Dual Dig Level II (2006)

1. Find the equation of the parabola in the usual xy-plane that passes through the points $(0,2)$, $(1,1)$, and $(-1,7)$.

2. Find the equation (in factored form) of the cubic function of x that has 5, 2, and -4 as zeros and a graph with y-intercept 20.

3. Give the remainder when $2x^4+10x^2-6x+2$ is divided by $2x^2+6$.

4. It takes Larry 9 more hours than Ken to grade a stack of math Final Exams. Working together they can grade all of the papers in 20 hours. How long does it take Larry to grade the papers working alone?

5. A, B, and C are vertices of a cube such that AB is a diagonal of one face of the cube, and BC is a diagonal of an adjacent face. Find the measure of Angle ABC in degrees.

6. The fraction $\dfrac{2x^2+8x+6}{x^2+2x}$ can be expressed as a sum in the form $\dfrac{A}{x}+\dfrac{Bx+C}{x+2}$, where A, B, and C are integers. Find the value of C.

7. The degree measures of the five interior angles of a pentagon are the first five terms in an arithmetic sequence. Find the measure of the third smallest angle (i.e., the "middle" angle) in degrees.

8. How many positive integers less than 100 have no positive odd integer divisors other than 1?

9. Consider a square with perimeter 32 inches. A 2nd (new) square is formed inside the original square by joining the midpoints of the original square. A 3rd square is formed by joining the midpoints of the 2nd square, and a 4th square is formed by joining the midpoints of the 3rd square. What is the total perimeter of all four squares?

10. Find the determinant: $\begin{vmatrix} 3 & 12 & -90 \\ 0 & 4 & 700 \\ 0 & 0 & 5 \end{vmatrix}$.

11. Factor completely as a product of linear factors: x^3-3+3x^2-x.

12. Tweedledee randomly tosses two fair coins and then (truthfully) tells Tweedledum that at least one of them came up "heads." Given this information, what is the probability that both coins came up "heads"?

13. The five students in a class take a test, and their average score is 64 points. After discovering that the top-scoring student had cheated, the teacher computes the average of the four other scores and finds that that average is 60 points. What score did the cheating student originally get on the test?

<u>(TURN OVER SHEET)</u>

14. A diagonal of a regular polygon is any line segment whose endpoints are two distinct vertices of the polygon and is not a side of the polygon. How many diagonals does a 16-sided regular polygon have?

15. f is a linear function such that $f(x) = mx + b$ for some real numbers m and b. It is known that f is its own inverse, but it is **not** the identity function given by $f(x) = x$. What must m be?

16. Simplify the number: $\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\ldots}}}}$. Hint: Consider the square of the number.

17. Find the difference between the circumference of a circle of radius 1 centimeter and the perimeter of a regular hexagon inscribed within the circle. (By "inscribed," we mean that all six vertices of the hexagon lie on the circle.) Round off your answer to the nearest hundredth of a centimeter.

18. Find the digit in the ones place of the number given by: $0! + 1! + 2! + \ldots + 2006!$

19. At a particular school, each student must study French and/or German. $\frac{1}{3}$ of the students who study French also study German, and $\frac{1}{4}$ of the students who study German also study French. What fraction of the students at the school study German?

20. Three players, *A*, *B*, and *C*, take turns rolling a standard six-sided die. The game ends as soon as somebody rolls a "6" on the die, in which case that player wins the game. Player *A* rolls first, then *B*, then *C*, then *A* rolls again, and so forth, until someone wins. What is the exact probability that Player *A* will win this game?