MATH 150
<u>SECTION 4.6: OPTIMIZATION</u>

Note: These are guidelines, not official rules. You can be more creative!

SETTING UP THE PROBLEM

Step 1: Read the problem.

Step 2: Set up a diagram, a table, etc.

- Indicate what is given, and introduce variables as necessary.

Step 3: Write the primary equation (PE).

- Identify the variable that we are trying to maximize or minimize.
  For this handout, let's call this variable "A" for convenience.

- Write an equation that expresses A in terms of the other variables. Sometimes, you may want to write an "easier" equation involving A, first, and then solve for A.

Step 4: Write any relevant secondary equations (SEs).

- These are constraints imposed by other given information in the problem, basic relationships (for example, diameter = 2*radius, Pythagorean Thm.), etc.

- These equations indicate relationships among the variables, possibly including new variables that you introduce.

Step 5: Express A in terms of <u>only one</u> variable.

- Use the secondary equations to express the other variables in terms of <u>just one</u> variable, possibly a <u>new</u> variable that you have introduced.

- Substitute into the primary equation.
  If x, say, is the last remaining independent variable, you should get something of the form $A = f(x)$.

Step 6: Write the "feasible domain" of f.

- What are the values for x that "make sense" in the context of the problem?

  <u>Note</u>: Sometimes, only integer or whole-number values for x make sense, but we often ignore this restriction. Sometimes, we round off/up/down our answer.

- No zero denominators, negative radicands for even roots, etc.

MAXIMIZING / MINIMIZING f

What if the feasible domain is a closed interval, $[a, b]$?

If f is continuous on $[a, b]$, then we can look back to Section 4.1 and apply the Extreme Value Theorem (EVT). Then, f must have absolute maximum and minimum values on $[a, b]$. These must occur at critical numbers or (for "endpoint extrema") at a or b.

- Find the critical numbers of f in the interval (a,b).
- Evaluate $f(a)$, $f(c)$ for each relevant critical number c, and $f(b)$.
- The highest value among the aforementioned "candidates" is the absolute maximum value of f on the interval, and the lowest value corresponds to the absolute minimum value. There may be "ties" – we can have more than one absolute maximum point, for example, corresponding to same max value.

  Note: The First Derivative Test and the Second Derivative Test can be used as alternatives, but consider global behavior, not just local behavior!

What if the feasible domain is not a closed interval?

Then, the EVT does not apply, and we are not guaranteed to have absolute maximum or minimum values for f.

Again, find critical numbers of f in the feasible domain.

Some issues:

- If f is continuous on the feasible domain (assume the domain has no "gaps"), and if there is only one critical number there, then a local maximum [minimum] point (at the critical number) must also be the absolute maximum [minimum] point.
- Should you investigate what happens as $x \to \infty$ and/or as $x \to -\infty$?
- If the feasible domain is (a, b), could you consider the closed interval $[a, b]$ if f is continuous there? Otherwise, you may need to investigate the behavior of f as $x \to a^+$ and as $x \to b^-$.
- If there are discontinuities, investigate them.

Graphs may help!

ANSWER THE QUESTION(S)!!

- The problem may require you to give more than one final answer!
- Box in your final answer(s); make sure they're the ones asked for in the problem. Does (Do) your answer(s) make sense?
- Approximate or round off/up/down as appropriate.
- Include units as appropriate.