8.1: INVERSE TRIG FUNCS.

give angles ($\angle$s)

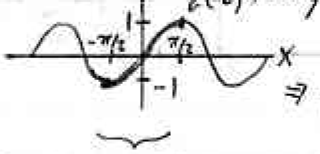
In apps

(A) $\sin^{-1}$, or arcsin

$f(x) = \sin x$ (D) $\angle \to \boxed{f} \to$ (R) sin value

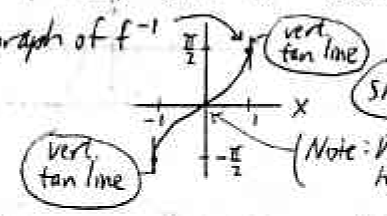
$f^{-1}(x) = \sin^{-1} x$ (D) sin value $\to \boxed{f^{-1}} \to$ (R) $\angle$

$(\frac{\pi}{2}, 1)$ on graph of f $\Rightarrow (1, \frac{\pi}{2})$ on graph of f^{-1}



fails HLT

$\Rightarrow f$ not 1-1, so...



I want to cut out a piece of the graph that does pass HLT + has same range (piece is as tall as overall graph) Changes concavity. If I have a pair of inverse funcs, the domain of one is the range of the other.

$\sin^{-1}$ ✗

(D) Restrict Domain to $[-\frac{\pi}{2}, \frac{\pi}{2}]$ → Domain $= [-1, 1]$ (D)

(R) Range still $[-1, 1]$ → Range $= [-\frac{\pi}{2}, \frac{\pi}{2}]$ (R)

RAD

For Memory:

covers each sin value once

$\sin(\frac{\pi}{6}) = \frac{1}{2}$

angle $\angle$ — sin value in $[-1, 1]$

$\sin^{-1}(\frac{1}{2}) = \frac{\pi}{6}$

sin value — angle in range $\angle$

$\sin^{-1}(3)$ is undefined

not a sin value

SKIP

what's your intuition?

$\sin(\sin^{-1}(x)) = x$

if a sin value
else undefined

Can you spit me out? $\sin^{-1}$ can spit out x

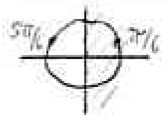
$\sin^{-1}(\sin(\theta)) = \theta$

if in $[-\frac{\pi}{2}, \frac{\pi}{2}]$
else work it out

Is this $\frac{5\pi}{6}$? Don't need for 1, 3?

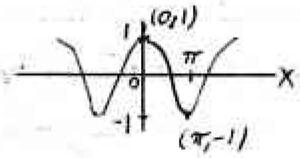
$\sin^{-1}(\sin(\frac{5\pi}{6})) = \sin^{-1}(\frac{1}{2}) = \frac{\pi}{6}$

not in $[-\frac{\pi}{2}, \frac{\pi}{2}]$

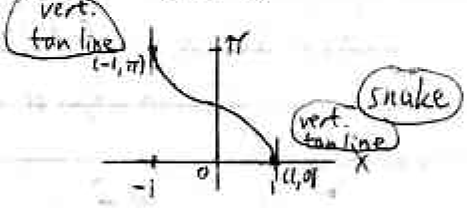


Ⓑ $\cos^{-1}$, or arccos

$\cos x$



$\cos^{-1} x$

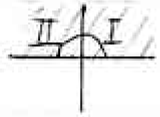


Rest. Domain $= [0, \pi]$ → Range $= [-1, 1]$

Range $= [-1, 1]$ → Domain $= [0, \pi]$

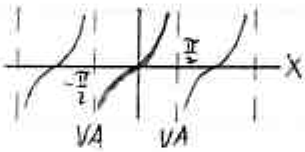
Domain $= [-1, 1]$

Range $= [0, \pi]$

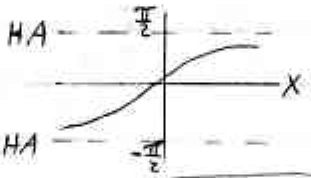


Ⓒ $\tan^{-1}$, or arctan

$\tan x$



$\tan^{-1} x$



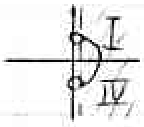
Unbounded snake approaches HAs

Rest. Domain $= \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ → Range $= \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Range $= \mathbb{R}$, or $(-\infty, \infty)$ → Domain $= \mathbb{R}$

Domain $= \mathbb{R}$

Range $= \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



covers each tan (slope) once

$\tan^{-1}(3) \approx 1.25$ (rads)

any real # is a tan value

Ps 1, 5, 7

$\left(\tan(\tan^{-1}(x)) = x \quad \forall \text{real } x\right)$

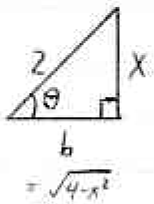
① Calc II

Like structure of quad. formula.

Ex Write $\cos(\sin^{-1}(\frac{x}{2}))$ as an algebraic ≈ rational, but roots also OK
expression in x.

$$\text{Let } \theta = \sin^{-1}(\tfrac{x}{2})$$
$$\Rightarrow \sin\theta = \tfrac{x}{2}$$

More complicated than this... See Trig. Works if we use restricted ranges for inv. trig funcs.



Don't worry about quadrants!

Pyth. Thm.:

$$x^2 + b^2 = (2)^2$$
$$b^2 = 4 - x^2$$
$$b = \pm\sqrt{4-x^2}$$

$2-x$?

$$\cos\theta = \frac{\sqrt{4-x^2}}{2}$$ ← can't split at "−"

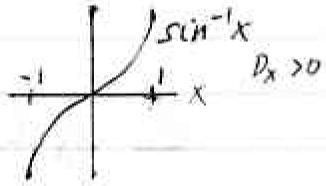
8.2: D_x, $\int$

Ⓐ D_x

useful in optim. probs involving Ls, $\int$ (put on a skirt!)

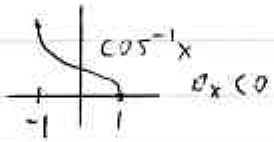
vert. tan lines at ± 1

$$\boxed{D_x(\sin^{-1}x) = \underbrace{\frac{1}{\sqrt{1-x^2}}}_{\text{algebraic!}} \quad \text{SHOCKING!!} \qquad \text{if } x \text{ is in } (-1,1)}$$

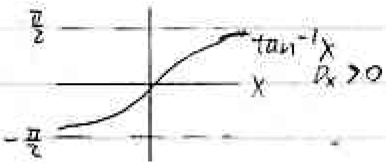


$\int$R145 $= -\sin^{-1}x + C$
$= \cos^{-1}x + C$

$$\boxed{D_x(\cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}} \qquad \text{if } x \text{ is in } (-1,1)}$$



$$\boxed{D_x(\tan^{-1}x) = \frac{1}{1+x^2} \qquad \text{for all real } x}$$



based on Swokowski's choice

$$\boxed{D_x(\sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}} \text{ or } \frac{1}{|x|\sqrt{x^2-1}} \qquad \underline{\text{if}}\ |x|>1}$$

depends on which range is used for $\sec^{-1}x$

Chain Rule applies

Ex $D_x(\sin^{-1} u) = \frac{1}{\sqrt{1-u^2}} \cdot \underbrace{D_x u}_{\text{tail}}$

Ex $D_x[\sin^{-1}(\underbrace{4x+1}_{=u})] = \frac{1}{\sqrt{1-u^2}} \cdot D_x u$

$= \frac{1}{\sqrt{1-(4x+1)^2}} \cdot \overbrace{D_x(4x+1)}^{=4}$

up to 17

$= \frac{4}{\sqrt{1-(4x+1)^2}}$

Optional

Prove $D_x(\overbrace{\sin^{-1} x}^{\text{"}\theta\text{"}}) = \frac{1}{\sqrt{1-x^2}}$

Assume x in $(-1,1)$.

Let $\theta = \sin^{-1} x$

$\Rightarrow \sin\theta = x$ $\quad$ θ in $(-\frac{\pi}{2}, \frac{\pi}{2})$ $\quad$ I, IV

Imp. Diff.

$D_x(\sin\theta) = D_x(x)$

$(\cos\theta)(D_x\theta) = 1$

$\cos\theta \neq 0$ in IV

Solve for $D_x\theta = \frac{1}{\cos\theta}$

Pyth. ID: $\sin^2\theta + \cos^2\theta = 1$

Algebraic relation $\Rightarrow$ $\cos\theta = \pm\sqrt{1-\sin^2\theta}$

$\cos\theta > 0$

IV, I θ in

$D_x\theta = \frac{1}{\sqrt{1-\sin^2\theta}}$ $\quad$ $\left(\cos\theta = \sqrt{1-x^2}\right)$

Go back to x.

$D_x(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

<u>Ex</u> $D_x[\tan^{-1}(\underbrace{x^4}_{=u})] = \frac{1}{1+u^2} \cdot \underbrace{D_x u}_{\text{tail}}$

$= \frac{1}{1+(x^4)^2} \cdot \underbrace{D_x(x^4)}_{4x^3}$

$= \boxed{\frac{4x^3}{1+x^8}}$

You don't have to go thru there formalities. We can do it directly $D_x(\tan^{-1}(\text{blah}))$

<u>Ex</u> $D_x[\tan^{-1}(\ln x)] = \frac{1}{1+(\ln x)^2} \cdot \underbrace{\frac{1}{x}}_{\text{tail}}$

(A) D_x (inv. trig fs)
(B) $\int$ Use rules to $\int$ funcs we couldn't $\int$ before.
8.9(i) no typo

(B) $\int$

$D_x(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}} \Rightarrow \boxed{\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x + C}$

$D_x(\cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}} \Rightarrow \int -\frac{1}{\sqrt{1-x^2}}\,dx = \cos^{-1}x + C$

or $-\sin^{-1}x + C$

$D_x(\tan^{-1}x) = \frac{1}{1+x^2} \Rightarrow \boxed{\int \frac{1}{1+x^2}\,dx = \tan^{-1}x + C}$

Swok. Range
Larson

$D_x(\sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}} \Rightarrow \boxed{\int \frac{1}{x\sqrt{x^2-1}}\,dx = \sec^{-1}x + C}$

or $|x|$ if using another book's range for $\sec^{-1}x$ — or $|x|$

Keep on board

Formula avail in book: $\int \frac{1}{\sqrt{a^2-u^2}}\,du = \sin^{-1}\left(\frac{u}{a}\right) + C$

Ex $\int \frac{1}{\sqrt{9-x^2}}\,dx$ — want 1

Use: $\int \frac{1}{\sqrt{1-u^2}}\,du = \sin^{-1}u + C$

$= \int \frac{1}{\sqrt{9\left(1-\frac{x^2}{9}\right)}}\,dx$

$= \int \frac{1}{3} \cdot \frac{1}{\sqrt{1-\frac{x^2}{9}}}\,dx$ — du; want u^2

Let $u = \frac{x}{3} \Rightarrow u^2 = \frac{x^2}{9}$
$du = \frac{1}{3}dx$

$= \int \frac{du}{\sqrt{1-u^2}}$

$= \sin^{-1}u + C$
$= \sin^{-1}\left(\frac{x}{3}\right) + C$

Good to Know: (Patterns)	$\int \frac{1}{\sqrt{9-x^2}}\,dx = \sin^{-1}\left(\frac{x}{3}\right) + C$ (No $\frac{1}{3}$)
	$\int \frac{1}{9+x^2}\,dx = \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + C$
	$\int \frac{1}{x\sqrt{x^2-9}}\,dx = \frac{1}{3}\sec^{-1}\left(\frac{x}{3}\right) + C$

$\sqrt{9} = 3$

DO IF
HAVE TIME

Ex $\int \frac{x}{x^4+25}\,dx$

Use: $\int \frac{1}{1+u^2}\,du = \tan^{-1}u + C$

$= \int \frac{x}{25+x^4}\,dx$

Want 1

$= \int \frac{1}{25} \cdot \frac{x}{\left(1+\frac{x^4}{25}\right)}\,dx$

want u^2

Let $u = \frac{x^2}{5}$ $\left(\Rightarrow u^2 = \frac{x^4}{25}\right)$
$du = \frac{2}{5}x\,dx$ $\left(\Rightarrow \frac{5}{2}\,du = x\,dx\right)$

$= \frac{1}{\cancel{25}_{5}} \cdot \frac{\cancel{5}}{2} \int \frac{\frac{2}{5}x}{1+u^2}\,dx$

du

$= \frac{1}{10} \int \frac{du}{1+u^2}$

$= \frac{1}{10}\tan^{-1}u + C$

$= \frac{1}{10}\tan^{-1}\left(\frac{x^2}{5}\right) + C$

8.3: HYPERBOLIC FUNCS.

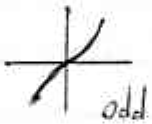
(A) Intro

$\sinh x$ = hyperbolic sine of x

like $\sin^2 x = \frac{1-\cos(2x)}{2}$

"sin is bad"

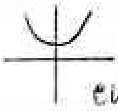
$$= \frac{e^x - e^{-x}}{2}$$



odd

cawsh

$$\cosh x = \frac{e^x + e^{-x}}{2}$$



even

hanging cables

catenary: $y = a\cosh\left(\frac{x}{a}\right)$

tansh
setch
cosetch
cotansh

$$\tanh x = \frac{\sinh x}{\cosh x}$$
$$\operatorname{sech} x = \frac{1}{\cosh x}, \text{ etc.}$$

4 are 1-1 except cosh, sech

Can $\doteq$ sinh^{-1}x, cosh^{-1}x $\to$ others

not periodic

$$\cosh^2 x - \sinh^2 x = 1$$

cos, sin are circular functions

If θ acute, $\theta = 2A$ for both

See my web site for more info.

cosh, sinh are hyperbolic functions

(B) D_x

What's $D_x(\sin x)$? For 151...

$D_x(\sinh x) = \cosh x$ $\quad$ $D_x(\cosh x) = \sinh x$

NO "$-$"

$D_x(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$ $\quad$ $D_x(\operatorname{csch} x) = -\operatorname{csch} x \coth x$

"$-$"

$D_x(\tanh x) = \operatorname{sech}^2 x$ $\quad$ $D_x(\coth x) = -\operatorname{csch}^2 x$

Chain Rule applies

$D_x(\sinh u) = (\cosh u)(D_x u)$, etc.

Ex $D_x[\cosh(4x)] = \sinh(4x) \cdot 4$
$= 4\sinh(4x)$

Overall, we taking a deriv of a ... power

Ex $D_x[\coth^3(2x)] = D_x[\coth(2x)]^3$
$= 3[\coth(2x)]^2 \underbrace{D_x[\coth(2x)]}_{-\operatorname{csch}^2(2x)\cdot 2}$
$= -6\coth^2(2x)\operatorname{csch}^2(2x)$

Ⓒ $\int$

As expected, but

$$\int \sinh x \, dx = \cosh x + C$$

NO "−"

$$\int \operatorname{sech} x \tanh x \, dx = -\operatorname{sech} x + C$$

"−"

Ex $\int x \operatorname{sech}^2(7x^2+1)\,dx$

Let $u = 7x^2+1$
$du = 14x\,dx$

$$= \frac{1}{14}\int \underbrace{14x \operatorname{sech}^2 u \, dx}_{du}$$

$$= \frac{1}{14}\int \operatorname{sech}^2 u \, du$$
$$= \frac{1}{14}\tanh u + C$$
$$= \frac{1}{14}\tanh(7x^2+1) + C$$

MISI $\int$ by Parts

$\int \ln x \, dx$