So we can refine / better interpolate our graph.

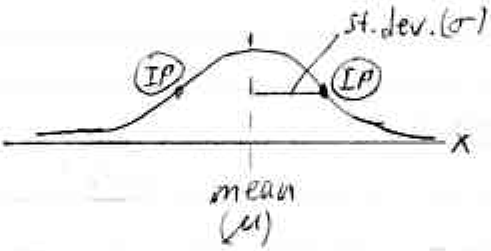
(3.2): GRAPHING USING f', f''

(A) Intro

Ex (Normal Curve)

upper div. bio class - 300/1000 Grader

Discuss in order: CU/CD ⇒ IP ⇒ σ



Concave Up (CU) ∪ Concave Down (CD) ∩ (CU)

How to Ace vi
Up like a cup ☺
Down like a frown ☹

(IP) = Inflection Point
where concavity changes CU ⇄ CD
and f is cont.

In 96, 116, you studied exp'l growth models of pop. Realistic? Malthus
1960s
Earth: L = 10B
Sci-fi

Ex (Population: Logistic Model)

If deriv. (+) ⇒ big daddy (↗)

f ↗ at ↗ rate
⇒ f'' (+)
⇒ f' (↗)
⇒ f graph (CU)

f ↗ but at ↘ rate
⇒ f'' (−)
⇒ f' (↘)
⇒ f graph (CD)

Analogy:
f is to f' as
f' f''

go together (Word association)

p. vi Concavity (How to Ace Calculus): f'' ☺ Up like a cup f'' ☹ Down like a frown

If go past IP, yes, sales will ↗, but you don't get the bang for your buck that you used to.

Ex (Econ)

Pt. of diminishing returns

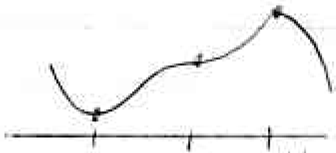
My term

(B) PINs, IPs

Possible Inflection Numbers

A CN is a # in the domain of f where f′ is 0 or DNE.	A PIN " " f″ "
CNs are the only places where R. Max./Min. Pts. can appear.	PINs " IPs "

Special pts. for f′ vs. f″



CN CN CN

0 0 DNE

f' ⊖ ⊕ ⊕ ⊖

⇒ f ↘ ↗ ↗ ↘

⇒ 1st DT R. Min Neither R. Max

Give them this info.

Is this an IP?

PIN PIN

0 DNE

f'' ⊕ ⊖ ⊖

⇒ f graph CU CD CD

⇒ IP no change

not IP

© Graphing f

Ex (#12: 2nd Ed / 3rd Ed)

Graph $f(x) = x^4 + 8x^3 + 18x^2 + 8$

Step 1 Domain $= \mathbb{R}$

Step 2 Find f', CNs

A CN is a # in Dom(f) where f' is what or what?

Where is f' 0 or DNE?

$$f'(x) = \underbrace{4x^3 + 24x^2 + 36x}_{\text{never DNE}} \overset{\text{set}}{=} 0$$

$$4x(x^2+6x+9) = 0$$

$$4x(x+3)^2 = 0$$

$$\Downarrow \qquad \Downarrow$$

CNs: 0 $\;-3$ in domain of f

Step 3 Sign Diagram for f'
Classify pts. at CNs.

Sampling sign of f' at test values

To perform a sign analysis, would we rather look at a sum or a product.

$$f'(x) = (4)(x)\underbrace{(x+3)^2}$$

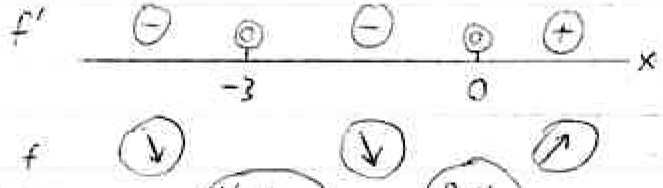
$$f'(-4) = (+)(-)(+) = -$$

$$f'(-1) = (+)(-)(+) = -$$

$$f'(1) = (+)(+)(+) = +$$

( )² never − can't be 0 at test values (not CNs)
( )² ⇒ +

Clean up, elim. test values.



Write "Pt." Later: "IP"s

There are always higher, lower pts. in a neigh

Why else is (0,8) signif.?

At CNs: Neither like — ; R. Min. Pt. like ∨

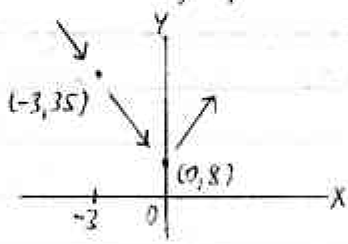
$(-3, f(-3))$ — $(-3, 35)$

$(0, f(0))$ — $(0, 8)$ also y-int.

NEW

My idea. A way to gather your thoughts before plunging into f" — drowning in quicksand

Step 4 "Skeleton graph" for f, y-int



Before we looked for CNs, now... what?

Step 5 Find f", PINs

Where is f" 0 or DNE?

Standard Poly form easier to D_x than factored form. What Rule would you have to use?

$$f'(x) = 4x^3 + 24x^2 + 36x$$

$$f''(x) = 12x^2 + 48x + 36 \overset{\text{set}}{=} 0$$

never DNE

$$12(x^2 + 4x + 3) = 0$$

$$12(x+3)(x+1) = 0$$

PINs: -3 $\quad -1$

in domain of f ✓

Step 6 Sign Diagram for f''
Find IPs

-4 -2 0 — x

-3 PIN -1 PIN

$$f''(x) = (12)(x+3)(x+1)$$

or $f''(x) = \ldots 36$

$$f''(-4) = (+)(-)(-) = (+)$$
$$f''(-2) = (+)(+)(-) = (-)$$
$$f''(0) = (+)(+)(+) = (+)$$

What does f'' tell us about

f' tells us…
(They combine effect of f', f'')

f'': $(+)$ 0 $(-)$ 0 $(+)$ — x; -3, -1

f graph: CU, CD, CU

change, change

At PINs: IP, IP

$(-3, f(-3))$ $(-1, f(-1))$

$(-3, 35)$ $(-1, 19)$

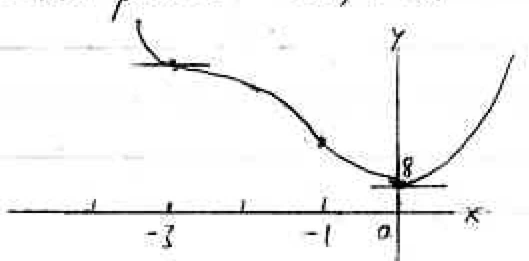
Plug into big daddy f!

(happened to be at a CN;
PINs may/may not be CNs, vice versa)

IPs don't have to be CPs

Step 7 Sketch graph of f.
Use skeleton from Step 4.
Label pts. at CNs, IPs.

Bols poor at combining f', f'' info.



$(-3, 35)$ CN, IP $(-1, 19)$ IP $(0, 8)$ CN

CU | CD | CU | CU

Windows
between, beyond CNs, IPs, discont's.

Bonus observations:
$f(x) = 0$ has no sol'ns.
(f has no roots/zeros)
(Graph has no x-ints.)
Range of $f = [8, \infty)$

Draw a line where sign of f', f'' may change

Windows: Unlike Microsoft's that work
behavior between, beyond CNs, PINs (IPs)

Ex Graph $f(x) = \sqrt[3]{x^2}$
$= x^{2/3}$

Step 1 Domain $= \mathbb{R}$ ($\sqrt[3]{\ }$ doesn't cause trouble unless does)

Step 2 $f'(x) = \frac{2}{3}x^{-1/3}$

$= \frac{2}{3(\sqrt[3]{x})}$ never 0

DNE at 0

CN: 0 in dom. of fn

Step 3

f': $\ominus$ on $x<0$, $\oplus$ on $x>0$; points -1, 0, 1; 0: CN

$f'(-1) = \frac{2}{3(\sqrt[3]{-1})} = \ominus$

$f'(1) = \frac{2}{3(\sqrt[3]{1})} = \oplus$

f: $\searrow$ $\nearrow$

R. Min Pt. $(0,0)$

Step 4

Step 5 $f''(x) = -\frac{2}{9}x^{-4/3}$

$= -\frac{2}{9(\sqrt[3]{x})^4}$ never 0

DNE at 0

PIN: 0 in domain of fn

Step 6

$\ominus$ on $x<0$, $\ominus$ on $x>0$; points -1, 0, 1; 0: PIN

$f''(-1) = -\underbrace{\frac{2}{9(\sqrt[3]{-1})^4}}_{+} = \ominus$

f graph: CD CD

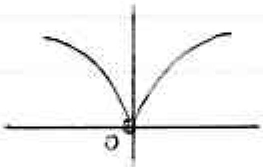
no change

Not IP

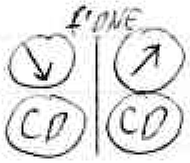
$f''(1) = -\frac{2}{9(\sqrt[3]{1})^4} = \oplus$

Step 7

Y cusp
vert tan line



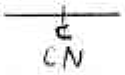
(0,0)
CN



We'll use 2nd DT in 3.3

(D) 2nd Deriv. Test (2nd DT)

also used to classify pts. at CNs

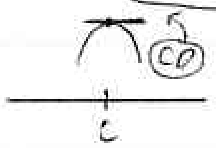


Assume $f'(c)=0$ (if DNE ⇒ Use 1st DT)

What's $f''(c)$?

What do we know about graph of f?
$f'(c)=0$ ⇒ we know we have a horiz tan line

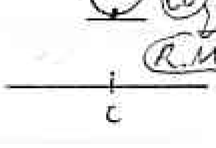
① If ⊖ R. Max. Pt.



Wordplay fails!
(You'd think ⊖ implies "Min." but NO! Think concavity, 1st.)

1st DT: use ↗, ↘
2nd: use concavity, fact we have horiz. tan line

② If ⊕



R. Min. Pt.

Much like Spears

③ If 0 or DNE ⇒
2nd DT is useless!
Use 1st DT.