(2.3): D_x SHORTCUTS

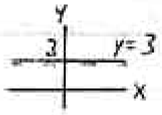
Come from limit def'n.

(A) Intro

How do you graph $f(x)=3$ or $y=3$

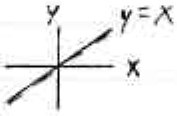
$D_x(3) = 0$

constant

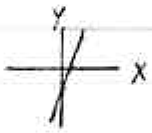


$f(x)=3$
$\Rightarrow f'(\text{at any real } \#) = 0$

$D_x(x) = 1$



$D_x(\underbrace{2x-3}_{\substack{mx+b \\ \text{(linear)}}}) = \underbrace{2}_{m}$



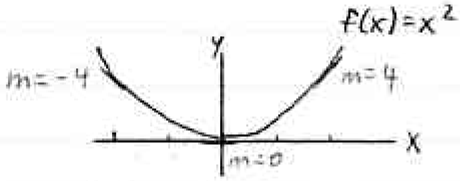
4 people on the deck of a boat

Power Rule
If n is any real constant,
$D_x(x^n) = nx^{n-1}$ "Man overboard"

Slope changes if x changes

Ex $D_x(x^2) = 2x^1$
$= 2x$

How do you graph



$f'(x) = 2x$ | $f'(-2) = 2(-2) = -4$ $\quad f'(0) = 0$ $\quad f'(2) = 4$

or $\left.\frac{df}{dx}\right|_{x=-2} = -4$

evaluated at

$f(x)$	$f'(x)$
x^2	$2x$
x^3	$3x^2$
x^4	$4x^3$

$$\underbrace{\sqrt[5]{x^3}}_{\text{radical form}} = \underbrace{x^{\frac{3}{5}}}_{\text{exponential form}} \qquad \frac{3}{5}x^{\frac{3}{5}-1} = \frac{3}{5}x^{\frac{3}{5}-\frac{5}{5}}$$

How can I rewrite this so that I get rid of "-"

$$= \frac{3}{5}x^{-\frac{2}{5}}$$

$$\text{or } \frac{3}{5x^{2/5}} \quad \text{(w/only positive exponents)}$$

$$\text{or } \frac{3}{5(\underbrace{\sqrt[5]{x^2}}_{\text{or } (\sqrt[5]{x})^2})} \quad \text{(radical form)}$$

$f(x)$	$f'(x)$
$\frac{1}{x} = x^{-1}$	$(-1)x^{-1-1} = -x^{-2}$ or $-\frac{1}{x^2}$

2 stays on top

$$\frac{1}{x^2} = x^{-2} \qquad -2x^{-2-1} = -2x^{-3} \text{ or } -\frac{2}{x^3}$$

$$\frac{1}{\sqrt{x}} = \frac{1}{x^{1/2}} = x^{-\frac{1}{2}} \qquad -\frac{1}{2}x^{-\frac{1}{2}-1} = -\frac{1}{2}x^{-\frac{1}{2}-\frac{2}{2}}$$

$$= -\frac{1}{2}x^{-\frac{3}{2}}$$

$$\text{or } -\frac{1}{2x^{3/2}}$$

$$\text{or } -\frac{1}{2\sqrt{x^3}}$$

or $x\sqrt{x}$ ($x>0$ assumed)

(B) D_x is a Linear Operator

(1) The deriv. of a sum = The sum of the derivs.
(2) (difference) (difference)
(3) Constant factors "pop out"

Can D_x term-by-term

If limits $\exists$

$\lim_{x \to \#}$ also linear

<u>Ex</u> $f(x) = 5x^7 - 4x^2 + 10$

(Hard Way:) lim def'n of f' or...

$$f'(x) = D_x(5x^7 - 4x^2 + 10)$$

$$\overset{(1),(2)}{=} D_x(5x^7) - D_x(4x^2) + D_x(10)$$

$$\overset{(3)}{=} 5 \cdot D_x(x^7) - 4 \cdot D_x(x^2) + 0$$

$$\overset{\text{Power}}{=} 5(7x^6) - 4(2x)$$

can skip

$$= \boxed{35x^6 - 8x}$$

$$\underset{35}{\cancel{5}}x^{\cancel{7}\,6} - \underset{8}{\cancel{4}}x^{\cancel{2}\,1} + \cancel{10}$$

Irritating

$D(\text{sum}) = \text{sum of } D\text{s}$

Ex $f(x) = -\dfrac{4}{x^3} + \dfrac{2}{\sqrt[4]{x}} + \dfrac{5}{9}x^3 + \pi$

Rewrite $\Rightarrow$ Powers, $\approx$ Poly.

$$= -4x^{-3} + 2x^{-\frac{1}{4}} + \frac{5}{9}x^3 + \underbrace{\pi}_{\text{constant}}$$

$$f'(x) = 12x^{-4} + 2(-\tfrac{1}{4})\underbrace{x^{-\frac{1}{4}-1}}_{\substack{= x^{-\frac{1}{4}-\frac{4}{4}} \\ = x^{-\frac{5}{4}}}} + \frac{5}{\cancel{9}_3}(\cancel{3}x^2) + 0$$

$$= \boxed{12x^{-4} - \frac{1}{2}x^{-\frac{5}{4}} + \frac{5}{3}x^2 \quad \text{or} \quad \frac{12}{x^4} - \frac{1}{2x^{5/4}} + \frac{5}{3}x^2}$$

Ex $f(x) = 2\sqrt{x} + \frac{3}{\sqrt{x}}$

Find $f'(9)$. (i.e., find $\left.\frac{df}{dx}\right|_{x=9}$.)

$$f(x) = 2x^{\frac{1}{2}} + 3x^{-\frac{1}{2}}$$

D_x 1st, then plug in $x=9$.

$$f'(x) = 2\left(\tfrac{1}{2}x^{-\frac{1}{2}}\right) + 3\left(-\tfrac{1}{2}x^{-\frac{3}{2}}\right)$$

$$= x^{-\frac{1}{2}} - \frac{3}{2}x^{-\frac{3}{2}}$$

all but 43
2nd: all but 33

$$= \frac{1}{x^{1/2}} - \frac{3}{2x^{3/2}} \quad \text{or} \quad \frac{1}{\sqrt{x}} - \frac{3}{2\underbrace{(\sqrt{x})^3}_{\text{Easier than } \sqrt{x^3} \text{ to eval at } 9}}$$

$$f'(9) = \frac{1}{\sqrt{9}} - \frac{3}{2(\sqrt{9})^3} = \frac{1}{3} - \frac{\cancel{3}^{1}}{2(\cancel{3})^{3}_{(3)^2}}$$

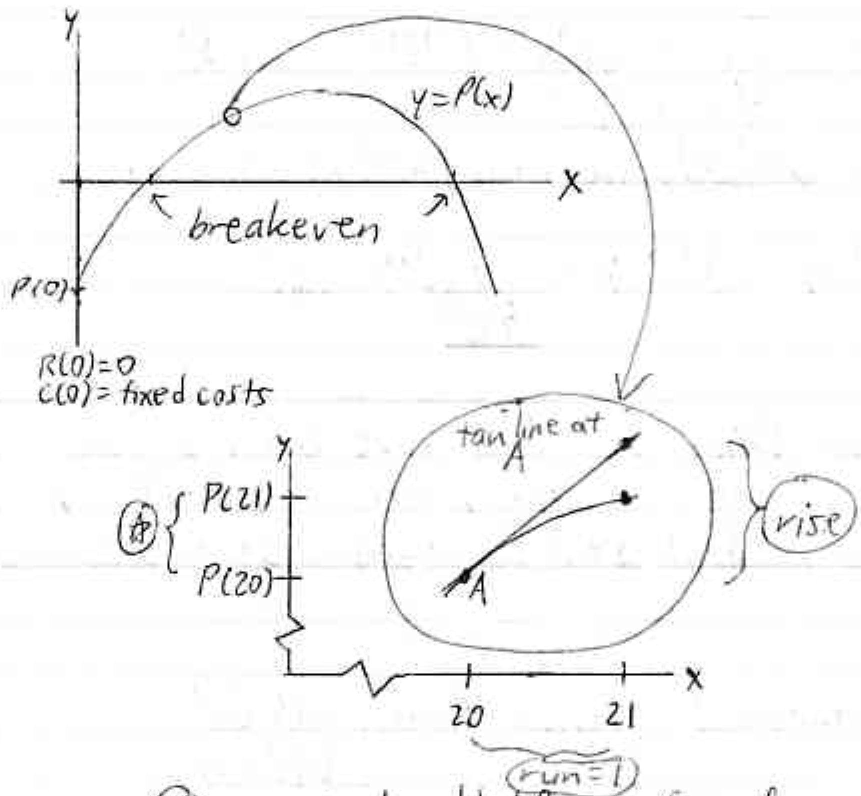
$$= \frac{1}{3} - \frac{1}{18} = \frac{6}{18} - \frac{1}{18} = \boxed{\frac{5}{18}}$$

Ⓒ Marginals

If x dolls, say, are produced and sold,

$R(x)$ = total revenue (\$ in)
$C(x)$ = cost (\$ out)
$P(x)$ = profit

$$\boxed{P(x) = R(x) - C(x)}$$



can't get something for nothing

indep. of produc. level

☆ = actual add't'l profit from producing and selling 1 more doll beyond 20.

$P(20)$, $P(21)$ can be hard to eval.
We hope rise $\underset{\text{approx.}}{\approx}$ ☆

In fact, rise $= P'(20)$

$P'(20) =$ slope of tan line at A

slope is $\frac{\text{what}}{\text{what}}$ What's the run?

$= \dfrac{\text{rise}}{\text{run}} \leftarrow 1$

$=$ rise

Ex $P(x) = -2x^2 + 120x - 1000$ (in \$)
$P'(x) = -4x + 120$
$= \underline{MP(x)\text{, the marginal profit func.}}$

$P'(20)$ or $MP(20) = -4(20) + 120$
$= \boxed{40}$

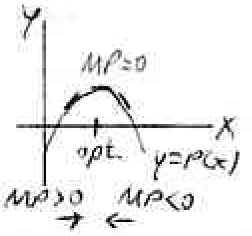
Interp.: When 20 dolls have been produced and sold, the profit increases by about \$40 for each addt'l doll.

The quality of this approx. often declines down-the-line.

$R'(x) =$ marginal revenue func., $MR(x)$
$C'(x) =$ marginal cost func., $MC(x)$

local vs. global

(If time) If you sell all you produce



What if

Derivs. are: slopes
rates
marginals