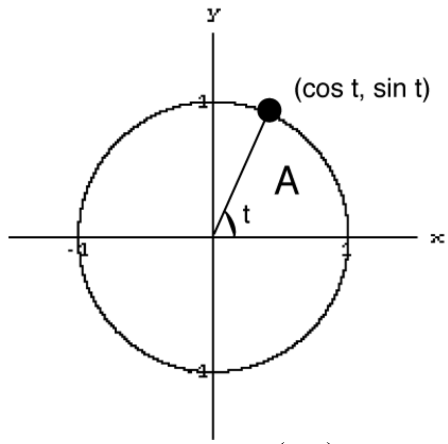
8.3: HYPERBOLIC FUNCTIONS

Circular functions (cos, sin)

Let t be a parameter measured in radians; you will study parametrizations in Chapter 13 in Calc II. ($x = \cos t$. $y = \sin t$. Remember, $\cos^2 t + \sin^2 t = 1$.) As t varies, we sweep out the unit circle given by $x^2 + y^2 = 1$.

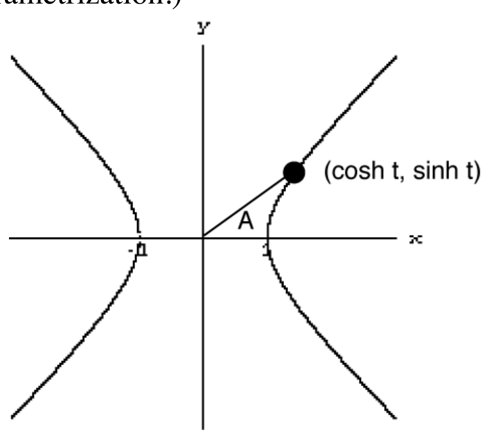


The area $A = \frac{t}{2}$ $(0 \le t \le 2\pi)$. Why?: $A = \underbrace{\left(\frac{t}{2\pi}\right)}_{\substack{t \text{ radians} \\ \text{corresponds to} \\ \text{what fraction} \\ \text{of a full} \\ \text{revolution?}}} \underbrace{(\pi)}_{\substack{\text{area} \\ \text{of a} \\ \text{unit} \\ \text{circle}}} = \frac{t}{2}$

Hyperbolic functions (cosh, sinh)

Let t be a parameter measured in "hyperbolic radians."

Consider the "unit hyperbola" given by $x^2 - y^2 = 1$, or, in particular, just the right branch of it. ($x = \cosh t \ge 1$. $y = \sinh t$. Remember, $\cosh^2 t - \sinh^2 t = 1$. In Chapter 13 in Calc II, you will study this parametrization.)



The area $A = \frac{t}{2}$, just as with the circular functions.
(If our point is below the x-axis, then A is a signed area.)

The area issues (not the angles) indicated above provide the connection between the circular and hyperbolic functions.