CH. 2

2.1: LIMITS and CONTINUITY

A Functions (1.3)

Ex Let f be the function defined by the rule $f(x) = \dfrac{x^2-16}{x-4}$

How do we eval?

A fraction bar automically groups N, D

① $$\underbrace{f(3)}_{\text{"f of 3" NOT "f times 3"}} = \frac{(3)^2-16}{(3)-4} \quad \begin{matrix}\leftarrow \text{work out} \\ \leftarrow \text{before } \div\end{matrix}$$

$$= \frac{-7}{-1}$$

$$= 7$$

$$\boxed{f(3) = 7}$$

cigarette

$$\begin{matrix}\text{Input} \\ 3\end{matrix} \Rightarrow \boxed{\begin{matrix} f \\ \text{button}\end{matrix}} \Rightarrow \begin{matrix}\text{Output} \\ 7\end{matrix}$$

② $$f(3.9) = \frac{(3.9)^2-16}{(3.9)-4}$$

$$\boxed{f(3.9) = 7.9}$$

$$3.9 \Rightarrow \boxed{f} \Rightarrow 7.9$$

③ $f(4) = \dfrac{(4)^2-16}{(4)-4}$ ← ÷ by 0 is "illegal"

$\boxed{f(4) \text{ is undefined}}$

Carly's calc.
the finger

$4 \Rightarrow \boxed{f} \Rightarrow$ "ERROR!"

$f(x) = \dfrac{x^2-16}{x-4}$ is defined for

$\{x \mid x \neq 4\}$.

the set of all real values "x" | such that | x is not 4

what do you call the set of legal inputs?
[illegible] outputs?
Range(x^2): nonneg.

This is the [natural] domain of f.

We never have a calc. button $\boxed{f}$

Input in nat. dom. $\Longrightarrow$ $\boxed{\text{func. } f}$ $\Longrightarrow$ Exactly 1 real output (in the range of f)

Table of f values

IQ Test
If I input 3.99 we guess...
Let's confirm

x	f(x)	
3	7	⇒ (3,7) on graph
3.9	7.9	
4	und.	⇒ No point (literally)

Nice trick: Simplify 1st!

$$f(x) = \frac{x^2-16}{x-4} \leftarrow \text{Factor} \quad , x \neq 4$$

$$= \frac{(x+4)\cancel{(x-4)}}{\cancel{x-4}} \quad , x \neq 4$$

We can cancel (the $(x-4)$ factors) if $x \neq 4$.

$$= x+4 \quad , x \neq 4$$

gives simple yet complete descrip. of f

$$f(x) = \frac{x^2-16}{x-4} = \begin{cases} x+4 & , x \neq 4 \\ \text{und.} & , x = 4 \end{cases}$$

Table Easy!

x	f(x)		Graph
3	3+4 = 7	⇒	(3, 7)
3.9	3.9+4 = 7.9	⇒	(3.9, 7.9)
3.99	7.99	⇒	(3.99, 7.99)
4	und.	⇒	No point

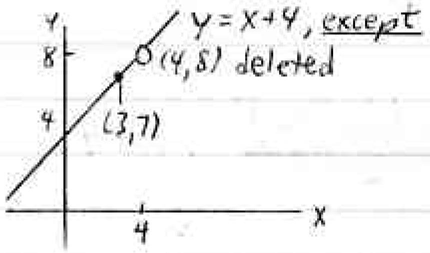
literally

Why?

Graph $y = f(x)$

independent variable

dependent



Ⓑ Limits

Old Ex $f(x) = \dfrac{x^2-16}{x-4}$

We should be able to say SOMETHING...

We can't say "$f(4) = 8$."

We can say

"$\lim\limits_{x \to 4} f(x) = 8$."

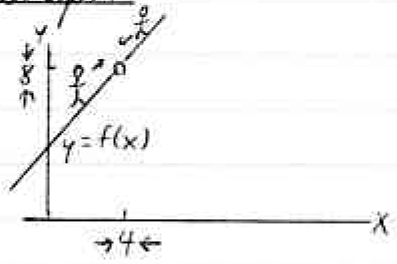
i.e., "The limit of $f(x)$ as x approaches 4 is 8."

i.e., "$f(x) \to 8$ as $x \to 4$."

Ways to see this

① Graph

skinny lovers but alas can never meet
Bill+Hillary
Siegfried+Roy

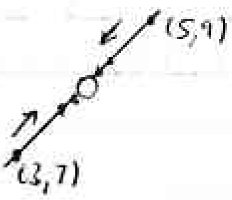


② Tables

We'll talk more about this later.

x	f(x)
3	7
3.9	7.9
3.99	7.99
as ↓	then ↓
4	8
↑	↑
4.01	8.01
4.1	8.1
5	9

x	f(x)
5	9
4.1	8.1
4.01	8.01
↓	↓
4	8



③ Simplifying 1st

$$\lim_{x\to4}\frac{x^2-16}{x-4} = \lim_{x\to4}\frac{(x+4)\cancel{(x-4)}}{\cancel{x-4}}$$

☆

$$= \lim_{x\to4}(x+4)$$

We can see E

Plug in $x = 4$.

$= 4 + 4$

$= \boxed{8}$

A Why is this OK?

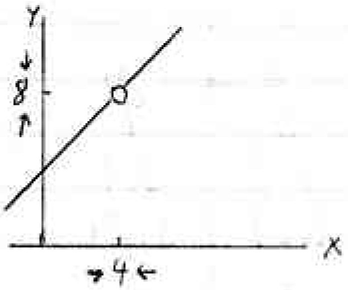
$y = \dfrac{x^2 - 16}{x - 4}$

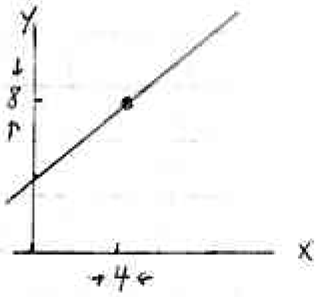
$y = x + 4$

Except here, the lines meet.

You're filling in the dot.

lim doesn't care.





In both cases, as $x \to 4$, then $y \to 8$.
What happens at $x = 4$ is technically irrelevant!

Up to 27
2nd: 25

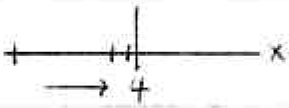
© One-Sided Limits

Old Ex $f(x) = \dfrac{x^2-16}{x-4}$

Table 1

x	$f(x)$
3	7
3.9	7.9
3.99	7.99
$\downarrow$	$\downarrow$
4	8

As $x \to 4$ from the left (from lower values), then $f(x) \to 8$.



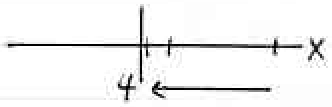
$$\boxed{\lim_{x \to 4^-} f(x) = 8}$$ ☆

from the left

Table 2

x	$f(x)$
5	9
4.1	8.1
4.01	8.01
$\downarrow$	$\downarrow$
4	8

As $x \to 4$ from the right (from higher values), then $f(x) \to 8$.



$$\boxed{\lim_{x \to 4^+} f(x) = 8}$$ ★★

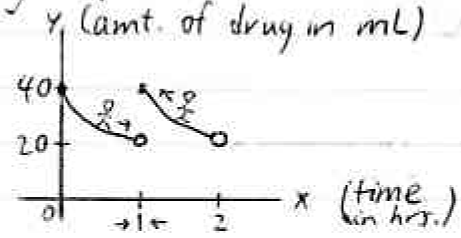
from the rt.

★ = ★★, so $\boxed{\lim_{x \to 4} f(x) = 8}$ (Two-sided limit exists.)

Finally... Got your att'n

Ex (Drug Injections)

Fix Dr. Banner, Kevork

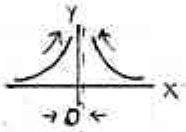


$$\boxed{\begin{array}{c} \lim_{x \to 1^-} f(x) = 20 \\ \hline \lim_{x \to 1^+} f(x) = 40 \end{array}}$$ $\neq$, so $\boxed{\lim_{x \to 1} f(x) \text{ does not exist (DNE)}}$

Up to 35 2nd: 33

① Limits and Infinity (∞)

Ex $f(x) = \frac{1}{x^2}$



(Can do Table)

x	f(x)
-1	1
-0.1	100
-0.01	10000
0	und.
0.01	10,000
0.1	100
1	1

Vertical Asymptote (VA)
a line the graph approaches

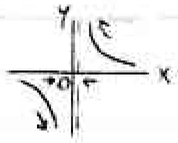
Is ∞ a real #?

$\boxed{\begin{array}{l} \lim\limits_{x \to 0^-} f(x) = \infty \\ \lim\limits_{x \to 0^+} f(x) = \infty \end{array}}$ Match, so $\boxed{\lim\limits_{x \to 0} f(x) = \infty}$

$\infty, -\infty$ are special cases of DNE.

Ex $f(x) = \frac{1}{x}$

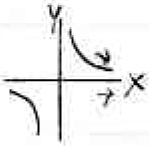
Hell



$\boxed{\begin{array}{l} \lim\limits_{x \to 0^-} f(x) = -\infty \\ \lim\limits_{x \to 0^+} f(x) = \infty \end{array}}$ Do not match, so $\boxed{\lim\limits_{x \to 0} f(x) \text{ DNE}}$ ($\infty, -\infty$ inappropriate)

up to 51
2nd:49

$\lim\limits_{x \to \infty} f(x) = 0$



w/out...?

(E) Continuity

$\approx$ can trace graph w/out lifting pencil

A func. f is continuous at c $(x=c)$

$\Longleftrightarrow$ if and only if

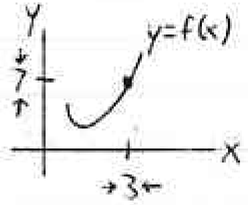
(1) $f(c)$ is defined ("There is a point.")

(2) $\lim_{x\to c} f(x)$ exists

(3) $\lim_{x\to c} f(x) = f(c)$ ((1),(2) are $=$)

Otherwise, f is discontinuous at c.

Ex



(1) $f(3) = 7$

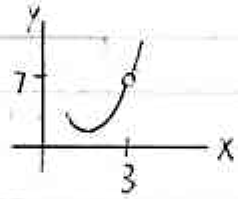
(2) $\lim_{x\to 3} f(x) = 7$

(3): (1) = (2) ✓

, so $\boxed{f \text{ is cont. at } 3}$

HW: Know how to read a problem 1,2,3

Ex

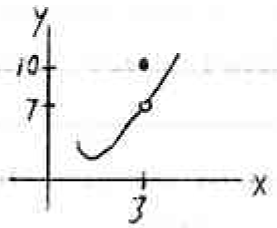


Is the lim 7?

If either (1),(2) FAILS, it's pointless to consider (3)

(1) FAILS: $f(3)$ is und. $\rightarrow$ $\boxed{f \text{ is discont. at } 3}$

NOTE: (2) OK: $\lim_{x\to 3} f(x) = 7$

Ex 

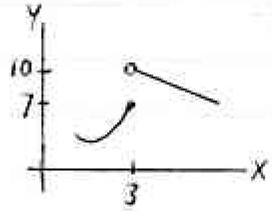
On test, ①,②,③

① OK $f(3)=10$

② OK $\lim_{x\to 3} f(x)=7$

$\neq$ ③ FAILS $\Rightarrow$ $\boxed{f \text{ is discont. at } 3}$

on that day...

Ex 

① OK: $f(3)=7$, BUT

$\infty, -\infty$ also bad (DNE)

$\lim_{x\to 3^-} f(x)=7$

$\lim_{x\to 3^+} f(x)=10$

$\neq$ $\Rightarrow$ $\lim_{x\to 3} f(x)$ DNE ② FAILS

$\Downarrow$

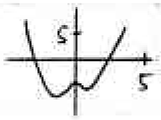
$\boxed{f \text{ is discont. at } 3}$

up to 57
2nd: 57

f is <u>continuous [everywhere]</u> $\Leftrightarrow$
f is cont. at <u>all</u> real values

<u>Ex</u> Polynomial funcs. are cont. e-where.

<u>Ex</u> $f(x)=2x^4-3x^2+x-4$



$\boxed{f \text{ is cont. e-where.}}$

More generally, rational funcs. are cont. on their domain.

$f(x)$ can be written as $\frac{\text{poly}}{\text{poly}}$ ← not 0

Ex $f(x) = \dfrac{3x^2-10}{x^3-x^2-2x}$ ← Factor

$= \dfrac{3x^2-10}{x(x^2-x-2)}$

$= \dfrac{3x^2-10}{x(x+1)(x-2)}$

If I have 3 #s, and I say product ≠ 0

$f(x)$ is und. $\Leftrightarrow$ Denom. $= 0$

($\Leftrightarrow$ Any factor of the denom. $= 0$)

$\Leftrightarrow x = 0$ or $x+1=0$ or $x-2=0$

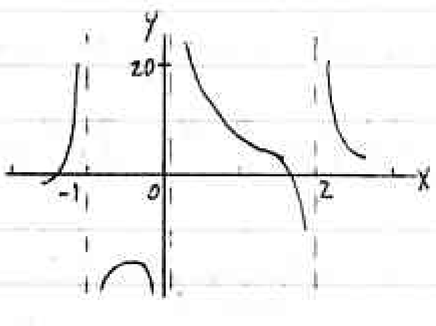
$x=-1$ $\quad$ $x=2$

Domain $= \{x \mid x \neq 0,\ x \neq -1,\ x \neq 2\}$

Also, where f is cont.

Up to 70

$\boxed{f \text{ is discont. at } x=0,\ x=-1,\ x=2}$



(F) Back to Limits

Circular reasoning

If f is cont. at $c \overset{\text{by def'n}}{\Longrightarrow} \lim\limits_{x \to c} f(x) = f(c)$

(Direct sub works: Plug in $x = c$.)

$\frac{1}{x}$

What happens to graph $x \leftarrow x-1$

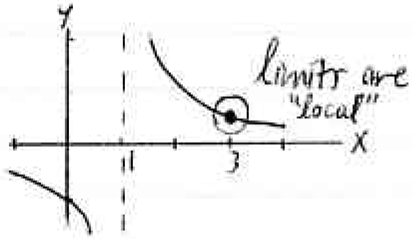
Asym at 1 instead of 0.

Ex $\lim\limits_{x \to 3} \frac{1}{x-1}$

rat'l cont. at 3
$\Rightarrow$ Plug in $x = 3$

$= \frac{1}{(3)-1}$

$= \boxed{\frac{1}{2}}$

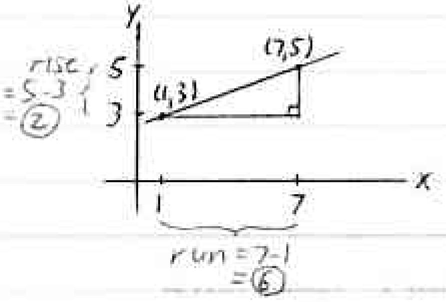


Here, it helps to know what happens at $x = 3$.

Book: Rates 1st (3rd)

(2.2): SLOPES, DERIVATIVES, RATES OF CHANGE

(A) Slope



slope "m" of the line

$= \frac{rise}{run} \quad \left(\text{or } \frac{\Delta y}{\Delta x}\right)$

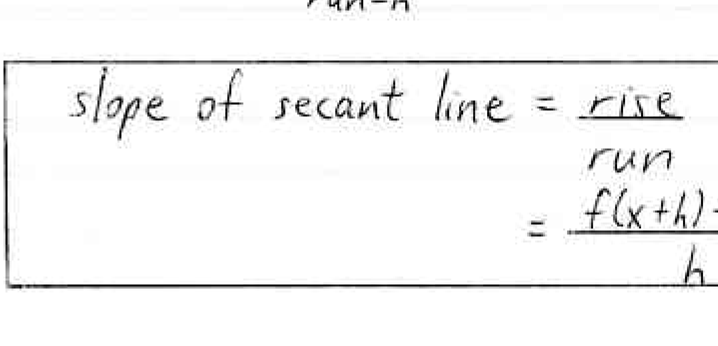
delta: change in

$= \frac{2}{6}$

$= \boxed{\frac{1}{3}}$

What is the "slope" at point P?

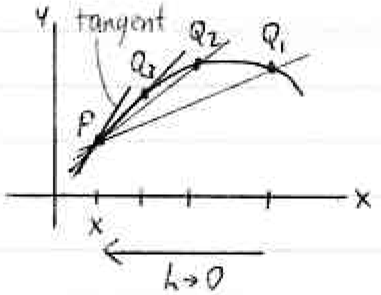
Book: (Latin)
secare to cut
tangere to touch



$$\boxed{\text{slope of secant line} = \frac{rise}{run} = \frac{f(x+h)-f(x)}{h}}$$

Let $h \to 0$

magic red line



As $h \to 0$, the secant lines approach the <u>tangent line</u> to the curve at P.

Slope of tangent line at P

$$= \lim_{h \to 0} (\text{slopes of secant lines})$$

$$= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \text{, if it exists}$$

We call this $f'(x)$, the <u>derivative</u> of f at x.

"prime"

Ⓑ Finding f', the Derivative Func. of f

Ex $f(x) = 3x^2 + 2x$
Find $f'(x)$ using the limit def'n of deriv.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

If $f(x) = 3x^2 + 2x$
$\Rightarrow f(x+h) = 3(x+h)^2 + 2(x+h)$
We replaced the "x"'s with "$(x+h)$"'s.

Don't forget!

$$= \lim_{h \to 0} \frac{[3(x+h)^2 + 2(x+h)] - [3x^2 + 2x]}{h}$$

$$= \lim_{h \to 0} \frac{3(x+h)^2 + 2(x+h) - 3x^2 - 2x}{h}$$

$$= \lim_{h \to 0} \frac{3(x^2 + 2xh + h^2) + 2x + 2h - 3x^2 - 2x}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{3x^2} + 6xh + 3h^2 + \cancel{2x} + 2h - \cancel{3x^2} - \cancel{2x}}{h}$$

$$= \lim_{h \to 0} \frac{6xh + 3h^2 + 2h}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(6x + 3h + 2)}{\cancel{h}}$$

(h is never 0)

$$= \lim_{h \to 0} (6x + \overset{\text{sub} \to 0}{3h} + 2)$$

25-41 2nd 9-25

$$= \boxed{6x + 2}$$

We have <u>differentiated</u> f.

Ⓒ Notation

If $f(x) = 3x^2 + 2x \Rightarrow f'(x) = 6x + 2$

<u>or</u> $\underbrace{D_x (3x^2 + 2x)}_{\text{The deriv. of } 3x^2+2x \text{ with respect to } x} = 6x + 2$

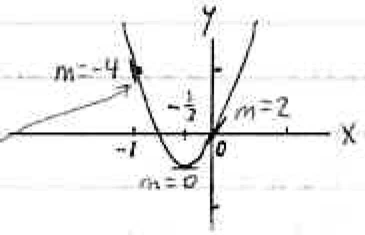
<u>or</u> $\frac{d}{dx}(3x^2 + 2x) = 6x + 2$

<u>or</u> if $y = 3x^2 + 2x$

like $\frac{\Delta y}{\Delta x}$ $\Rightarrow y'$ (or $\frac{dy}{dx}$) $= 6x + 2$

① Derivs. are Slopes

$$f(x) = 3x^2 + 2x$$



$$f'(x) = 6x + 2 \quad \text{(slope func.)}$$

What do you think

$$f'(-1) = 6(-1) + 2 = -4$$
$$f'(-\tfrac{1}{3}) = 0$$
$$f'(0) = 2$$

The slope of the tangent line to the point w/ x-coord. -1 is -4.

Ex Find eqs. for the tangent line at $x = -1$.

What's the point?

How do I find the y-coord

$$f(-1) = 3(-1)^2 + 2(-1) = 1$$

$$\overset{x}{(-1}, \overset{y}{1)}$$

What are forms for the eq. of a line?

Point-slope form

$$y - y_1 = m(x - x_1)$$

slope $= f'(-1) = -4$

$$\text{Pt. } (x_1, y_1) = (-1, 1)$$

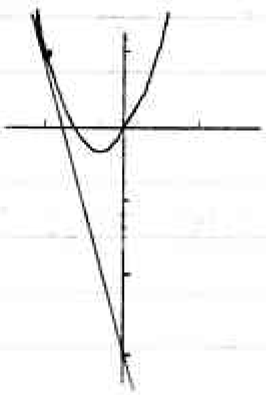
$$\boxed{\begin{aligned} &y-1=-4(x-(-1)) \\ \text{or } &y-1=-4(x+1)\end{aligned}}$$

Slope-intercept form $(y=mx+b)$

$$y=-4(x+1)+1$$
$$y=-4x-4+1$$

$$\boxed{y=-4x-3}$$

25-45
1, 3, 7
2nd: 9-24, 1, 3, 7
The graph of an eq. consists of all pts. that satisfy the eq.



or $y=mx+b$
$1=-4(-1)+b$
$\Rightarrow b=-3$

$y=-4x-3$

Ⓔ Slopes as Rates of Change

yen
Sm future salary - \$/yr.

Ex Your holdings (co. stock) in dollars after x years on the job is given by

How much to start?

$$f(x)=500x^2+20{,}000 \quad (\text{for } 0 \le x \le 4)$$

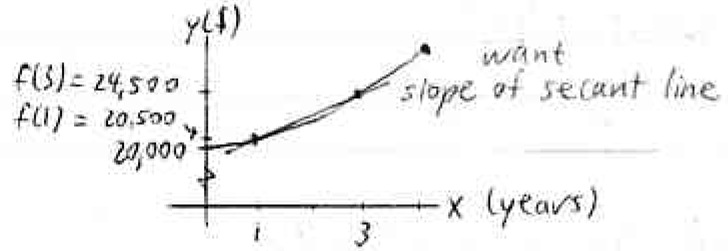
Fired for smithy place

① Find the average rate of change of your holdings between $x=1$ and $x=3$.

Holdings changing every millisec.
Sit on your butt!

Not to scale

Don't have to draw



Kink

How do we find y-coords?

$$f(1) = 500(1)^2 + 20{,}000$$
$$= 20{,}500$$

$$f(3) = 500(3)^2 + 20{,}000$$
$$= 24{,}500$$

Avg. rate of change bet. $x=1$, $x=3$

$$= \frac{f(3)-f(1)}{3-1} \quad \frac{\text{rise}}{\text{run}} \quad \left(\begin{matrix}\text{Difference}\\ \text{quotient}\end{matrix}\right)$$

$$= \frac{24{,}500 - 20{,}500}{2}$$

Your ↗ 4000$ in 2 yr.

$$= \frac{4000}{2}$$

$$= 2000 \ \frac{\$}{\text{year}} \quad \begin{matrix}\leftarrow y \text{ unit}\\ \leftarrow x \text{ unit}\end{matrix}$$

all but 17, 59
2nd: all but 45
Write it so Cousin Eddie can understand.

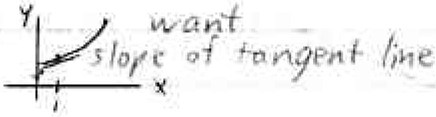
$\boxed{\text{On avg., your holdings increased at the rate of } 2000 \ \frac{\$}{\text{year}} \text{ between } x=1 \text{ and } x=3.}$

(Interpreting your answer.)

② Find the instantaneous rate of change of your holdings at $x=1$.

We want $f'(1)$.

We want slope of what line?



$$f(x) = 500x^2 + 20{,}000$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

We can plug in $x=1$.
May want general $f'(x)$ formula to find $f'(2), f'(3), \ldots$

$$f'(1) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$$

$$= \lim_{h\to 0} \frac{[500(1+h)^2 + 20{,}000] - [500(1)^2 + 20{,}000]}{h}$$

$$= \lim_{h\to 0} \frac{500(1+2h+h^2) + 20{,}000 - 500 - 20{,}000}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{500} + 1000h + 500h^2 + \cancel{20{,}000} - \cancel{500} - \cancel{20{,}000}}{h}$$

$$= \lim_{h \to 0} \frac{1000h + 500h^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(1000 + 500h)}{\cancel{h}}$$

$$= \lim_{h \to 0} (1000 + \underbrace{500h}_{\to 0})$$

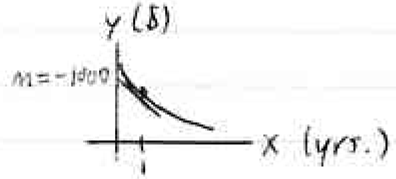
$$= 1000 \ \frac{\$}{\text{year}}$$

why is your avg. bet. x=1, x=3 higher

What if we had

$$\boxed{\text{Your holdings increased at the rate of } 1000 \ \frac{\$}{\text{year}} \text{ at } x=1.}$$

<u>Ex</u> Enron



$$f'(1) = -1000 \ \frac{\$}{\text{year}}$$

$$\boxed{\text{Your holdings } \underline{\text{decreased}} \text{ at the rate of } 1000 \ \frac{\$}{\text{year}} \text{ at } x=1.}$$

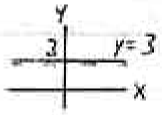
2.3: D_x SHORTCUTS

Come from limit def'n.

A Intro

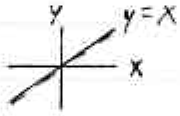
How do you graph $f(x)=3$ or $y=3$

$D_x(3) = 0$
constant

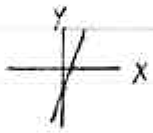


$f(x)=3$
$\Rightarrow f'(\text{at any real } \#) = 0$

$D_x(x) = 1$



$D_x(\underbrace{2x-3}_{\substack{mx+b \\ \text{(linear)}}}) = \underbrace{2}_{m}$



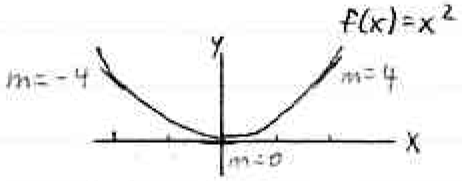
4 people on the deck of a boat

> **Power Rule**
> If n is any real constant,
> $D_x(x^n) = nx^{n-1}$ "Man overboard"

Slope changes if x changes

Ex $D_x(x^2) = 2x^1$
$= 2x$

How do you graph



$f'(x) = 2x$ | $f'(-2) = 2(-2) = -4$ $\quad f'(0) = 0$ $\quad f'(2) = 4$

or $\left.\dfrac{df}{dx}\right|_{x=-2} = -4$

↑ evaluated at

$f(x)$	$f'(x)$
x^2	$2x$
x^3	$3x^2$
x^4	$4x^3$
$\underbrace{\sqrt[5]{x^3}}_{\text{radical form}} = \underbrace{x^{\frac{3}{5}}}_{\text{exponential form}}$	$\frac{3}{5}x^{\frac{3}{5}-1} = \frac{3}{5}x^{\frac{3}{5}-\frac{5}{5}}$

How can I rewrite this so that I get rid of "–"

$= \frac{3}{5}x^{-\frac{2}{5}}$

or $\frac{3}{5x^{2/5}}$ (w/only positive exponents)

or $\frac{3}{5(\underbrace{\sqrt[5]{x^2}}_{\text{or } (\sqrt[5]{x})^2})}$ (radical form)

$f(x)$	$f'(x)$
$\frac{1}{x} = x^{-1}$	$(-1)x^{-1-1} = -x^{-2}$ or $-\frac{1}{x^2}$
$\frac{1}{x^2} = x^{-2}$	$-2x^{-2-1} = -2x^{-3}$ or $-\frac{2}{x^3}$
$\frac{1}{\sqrt{x}} = \frac{1}{x^{1/2}} = x^{-\frac{1}{2}}$	$-\frac{1}{2}x^{-\frac{1}{2}-1} = -\frac{1}{2}x^{-\frac{1}{2}-\frac{2}{2}}$

2 stays on top

$= -\frac{1}{2}x^{-\frac{3}{2}}$

or $-\frac{1}{2x^{3/2}}$

or $-\frac{1}{2\sqrt{x^3}}$

or $x\sqrt{x}$ ($x>0$ assumed)

(B) D_x is a Linear Operator

(1) The deriv. of a sum = The sum of the derivs.
(2) (difference) (difference)
(3) Constant factors "pop out"

Can D_x term-by-term

If limits $\exists$ $\lim\limits_{x \to \#}$ also linear

Ex $f(x) = 5x^7 - 4x^2 + 10$

(Hard Way:) lim def'n of f' or...

$$f'(x) = D_x(5x^7 - 4x^2 + 10)$$

$$\overset{(1),(2)}{=} D_x(5x^7) - D_x(4x^2) + D_x(10)$$

$$\overset{(3)}{=} 5 \cdot D_x(x^7) - 4 \cdot D_x(x^2) + 0$$

$$\overset{\text{Power}}{=} 5(7x^6) - 4(2x)$$

can skip

$$= \boxed{35x^6 - 8x}$$

$$\underset{35}{\cancel{5}}x^{\cancel{7}\,6} - \underset{8}{\cancel{4}}x^{\cancel{2}\,1} + \cancel{10}$$

Irritating

D(sum) = sum of Ds

Ex $f(x) = -\dfrac{4}{x^3} + \dfrac{2}{\sqrt[4]{x}} + \dfrac{5}{9}x^3 + \pi$

Rewrite ⇒ Powers, ≈ Poly.

$$= -4x^{-3} + 2x^{-\frac{1}{4}} + \frac{5}{9}x^3 + \underbrace{\pi}_{\text{constant}}$$

$$f'(x) = 12x^{-4} + 2\left(-\tfrac{1}{4}\right)\underbrace{x^{-\frac{1}{4}-1}}_{\substack{=x^{-\frac{1}{4}-\frac{4}{4}} \\ =x^{-\frac{5}{4}}}} + \frac{5}{\cancel{9}_3}\left(\cancel{3}x^2\right) + 0$$

$$= 12x^{-4} - \frac{1}{2}x^{-\frac{5}{4}} + \frac{5}{3}x^2$$

or $\dfrac{12}{x^4} - \dfrac{1}{2x^{5/4}} + \dfrac{5}{3}x^2$

Ex $f(x) = 2\sqrt{x} + \frac{3}{\sqrt{x}}$

Find $f'(9)$. (i.e., Find $\left.\frac{df}{dx}\right|_{x=9}$.)

$$f(x) = 2x^{\frac{1}{2}} + 3x^{-\frac{1}{2}}$$

D_x 1st, then plug in $x=9$.

$$f'(x) = 2\left(\tfrac{1}{2}x^{-\frac{1}{2}}\right) + 3\left(-\tfrac{1}{2}x^{-\frac{3}{2}}\right)$$

$$= x^{-\frac{1}{2}} - \frac{3}{2}x^{-\frac{3}{2}}$$

all but 43
2nd: all but 33

$$= \frac{1}{x^{1/2}} - \frac{3}{2x^{3/2}} \quad \text{or} \quad \frac{1}{\sqrt{x}} - \frac{3}{2\underbrace{(\sqrt{x})^3}_{\text{Easier than } \sqrt{x^3} \text{ to eval at } 9}}$$

$$f'(9) = \frac{1}{\sqrt{9}} - \frac{3}{2(\sqrt{9})^3} = \frac{1}{3} - \frac{\cancel{3}^{1}}{2(3)^{3}_{(3)^2}}$$

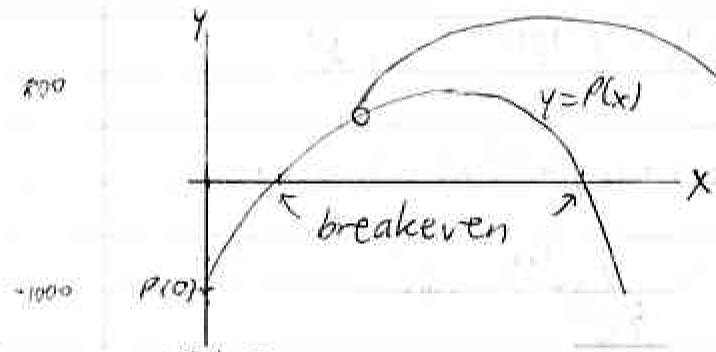
$$= \frac{1}{3} - \frac{1}{18} = \frac{6}{18} - \frac{1}{18} = \boxed{\frac{5}{18}}$$

Ⓒ Marginals

If x dolls, say, are produced and sold,

$R(x)$ = total revenue (\$ in)
$C(x)$ = cost (\$ out)
$P(x)$ = profit

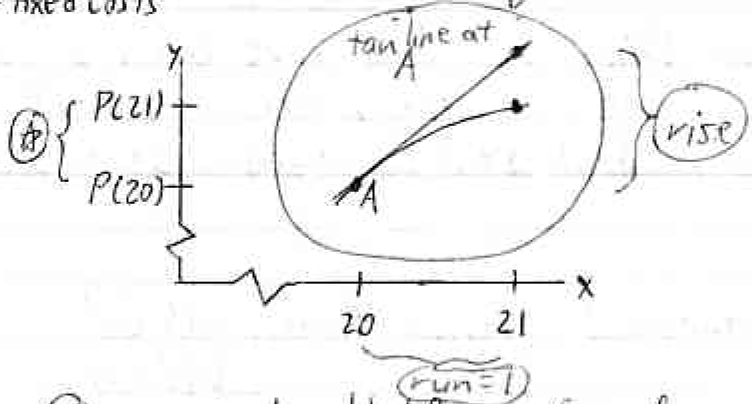
$$\boxed{P(x) = R(x) - C(x)}$$



R(0)=0
C(0) = fixed costs

can't get something for nothing

indep. of product. level



☆ = actual addt'l profit from producing and selling 1 more doll beyond 20.

P(20), P(21) can be hard to eval.
We hope rise $\approx$ ☆
approx.

In fact, rise $= P'(20)$

$P'(20) =$ slope of tan line at A

slope is $\frac{\text{what}}{\text{what}}$
What's the run?

$= \frac{\text{rise}}{\text{run}} \leftarrow 1$

$=$ rise

Ex $P(x) = -2x^2 + 120x - 1000$ (in \$)
$P'(x) = -4x + 120$
$=$ $MP(x)$, the marginal profit func.

$P'(20)$ or $MP(20) = -4(20) + 120$
$= \boxed{40}$

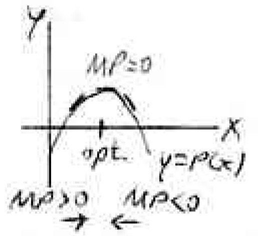
Interp.: When 20 dolls have been produced and sold, the profit increases by about \$40 for each addt'l doll.

The quality of this approx. often declines down-the-line.

$R'(x) =$ marginal revenue func., $MR(x)$
$C'(x) =$ marginal cost func., $MC(x)$

local vs. global

(If time) If you sell all you produce



What if

Derivs. are: slopes
rates
marginals

REVIEW 2.1-2.3 (OUTLINE)
(1.3 FUNCTIONS)
2.1 LIMITS

Using Tables, Graphs
Using Algebra to Simplify
Using Direct Sub ("Plugging in if you can")

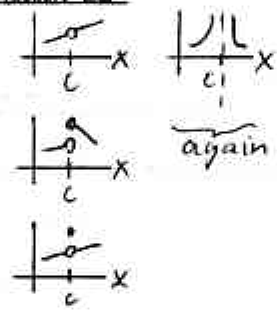
One-Sided Limits, Notation ($^+$, $^-$)

Knowing when to write ∞, $-\infty$, or DNE.
$\lim_{x \to \infty \text{ or } -\infty}$

CONTINUITY

Definition of cont. at a point (at c).

① f(c) exists / is defined
② $\lim_{x \to c} f(x)$ exists
③ They're =.

Failures


Where is f discont.? Why?

(2.2) f', THE DERIVATIVE OF f

Limit definition of f'

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad \left(\begin{array}{c}\text{Know and} \\ \text{use!}\end{array}\right)$$

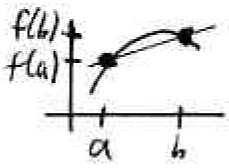
f' as slope (of tangent line) (+, 0, −)
instantaneous rate of change
marginal (2.3)

Notation (different types, like D_x, $\frac{d}{dx}$, etc.)

Evaluating $f'(9)$, for example. Notation such as $\left.\frac{df}{dx}\right|_{x=9}$

Finding Eqs. of Tangent Lines

Average Rate of Change of f from a to b

= Slope of secant line : 

$$= \frac{f(b) - f(a)}{b - a}$$

(2.3) D_x SHORTCUTS

Rewrite $f(x)$ so it looks like a poly.
Radical form ⇒ Exponential form
Ex $\sqrt{x} \Rightarrow x^{1/2}$

$D_x(\text{constant}) = 0$
$D_x(x^n) = nx^{n-1}$ (Power Rule)

Linearity
Can D_x term-by-term
Constant factors pop out.

Rewriting answers using only "+" exponents or radical form.

Evaluating $f'(9)$, for example.

Marginals

Word Problems

Interpreting answers (including marginals)

Units, of rate of change (incl. units) (say, f') : y/x $\frac{y \text{ units}}{x \text{ units}}$

$P = R - C$ (Profit = Revenue − Cost)