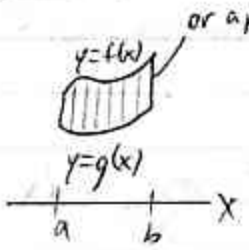
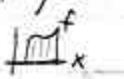
(6.1) Area



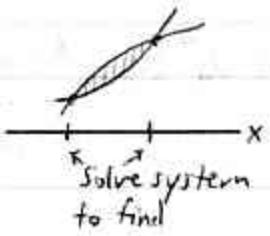
$A = \int_a^b [f(x) - g(x)]\,dx$ (∫ lengths of line segments)

top − bot

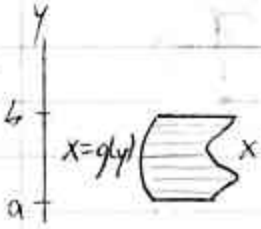
If g = 0



flush against axis, no gap



If left-right... flips sign. Do it right!



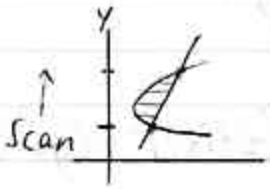
$A = \int_a^b [f(y) - g(y)]\,dy$

right − left

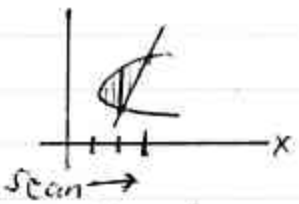
If g = 0

If you sort x, what would you have to do?

Also depends what f, g look like. What can you solve for?



vs.



$\int dy$

$\int dx + \int dx$

↑ Better?

Maybe not if hard to take eqn. and write as $x = f(y)$.

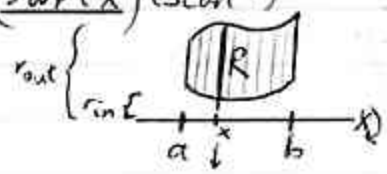
(6.2/6.3) Volumes of Solids of Revol.

Given: Axis of rev.
Know how scan ⇔ Know method

(6.2) Disk / Washer

regardless of method

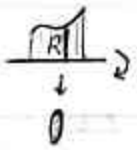
(dx)(Surf x) (Scan →)



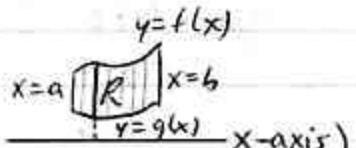
$$V = \int_a^b (\pi r_{out}^2 - \pi r_{in}^2)\, dx$$

(r_{out}, r_{in}: funcs. of x)

If = 0 ⇒ Disk

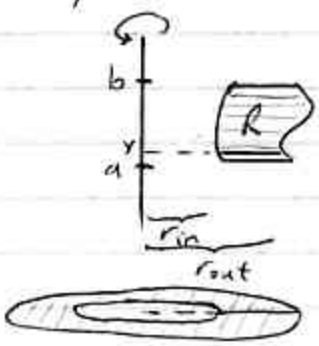


If



$$V = \int_a^b (\pi[f(x)]^2 - \pi[g(x)]^2)\, dx$$

(dy)(Surf y) (Scan ↑)

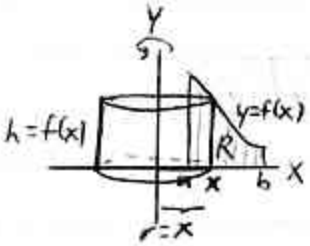


$$V = \int_a^b (\pi r_{out}^2 - \pi r_{in}^2)\, dy$$

(r_{out}, r_{in}: funcs. of y)

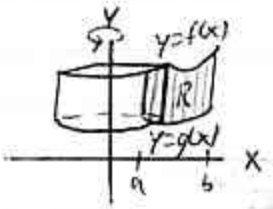
(6.3) Cyl. Shells

Lateral Surface Area $= 2\pi r h$
(funcs. of x (or y))

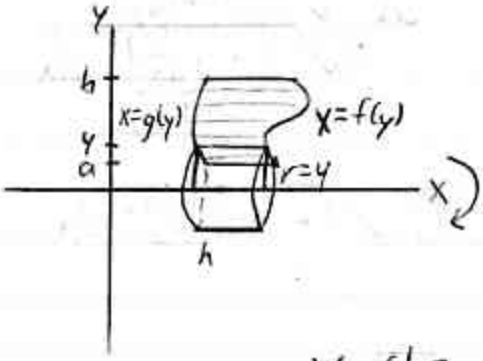


$$V = \int_a^b 2\pi x f(x)\,dx$$

What's diff. about this cyl?



$$V = \int_a^b 2\pi x \overbrace{[f(x)-g(x)]}^{h,\text{ or } h(x)}\,dx$$
top − bot



$$V = \int_a^b 2\pi y \overbrace{[f(y)-g(y)]}^{h}\,dy$$
right − left

Weird axis
Draw cylinder (doesn't affect)
Find radius, height.
Use right-left or top-bottom (← for nonnegative linear measures)

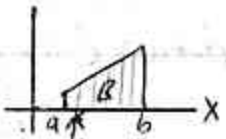
shifted axis of revol, affects radius

(6.2 also) r_{in}, r_{out}

Axis of revol. won't cross R unless you can exploit symmetry. (cutting symmetry optional)

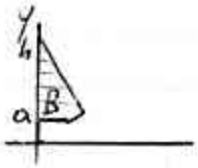
(does what top does)

(6.4) Volumes by General Cross-Sections



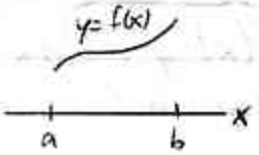
slot for cross-sec of area $A(x)$

$$V = \int_a^b A(x)\,dx$$

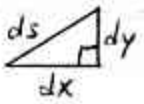


$$V = \int_a^b A(y)\,dy$$

(6.5) Arc Length



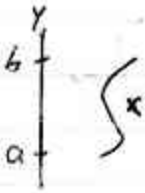
$$L = \int_{x=a}^{x=b} ds$$



$$ds = \sqrt{(dx)^2 + (dy)^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$$

$\frac{dy}{dx} \approx f'(x)$

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\,dx$$



"→" means "replaced by," not "limit"

$x \to y$ in $dx \to dy$

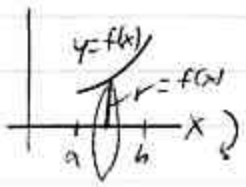
$$L = \int_{y=a}^{y=b} ds$$

$$ds = \sqrt{(dx)^2 + (dy)^2} = \sqrt{\left(\frac{dx}{dy}\right)^2 + 1}\,dy$$

$\frac{dx}{dy} \approx f'(y)$

$$L = \int_a^b \sqrt{1 + [f'(y)]^2}\,dy$$

Surfaces of Revol.

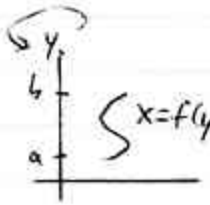


$$S = \int_{x=a}^{x=b} \underbrace{C(x)}_{\text{circum.}} ds$$

$= 2\pi r$

$$= \int_{x=a}^{x=b} 2\pi f(x)\, ds$$

$\sqrt{\ }\, dx$



$x \to y$

$\sqrt{\ }\, dy$

SKIP

(6.6) Work

Motor Ex

(6.7) Centers of Mass