LESSON 7: MEASURES OF RELATIVE STANDING OR POSITION (SECTION 3-4)

How high or low is a data value relative to the others? We want standardized measures that will work for practically all populations involving quantitative data.

PART A: z SCORES

\[ z = \frac{x - \text{mean}}{\text{SD}} \]  

Idea:  
\[ \Rightarrow \text{new mean} = 0 \]  
\[ \Rightarrow \text{new SD} = 1 \]

We are transforming the original data set of x-values into a new data set of z-values.

\[ x_1 \quad x_2 \quad x_3 \quad \text{etc.} \]
\[ \downarrow \quad \downarrow \quad \downarrow \]
\[ z_1 \quad z_2 \quad z_3 \quad \text{etc.} \]

Subtracting off the mean from all of the original x-values recenter the data set so that the new mean will be 0. Then, dividing by the SD rescales the data set so that the new SD will be 1.

Notation

| Population (Size N) | \[ z = \frac{x - \mu}{\sigma} \] |
| Sample (Size n) | \[ z = \frac{x - \bar{x}}{s} \] |

Round off z scores to two decimal places.

They have no units, because we divide by the SD. We will be able to use z scores in many different applications where different units are involved. If we are studying heights, for instance, we may use inches, feet, meters, etc. and still obtain the same z scores.
### z Score Rule of Thumb for “Unusual” Data Values

Data values whose corresponding $z$ scores are either less than $-2$ or greater than $2$ (i.e., $z < -2$ or $z > 2$) are often considered “unusual.”

In other words, data values that are more than 2 SDs from the mean are often considered “unusual.”

<table>
<thead>
<tr>
<th>Class 1</th>
<th>Class 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z = \frac{70 - 55}{5} = 3.00$</td>
<td>$z = \frac{70 - 60}{10} = 1.00$</td>
</tr>
</tbody>
</table>

A score of 70 points is more impressive in Class 1; in fact, it is “unusually” high in that class.
PART B: FRACTILES, OR QUANTILES

We will look at three types of these.

Percentiles

Example

Because 92% of the scores lie below 85 points, we can say that the 92\textsuperscript{nd} percentile of the data is 85: \( P_{92} = 85 \text{ points} \).
Deciles

Think: Dimes.

1st decile = \( D_1 = P_{10} \)
2nd decile = \( D_2 = P_{20} \)

\vdots

9th decile = \( D_9 = P_{90} \)

\[ \text{Min} \quad D_1 \quad P_{10} \quad \ldots \quad P_{90} \quad \text{Max} \]

Quartiles

Think: Quarters.

1st quartile = \( Q_1 = P_{25} \)
2nd quartile = \( Q_2 = P_{50} \) \( \approx \) Median
3rd quartile = \( Q_3 = P_{75} \)

\[ \text{Min} \quad Q_1 \quad Q_2 \quad Q_3 \quad \text{Max} \quad \text{Summary} \]
LESSON 8: BOXPLOTS (ALSO SECTION 3-4)

The Five-Number Summary can be represented graphically as a boxplot, or box-and-whisker plot.

Boxplots can help us compare different populations, such as men and women.

See the last page of the MINITAB handout.