CH.3

③.① : GRAPHING USING f'

Ⓐ Where is f Increasing (↗) or Decreasing (↘)?

Based on this graph of f, I'd like to study the sign of f' and how it changes as x changes. (figure out)

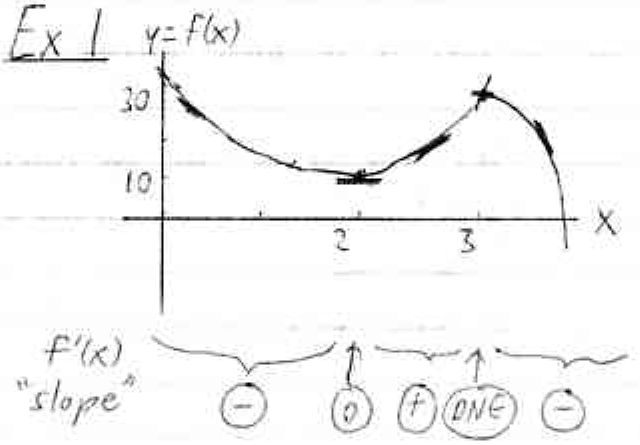
As far as the graph of f goes, derivs are what? Slopes of tan lines. Consider these tan lines. What sign will their slopes be?

not the point (?,3)

In 3.1, we'll go in reverse: we'll analyze the sign of f' and figure out where the graph ↗/↘.

What's the difference bet. an open/closed interval? ↗

Ex 1



$$f'(x) \underset{\oplus}{>} 0 \text{ on } \underbrace{(2,3)}_{\text{an open interval}} \Rightarrow f \nearrow \text{ on } (2,3)$$

$$f'(x) \underset{\ominus}{<} 0 \text{ on } (-\infty, 2) \underset{\substack{\uparrow \\ \text{union}}}{\cup} (3, \infty) \Rightarrow f \downarrow \text{ there}$$

Ⓑ Relative Extrema

Point, not interval

Ex 1 (2,10) is a relative/local minimum point (R.Min.Pt.)

What exactly do we mean by a rel/local
which is the true value? actual math term
you can / find a neighb...
There's a piece of the graph corresp. to an open interval on which the pt. is the lowest pt.
If we're looking for these kinds of pts, it's like a crime investigation. We want to investigate pts. where we have a horiz tan line - suspects.
You could have $f'(3)=0$ for R.Max
Messed up - if a fight in an alley, you ✓ it out!

i.e., f has a R.Min. value of 10 at 2,

the lowest pt. in some neighborhood around it

$\boxed{f'(2)=0}$

(3,30) is a R.Max.Pt.

highest

$\boxed{f'(3) \text{ DNE}}$

Ⓒ Critical #s (CNs)

#s in the domain of f where f' is 0 or DNE

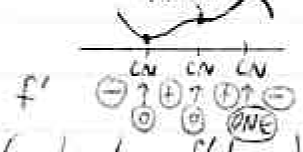
O.w., literally no pt.

"Critical Pt." used in Ch. 4

R.Min.Pt ⇒ CN here 1st
Here, horiz tan line, 2nd
but R.Max/Min Pt.?

Ex 1 2 and 3 are CNs.
(2,10) and (3,30) are the points at the CNs.

① The CNs of f are the only places where R. Max/Min. Pts. can appear.



② only places where f' can change what?
Book:
If f' cont.
(If f' changes smoothly)
Gelbaum 36:
f' ∃ but discont. at 0:
$f(x)=\begin{cases} x^2\sin\frac{1}{x}, & x\neq 0 \\ 0, & x=0 \end{cases}$
f ∃, f' DNE ⊃ CN
O.w. discont

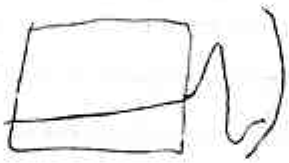
± 0 ±
↑
stops to get

② The CNs (and where f' discont.) are the only places where f' can change sign.

L3-2
3.1

You've graphed lines, parabolas, ... [illegible] Devil's Advocate or [illegible] Smart Aleck Let's just use graphing calc.? What kinds of x-coords should we be picking up? y? If we want to capture the "interesting" features of the graph?

① Graphing f

Graphing calc. $\boxed{\text{What window?}}$ y x (Don't want )

A 4th-deg. what! Book rushes thru mat'l; uses graphing calc. 4th-deg ☹ poly ☺

Ex (#18: 2nd Ed. / 3rd)

Graph $f(x) = -x^4 - 4x^3 - 4x^2 + 1$

Step 1 Find the domain of f

f is poly. ⇒ $\boxed{\text{Domain} = \mathbb{R} \quad (-\infty,\infty)}$

Where can we find rel. max/min pts.? When f' is ... or ... In an ER, you're in what condition?

Step 2 Find f', CNs

Where is f' 0 or DNE?

We want to solve this poly eq. What might we try to do? What's the GCF?

$$f'(x) = \underbrace{-4x^3 - 12x^2 - 8x}_{\text{never DNE} \atop \text{Factor!}} \overset{\text{set}}{=} 0$$

Just factor the left side! Don't ÷ by anything; you want to keep left side equal to f'(x).

$$-4x(x^2 + 3x + 2) = 0$$

$$-4x(x+2)(x+1) = 0$$

Zero factor property: If I have 4 #s and their product is 0... o.w. literally no pt. Up to 13

CNs: $\boxed{0}$ $\boxed{-2}$ $\boxed{-1}$

in domain of f ✓

"Sampling" f' at particular places to ✓ its sign.

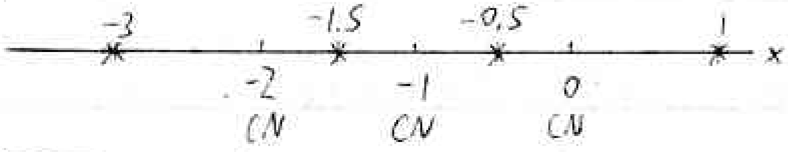
If f' ⊕ what does that tell us about f there? what kinds of pts?

Step 3 Use test values to Make a sign diagram for f'
Then,
①Find where f ↗, ↘.
② Classify pts. at CNs as R. Max. Pts., R. Min. Pts., or Neither

fenceposts
(The 3 CNs break the # line into 4 intervals.)

Idea

Pick a #. Can pick −π. From lowest to highest



f' is cont., so sign of f' at −3 is the same throughout (−∞, −2)

What do I know about f' on this interval? CNs are the only places where f' can change what? (Assuming f' cont.) Can f' change sign in here? Whatever sign f' is here will be the sign of f' everywhere in here.

$$f'(x) = -4x^3 - 12x^2 - 8x$$

or

$$(-4)(x)(x+2)(x+1)$$ ← factored form better for sign analysis.

Factored form often better for analyzing signs. Better products than sums.

+ or − ? # "−"s: even vs. odd 24

$$f'(-3) = (-4)(-3)(-3+2)(-3+1)$$ ← Can skip

$$= (-)(-)(-)(-)$$

$$= \textcircled{+}$$

If 2 terms "−", 1 "+", doesn't tell me anything. Maybe very "+".

$$f'(-1.5) = \big((-4)(-1.5)(-1.5+2)(-1.5+1)\big)$$

$$= (-)(-)(+)(-)$$

$$= \textcircled{-}$$

$$f'(-0.5) = (-)(-)(+)(+) = \textcircled{+}$$

$$f'(1) = (-)(+)(+)(+) = \textcircled{-}$$

Sign Diagram for f'

f'	⊕	⓪	⊖	⓪	⊕	⓪	⊖	x
		-2		-1		0		
f	↗		↘		↗		↘	

$$\boxed{\begin{array}{l} f \nearrow \text{ on } (-\infty,-2)\cup(-1,0) \\ f \searrow \text{ on } (-2,-1)\cup(0,\infty) \end{array}}$$

Book does → Me: what if pos?

Applying 1st DT.

At CNs: (Classify Pts.)

R. Max Pt.	R. Min Pt.	R. Max Pt.
$(-2, f(-2))$	$(-1, f(-1))$	$(0, f(0))$
$(-2, 1)$	$(-1, 0)$	$(0, 1)$

How do I find y-coords?

In 3.2, we'll have ex w/ neither

Plug into big daddy f, not f' (get 0)

$$f(-2) = -(-2)^4 - 4(-2)^3 - 4(-2)^2 + 1$$

not f' !!

Step 4 Sketch graph of f

Label pts. at CNs

Find y-intercept (if any)

What's often the easiest pt. to find on a graph if you have f(x)=... How do I find y int?

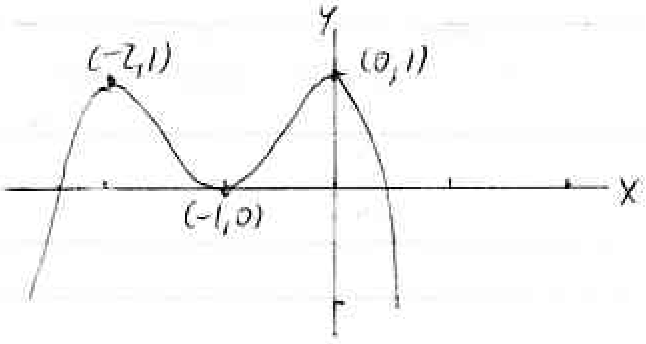
$$f(x) = -x^4 - 4x^3 - 4x^2 + 1$$

$$f(0) = \boxed{1}$$

0 happened to be a CN

Interpolating - guessing

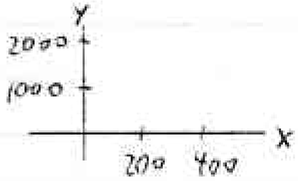
S-times this may not be so nice...



Bonus observation: Range = (-∞, 1]

x-ints: -1, $-1 \pm \sqrt{2}$ ≈ .414, -2.414

If necessary,

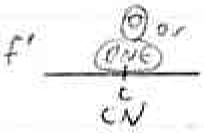


Can do HW

not in HW, but fair for test

Ⓔ 1st Deriv. Test (1st DT)

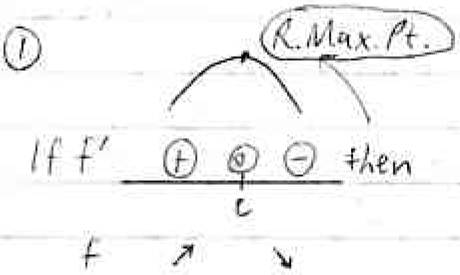
to classify pts. at CNs

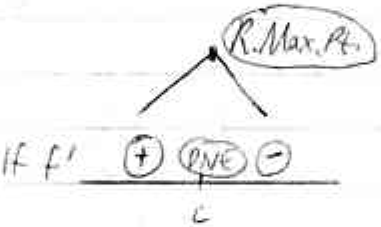


Assume f is cont. at c.

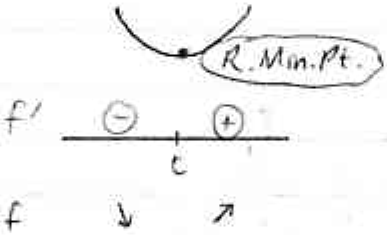
Do pic last

①





②



When do we have neither

③