CH.9: TWO POPULATIONS — not on tests

9-3 } MEANS FROM INDEP. SAMPLES

Ex

Pop. 1	Pop. 2
(Weights for U.S. Men in lbs.)	(... Women...)
Sample 1: Size n_1, Mean $\bar{x}_1$, SD s_1	Sample 2: n_2, $\bar{x}_2$, s_2
Mean $= \mu_1$	Mean $= \mu_2$

Ex Medical Study

Control (placebo) vs. Treatment (drug)

Need (Assumptions)

$n_1, n_2 > 30$ or Pops. ~~roughly~~ Normal

Test $H_0: \mu_1 = \mu_2$

Usual Test Statistic

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - \overbrace{(\mu_1 - \mu_2)}^{=0 \text{ under } H_0}}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$\#df = \begin{pmatrix} \text{smaller of} \\ n_1, n_2 \end{pmatrix} - 1$

SKIP

Note 1 If σ_1, σ_2 known $\Rightarrow$ Use z.

Note 2 If we assume $\sigma_1^2 = \sigma_2^2$, some use a pooled variance, s_p^2, though some statisticians warn against this.

$$\text{Test } t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}}$$

where

$$\overbrace{s_p^2}^{\text{weighted avg. of } s_1^2, s_2^2} = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)}$$ (better estimate of the common pop. variance than s_1^2 or s_2^2 alone)

$\# df = n_1 + n_2 - 2$ (higher than before)

$\Rightarrow$ t dist'n looks more like z, smaller SD than if we didn't do this;

$\Rightarrow$ "easier" to reject H_0

(9-4) MEANS FROM DEPENDENT SAMPLES (MATCHED PAIRS)

Ex Test the claim that a weight loss program for men is effective. $\alpha = 0.05$. (☆)

What 2 #s do we care about? 1#?

What do we put here? 1 the wrong way!

Man #	Before (lbs.)	After (lbs.)	Differences (d)
1	230	225	5
2	250	248	2
3	210	211	-1

Focus on

⇓

Sample mean
$\bar{d} = 2$
Sample SD
$s_d = 3$

Assumptions
$n > 30$ or
all differences roughly Normal (need here)

Test $H_0: \mu_d = 0$ (program has no effect)
(mean of all differences)

$H_A: \mu_d > 0$ (Claim)

If d = After − Before, $H_A: \mu_d < 0$

Usual Test Statistic

0 under H_0

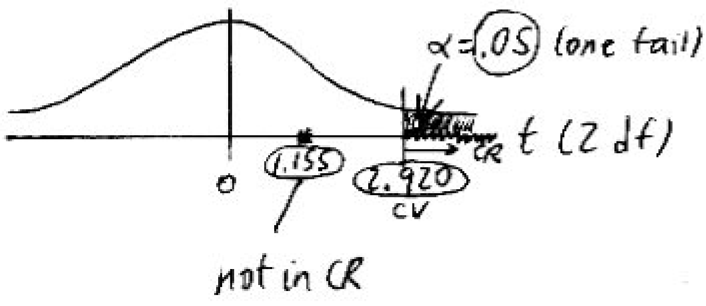
$$t = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}}$$

$\# df = n - 1$

SKIP IN CLASS

Here,
$$t = \frac{2-0}{\frac{3}{\sqrt{3}}}$$
$$\approx 1.155$$
$$n-1 = 3-1 = 2 \text{ df}$$

<u>Find CV, CR</u>



We do not reject Ho.

There is <u>insufficient</u> evidence in <u>support</u> of the claim that A.

9-2 } p_1, p_2 (Ugly!)

Not in Triola $\sigma_1^{(2)}, \sigma_2^{(2)}$

SKIP

$H_0: \sigma_1^2 = \sigma_2^2$

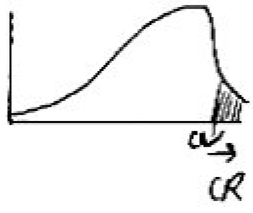
Test statistic

$F = \dfrac{s_1^2}{s_2^2}$ ← the larger of the two sample variances

CV and CR

In F tables, ✓ for desired α, use (cut in half for two-tailed test)

numerator $df_1 = n_1 - 1$
denominator $df_2 = n_2 - 1$



always right-tailed

$F(df_1, df_2)$

Note You won't see $H_A: \sigma_1^2 < \sigma_2^2$, because $s_1^2 \geq s_2^2$ by definition.