Dual Dig Level I (2013)

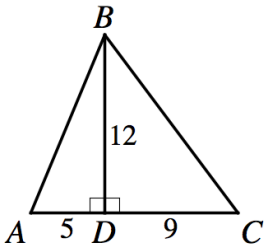
1. A wooden cube has a volume of 27 in^3 and is painted black on all sides. A very, very thin saw blade is used to cut the original cube into 27 cubes of side length 1 inch. What fraction of the cubes will have NO black paint on them whatsoever?

2. Simplify as a fraction: $\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\left(1-\frac{1}{5^2}\right)$

3. On Sunday, a gas company promises to increase its gas prices by 200% by that Friday at noon. If the company raises prices by 50% by Wednesday at noon, by what percent must prices rise from Wednesday at noon to Friday at noon if the company is to keep its promise?

4. Factor using integer coefficients: $2x^3-5x^2+6x-15$

5. What is the perimeter of triangle ABC below? $\overline{AD}$ is 5 inches long, $\overline{BD}$ is 12 inches long, and $\overline{CD}$ is 9 inches long. $\overline{BD}$ is perpendicular to $\overline{AC}$.



6. Solve for R: $2aR - 3R = 5bR + 6b$

7. Give the remainder when $3x^3-2x+4$ is divided by x^2-1.

8. A Norman window (depicted below) consists of a semicircular region with diameter forming the top of a rectangle of width x. If the height of the rectangular part of the window is 6 feet less than four times the width, write an expression in x for the <u>perimeter</u> of the entire window (in feet).

9. Three generations of the Foster family, two members from each generation, are going to the Del Mar Fair. The two members of the youngest generation receive a 50% discount as children. The two members of the oldest generation receive a 25% discount as senior citizens. The two members of the middle generation receive no discount. Grandfather Foster, whose senior ticket costs $6.00, is paying for everyone. How many dollars must he pay?

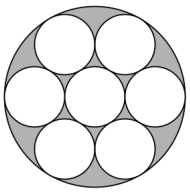
10. Simplify by writing as a single non-compound (non-complex) fraction: $1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+x}}}$

11. How many liters of pure water must be added to 12 liters of 30% antifreeze solution to produce a 20% solution?

12. For $f(x)=\dfrac{2}{x+1}$, find the difference quotient: $\dfrac{f(x+h)-f(x)}{h}$ in simplest form.

13. Find the equation of a parabola (in the form $y = ax^2 + bx + c$) with vertex at $(3, -8)$ and passing through the point (6, 10).

14. Find the sum of the areas of the shaded regions below. The seven white circular disks each has radius 1 inch. The innermost white disk is tangent to the six surrounding white disks, and those six disks are each tangent to the larger circle and to three neighboring white disks.



15. A sequence of a_n terms, starting with a_1, is defined by: $a_n=(-1)^{n+1}\cdot n$. The first six terms, for example, are given by: $1,-2,3,-4,5,-6$. Find the sum of the first 1000 terms of the sequence.

16. Ken competes in a triathlon. He averages 2 miles per hour in the 1/4-mile swim and 6 miles per hour in the 3-mile run. His goal is to finish the triathlon in 2 hours. To accomplish his goal, what must his average speed (in mph) be for the 15-mile bicycle ride?

17. In the magic square shown, the sums of the numbers in each row, column, and diagonal are the same. Five of the numbers are represented by v, w, x, y, and z. Find $y + z$.

v	24	w
18	x	y
25	z	21

18. Give one solution (x, y, z) to the system: $\begin{cases} x^3+y^3+z^3=495 \\ x+y+z=15 \\ xyz=105 \end{cases}$, where x, y, and z are positive integers.

19. You stand before three boxes, each with two coins. One box contains one gold coin and one silver coin. Another contains two gold coins. Another contains two silver coins. You do not know which box is which. You randomly select a box and randomly take out one of the coins. You see that the coin is gold. What is the probability that the other coin in the selected box is also gold?

20. Find the greatest integer exponent n for which 2013! is divisible by 2^n (meaning $2^n \cdot k = 2013!$ for some positive integer k). 2013! is "2013 factorial," which is obtained from: $(1)(2)(3)(4)\cdots(2013)$.