

# FACTORING PROBLEMS

DON'T TURN THIS IN. THE SOLUTIONS FOLLOW THE PROBLEMS.

a)  $x^2 + 9x + 20$

b)  $x^2 - 20x + 100$  (Hint: This is a Perfect Square Trinomial (PST).)

c)  $x^2 - 4x - 12$

d)  $3x^2 - 20x - 7$

e)  $4x^2 + 11x + 6$

f)  $2x^2 + 10x + 5$

g)  $-3x^2 - 6x - 3$

h)  $x^4 - 16$

i)  $a^3 - 3a + 2a^2b - 6b$  (Hint: Use Factoring by Grouping.)

j)  $4x^2 + 9y^2$

k)  $x^3 + 125y^3$

l)  $x^3 - 125y^3$

## Solutions

a)  $ax^2 + bx + c = x^2 + 9x + 20 = (x + 5)(x + 4)$

We want 5 and 4, because they have product =  $c = 20$  and sum =  $b = 9$ .  
(This is the  $a = 1$  case.) We can rearrange the factors:  $(x + 4)(x + 5)$ .

b)  $\underbrace{x^2}_{(x)^2} - 20x + \underbrace{100}_{(10)^2} = \underbrace{(x - 10)^2}_{\text{Check: } 2(x)(-10) = -20x}, \text{ or } (x - 10)(x - 10) = (x - 10)^2$   
PST?

c)  $x^2 - 4x - 12 = (x - 6)(x + 2)$

How do we know we need  $-6$  and  $+2$ ?

The constant term,  $c$ , is negative, so we need split signs: one "+" and one "-."

The middle coefficient,  $b$ , is negative, so the negative number is "heavier."

2-factorizations of $-12$ (which is $c$ ) What $\cdot$ What = $-12$ ?	Sum = $b = -4$ ?
$-12, +1$	No
<b><math>-6, +2</math></b>	<b>Yes - Can stop</b>
$-4, +3$	No

d)  $F + (O + I) + L = 3x^2 - 20x - 7 = (3x + 1)(x - 7)$

Reasoning:

$F$  = First product (product of the First terms)

$O$  = Outer product (product of the Outer terms)

$I$  = Inner product (product of the Inner terms)

$L$  = Last product (product of the Last terms)

$(3x \quad \quad)(x \quad \quad) \leftarrow$  Really only one way to split  $F = 3x^2$

$\downarrow$  Need  $L = -7$

$+7 \quad -1$

$-1 \quad +7 \leftarrow$  Makes  $O + I = 20x$ . We need  $O + I$  to be  $-20x$ ,

which is the middle term of the trinomial.

We're only off by a sign, so the trick is to

just change both signs. If you know this

trick, you only need to consider half as

many combinations.

$+1 \quad -7 \leftarrow$  Makes  $O + I = -20x$ . This works.

$-7 \quad +1$

"Soft" reasoning:  $b = -20$  is a "very negative" coefficient, so we may want to pair up the  $3x$  and the  $-7$  to form the outer product, since they form  $-21x$ .

Note: You could have also used Factoring by Grouping.

e)  $4x^2 + 11x + 6 = (4x + 3)(x + 2)$

Method 1: Trial-and-Error ("Guess") Method

$$\begin{array}{cc}
 ( & ) \\
 \mathbf{4x} & \mathbf{x} \\
 2x & 2x \\
 + 1 & + 6 \\
 + 6 & + 1 \\
 + 2 & + 3 \\
 + 3 & + 2
 \end{array}
 \left. \begin{array}{l} \\ \\ \\ \\ \\ \\ \\ \end{array} \right\} \begin{array}{l} F = 4x^2 \\ \\ L = 6; \text{ Need both " + " because of } + 11x \end{array}$$

Method 2: Factoring by Grouping

4 and 6 are not prime or "1," so we may prefer this method. We want two integers whose product is  $ac = (4)(6) = 24$  and whose sum is  $b = 11$ . We want 8 and 3; split the middle term accordingly.

$$\begin{aligned}
 4x^2 + 11x + 6 &= 4x^2 + \underbrace{8x + 3x}_{\text{OK to switch}} + 6 \\
 &= (4x^2 + 8x) + (3x + 6) \\
 &= 4x(x + 2) + 3(x + 2) \quad \leftarrow \text{"Local factoring"} \\
 &= (4x + 3)(x + 2) \quad \leftarrow \text{"Global factoring"}
 \end{aligned}$$

f)

$2x^2 + 10x + 5$  is **prime** or **irreducible over the integers** (i.e., cannot be broken down further using integer coefficients). None of these combinations work:

Reasoning:

$$\begin{array}{cc}
 (2x & ) \\
 + 1 & + 5 \\
 + 5 & + 1
 \end{array}
 (x & ) \leftarrow \text{Really only one way to split } F = 2x^2$$

↓ Need  $L = 5$ ; Need both " + " because of  $+ 10x$

or use the Test for Factorability (see [Notes 1.48](#)):

$$b^2 - 4ac = (10)^2 - 4(2)(5) = 100 - 40 = 60, \text{ which is neither a perfect square nor 0,}$$

and the GCF (Greatest Common Factor) = 1, so the polynomial is prime.

g)

$$-3x^2 - 6x - 3 = \underbrace{-3}_{\text{GCF}}(x^2 + 2x + 1)$$

You should usually factor out the GCF first.

$$= -3(x + 1)^2$$

h) We need to apply the Difference of Two Squares rule  $[a^2 - b^2 = (a + b)(a - b)]$  twice:

$$\begin{aligned} \underbrace{x^4}_{(x^2)^2} - \underbrace{16}_{(4)^2} &= \underbrace{(x^2 + 4)}_{\text{prime}} \left( \underbrace{x^2}_{(x)^2} - \underbrace{4}_{(2)^2} \right) \\ &= (x^2 + 4)(x + 2)(x - 2) \end{aligned}$$

i) Use Factoring by Grouping:

$$\begin{aligned} a^3 - 3a + 2a^2b - 6b &= (a^3 - 3a) + (2a^2b - 6b) \\ &= a(a^2 - 3) + 2b(a^2 - 3) \\ &= (a + 2b)(a^2 - 3) \end{aligned}$$

j)  $4x^2 + 9y^2$  is **prime**. The GCF = 1, and we have no rule for the Sum of Two Squares (for now...).

k) We need to apply the Sum of Two Cubes rule  $a^3 + b^3 = \underbrace{(a + b)}_{\text{"Expected factor"}} \left( \underbrace{a^2 - ab + b^2}_{\substack{\text{NOT} \\ -2ab}} \right)$  :  
The visible signs follow the pattern: same, different, "+"

$$\underbrace{x^3}_{(x)^3} + \underbrace{125y^3}_{(5y)^3} = (x + 5y)(x^2 - 5xy + 25y^2)$$

l) We need to apply the Difference of Two Cubes rule  $a^3 - b^3 = \underbrace{(a - b)}_{\text{"Expected factor"}} \left( \underbrace{a^2 + ab + b^2}_{\substack{\text{NOT} \\ +2ab}} \right)$  :  
The visible signs follow the pattern: same, different, "+"

$$\underbrace{x^3}_{(x)^3} - \underbrace{125y^3}_{(5y)^3} = (x - 5y)(x^2 + 5xy + 25y^2)$$