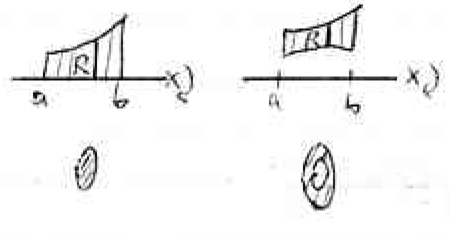
6.4: VOLUMES BY CROSS-SECTIONS

6.2: Disks, Washers for Solids of Revol. 6.4: Other solids, too.



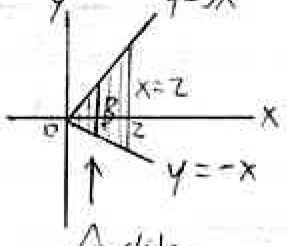
$V = \int_a^b \underbrace{A(x)}_{\substack{\text{area of}\\ \text{cross-sec}\\ \text{at } x}} dx$ or $V = \int_a^b A(y)\,dy$

Start x →

<u>Ex</u> B is bounded by (the graphs of) $y = 3x$, $y = -x$, and $x = 2$.
Find the volume of the solid that has B as its base if every cross-sec. by a plane $\perp$ to the x-axis is an equilateral triangle.

(Toys! Manips: Tri, Square Cross-secs)

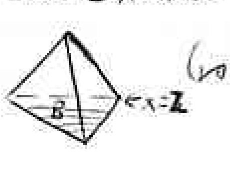
<u>Sketch B</u>



Δ slide stands here in this slot. Slots ⊥ x-axis.

Top vertices of Δs do form nice straight edge.
edge lies above
$y = \frac{3x-(-x)}{2}$
$y = x$

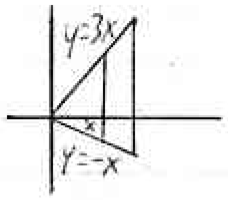
<u>Solid</u>



(rotated)

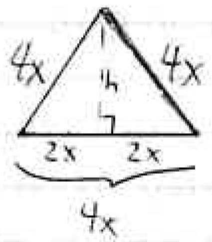
$\int$ can $\rightarrow$, so dx

Find $A(x)$

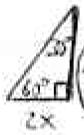


top - bot = $3x - (-x)$
$= 4x$

$A(x)$ = area of an equil. triangle of side length $4x$.



find h:



$h = 2x\sqrt{3}$, or $2\sqrt{3}x$

Long Way:

$$h^2 + (2x)^2 = (4x)^2$$
$$h^2 + 4x^2 = 16x^2$$
$$h^2 = 12x^2$$
$$h = \pm\sqrt{12x^2}$$
$$h = 2\sqrt{3}\,|x|$$
$$h = 2\sqrt{3}\,x \qquad (x>0)$$

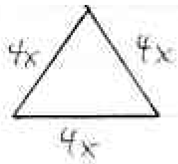
$$A(x) = \tfrac{1}{2}(\text{base})(\text{height})$$
$$= \tfrac{1}{2}(4x)(2\sqrt{3}x)$$
$$= 4\sqrt{3}x^2$$

I know 3 sides of a triangle.

Method 2 (Heron's Formula)

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

where s = semi-perimeter $= \frac{a+b+c}{2}$



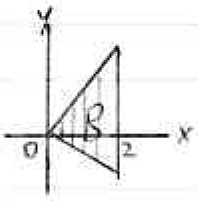
$$s = \frac{12x}{2} = 6x$$

$$\begin{aligned} A &= \sqrt{6x(6x-4x)(6x-4x)(6x-4x)} \\ &= \sqrt{6x(2x)(2x)(2x)} \\ &= \sqrt{48x^4} \\ &= 4\sqrt{3}\,x^2 \end{aligned}$$

Don't need !!

Method 3 $A = \frac{1}{2}(4x)(4x)\sin(60^\circ)$

Find V



$$\begin{aligned} V &= \int_0^2 A(x)\,dx \\ &= \int_0^2 4\sqrt{3}\,x^2\,dx \\ &= 4\sqrt{3}\left[\frac{x^3}{3}\right]_0^2 \\ &= 4\sqrt{3}\left[\frac{8}{3} - 0\right] \\ &= \frac{32\sqrt{3}}{3} \end{aligned}$$

(You)

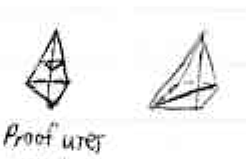
Easy: Base area = 2

Harder: Heron $2\sqrt{10}$, 8, $2\sqrt{2}$

Height: $4\sqrt{3}$

Vol. of a pyramid $= \frac{1}{3}$(Base area)(Height), like cone

(Figure out! Think a lot)



Proof uses similar triangles

either could rep the base area

or Base $= \frac{1}{2}(8)(4\sqrt{3}) = \frac{32\sqrt{3}}{2}$

Height 2

$\frac{1}{3}\left(\frac{32\sqrt{3}}{2}\right)(2)$

$A(y) = \frac{1}{2}bh$, regardless of tri shape

Pyramids w/ polygonal bases can be decomposed into pyr with tri bases

Write!

Works for curvy bases, too. Calculus idea

general cone

Cavalieri

One pyr w/ tri base → whole family