MATH 150 - CHAPTER 3 REVIEW

SECTION 3.1: INTRODUCTION

Secant lines (related to the graph of \( y = f(x) \))

slope = average rate of change of \( y \) with respect to \( x \) on \([a, a+h] \) or \([a,b]\)

\[slope = \frac{f(a+h) - f(a)}{h} \quad \text{or} \quad \frac{f(b) - f(a)}{b-a}\]

If \( h \approx 0 \), this = \( f'(a) \), if it exists.

Tangent lines

slope = instantaneous rate of change of \( y \) with respect to \( x \) at \( a \)

\[slope = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}\] if it exists. Use algebra to work through this!

\[= f'(a) \quad \text{(You may use our shortcuts unless you’re told to use the limit definition.)}\]

Slopes and equations of tangent lines

Rectilinear motion

Velocity = [Instantaneous] rate of change of position with respect to time.
\( v(t) = s'(t) \)

SECTION 3.2: \( f''(x) \)

Limit definition of the derivative

\[f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}\] if it exists. Use algebra to work through this!

Key cases when the derivative is “DNE”

Where there is a…

\( (m_L = \text{left - hand derivative}; m_R = \text{right - hand derivative}) \)

1) Discontinuity
2) Corner \( m_L \neq m_R \) (Maybe one is \( \pm \infty \), but not both.)
3) Vertical tangent line \( m_L \) is \( \infty \) or \(-\infty \), and \( m_R \) is \( \infty \) or \(-\infty \).
4) Cusp \( m_L \) and \( m_R \); one is \( \infty \); the other is \(-\infty \).
   This is a special case of 3).
SECTIONS 3.2 and 3.3: SHORTCUTS FOR DIFFERENTIATION

$c, m, b, and n$ are constants; $f$ and $g$ are functions of $x$.

<table>
<thead>
<tr>
<th>Derivative with respect to $x$</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>0</td>
</tr>
<tr>
<td>$mx + b$</td>
<td>$m$</td>
</tr>
<tr>
<td>$x^n$</td>
<td>$nx^{n-1}$</td>
</tr>
<tr>
<td>$cf$</td>
<td>$cf'$</td>
</tr>
<tr>
<td>$f + g$</td>
<td>$f' + g'$</td>
</tr>
<tr>
<td>$f - g$</td>
<td>$f' - g'$</td>
</tr>
<tr>
<td>$fg$</td>
<td>$f'g + fg'$</td>
</tr>
<tr>
<td>$\frac{f}{g}$</td>
<td>$\frac{gf' - fg'}{g^2}$</td>
</tr>
</tbody>
</table>

Solve $f'(x) = 0$. (Where are tangent lines horizontal?)
Slopes and equations of tangent lines and normal lines

SECTION 3.4: TRIG

RADIANT mode!!!

Prove $D_x (\sin x) = \cos x$, and $D_x (\cos x) = -\sin x$ using all of these:

1) The limit definition of the derivative

2) $\lim_{h \to 0} \frac{\sin h}{h} = 1$

3) $\lim_{h \to 0} \frac{\cos h - 1}{h} = 0$

Derive these using $D_x (\sin x) = \cos x$, $D_x (\cos x) = -\sin x$, the Quotient Rule, and/or the Reciprocal Rule:

$$D_x (\tan x) = \sec^2 x \quad D_x (\cot x) = -\csc^2 x$$
$$D_x (\sec x) = \sec x \tan x \quad D_x (\csc x) = -\csc x \cot x$$

Use the table above (in 3.2/3.3) and trig identities/formulas.

Solve $f'(x) = 0$ by solving a trig equation. (Where are tangent lines horizontal?)
Slopes and equations of tangent lines and normal lines
SECTION 3.5: DIFFERENTIALS and LINEAR APPROXIMATIONS

\[
f(x + \Delta x) \approx f(x) + f'(x)\Delta x
\]

Known exactly \(= f'(x)dx \) \(= \text{slope}[\text{run}], \)
where run = new \(x\) - old \(x\)
change in \(y\) along tangent line at \((x, f(x))\) “rise”

SECTION 3.6: CHAIN RULE

Let’s say \(u\) is a function of \(x\).

\[
\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}
\]

Generalized Power Rule

\[
D_u u^n = (nu^{n-1})(D_u u)
\]

Generalized Trig Rules

\[
D_u (\sin u) = (\cos u)(D_u u)
\]

\[
D_u (\sec u) = (\sec u \tan u)(D_u u), \text{ etc.}
\]

Think “Big Picture”: Overall, do I have a power, or a trig function? Use techniques and deal with issues discussed on the previous page.

SECTION 3.7: IMPLICIT DIFFERENTIATION

Implicitly differentiate functions
Given an equation, find \(y'\).
Slopes and equations of tangent lines and normal lines

SECTION 3.8: RELATED RATES

1) Read the problem
2) Diagram (general; nothing specific to the instant of interest)
3) Set up the key relevant formula
4) Know what is given and what must be found at the instant of interest
5) Implicitly differentiate the formula in 3)
6) Plug in values at the instant of interest
   Diagrams, the Pythagorean Theorem and/or other formulas may help.
7) Solve for the desired rate (don’t forget units)
   Remember issues of unit compatibility and DEG vs. RAD.
8) State your conclusion using good English. (I'll tell you when you have to do this.)