text{radius}} \underbrace{(4-x^2)}_{\text{height}}\, dx}$$

x-values

Axis of revol.

	∥ to x-axis	∥ to y-axis
Disk/Washer	dx	dy
Cyl. Shell	dy	dx

(dx): scan ||| (dy): scan ≡↑

(How does the scanner move over R? → ∫dx, ∫dy)

Challenge: Derive formula for the vol of a sphere using cyl. shells