Note: These are guidelines, not official rules. You can be more creative!

SETTING UP THE PROBLEM

Step 1: Read the problem.

Step 2: Set up a diagram, a table, etc.

• Indicate what is given, and introduce variables as necessary.

Step 3: Write the primary equation (PE).

• Identify the variable that we are trying to maximize or minimize.
  For this handout, let’s call this variable “A” for convenience.

• Write an equation that expresses A in terms of the other variables. Sometimes, you may want to write an “easier” equation involving A, first, and then solve for A.

Step 4: Write any relevant secondary equations (SEs).

• These are constraints imposed by other given information in the problem, basic relationships (for example, diameter = 2*radius, Pythagorean Thm.), etc.

• These equations indicate relationships among the variables, possibly including new variables that you introduce.

Step 5: Express A in terms of only one variable.

• Use the secondary equations to express the other variables in terms of just one variable, possibly a new variable that you have introduced.

• Substitute into the primary equation.
  If x, say, is the last remaining independent variable, you should get something of the form $A = f(x)$.

Step 6: Write the “feasible domain” of f.

• What are the values for x that “make sense” in the context of the problem?

  Note: Sometimes, only integer or whole-number values for x make sense, but we often ignore this restriction. Sometimes, we round off/up/down our answer.

• No zero denominators, negative radicands for even roots, etc.
**MAXIMIZING / MINIMIZING $f$**

**What if the feasible domain is a closed interval, $[a, b]$?**

If $f$ is continuous on $[a, b]$, then we can look back to Section 4.1 and apply the Extreme Value Theorem (EVT). Then, $f$ must have absolute maximum and minimum values on $[a, b]$. These must occur at critical numbers or (for “endpoint extrema”) at $a$ or $b$.

- Find the critical numbers of $f$ in the interval $(a, b)$.
- Evaluate $f(a)$, $f(c)$ for each relevant critical number $c$, and $f(b)$.
- The highest value among the aforementioned “candidates” is the absolute maximum value of $f$ on the interval, and the lowest value corresponds to the absolute minimum value. There may be “ties” – we can have more than one absolute maximum point, for example, corresponding to same max value.

**Note:** The First Derivative Test and the Second Derivative Test can be used as alternatives, but consider global behavior, not just local behavior!

**What if the feasible domain is not a closed interval?**

Then, the EVT does not apply, and we are not guaranteed to have absolute maximum or minimum values for $f$.

Again, find critical numbers of $f$ in the feasible domain.

Some issues:

- If $f$ is continuous on the feasible domain (assume the domain has no "gaps"), and if there is only one critical number there, then a local maximum [minimum] point (at the critical number) must also be the absolute maximum [minimum] point.

- Should you investigate what happens as $x \to \infty$ and/or as $x \to -\infty$?

- If the feasible domain is $(a, b)$, could you consider the closed interval $[a, b]$ if $f$ is continuous there? Otherwise, you may need to investigate the behavior of $f$ as $x \to a^+$ and as $x \to b^-$.

- If there are discontinuities, investigate them.

**Graphs may help!**

**ANSWER THE QUESTION(S)!!**

- The problem may require you to give more than one final answer!
- Box in your final answer(s); make sure they’re the ones asked for in the problem.
  - Does (Do) your answer(s) make sense?
- Approximate or round off/up/down as appropriate.
- Include units as appropriate.