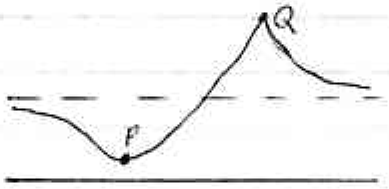
(3.3): OPTIMIZATION

(A) Absolute Extrema

Before:
R.Max/Min.: local behavior
Now, wider perspective.

Ex

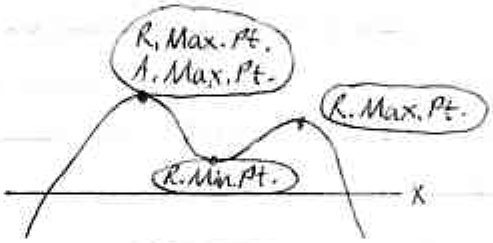


P is an absolute/global min pt. (A.Min Pt.)
lowest pt. everywhere (i.e., on domain of f)

Q (A.Max Pt.)
highest

King of the Hill vs. King of the World (Cameron)
Is this R.Max? A.Max?
Assume the graph continues as expected.

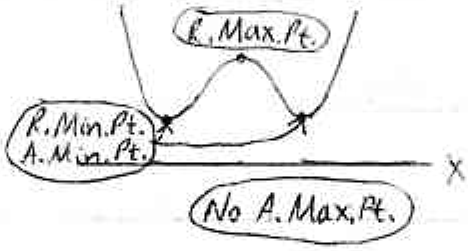
Ex



No A.Min.Pt.

Is this an A.Min.Pt.? Yes
Simpsons said they both get it

Ex

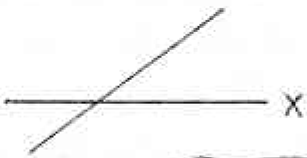


"Ties are OK"

We're not guaranteed an A.Max or Min. Pt.

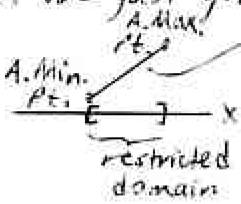
Let's try to set up conditions under which we're guaranteed...

Ex



No extrema

BUT if we just grabbed



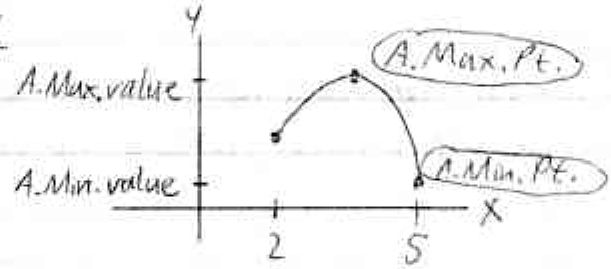
Not named in book

(B) Extreme Value Theorem (EVT)

Outside this interval, f can go nuts - Birchim hit frat days

> If f is cont. on a closed interval $[a,b]$, then there is guaranteed to be an A. Max. Pt. and an A. Min. Pt. on $[a,b]$
>
> restricted domain

Ex



f is cont. on $[2,5]$
$\Rightarrow$ EVT applies there.

What kinds of pts.

can

> Pts. at CNs and endpoints are the only pts. that can be A. Max/Min Pts.

Ex Find the abs. extreme values of
$$f(x) = 3x^4 - 8x^3 - 18x^2 + 1 \text{ on } [-2,2]$$

Step 1 EVT applies ✓

Where can I have R. Max/Min. Technically, can't have CN at a, b

No deriv. at endpts. - can't have R. Max/Min. No open ( ) around a, b

[-2, 2] fine we'll ✓ a, b anyway, but not CNs

Step 2 Find all CNs in $(-2,2)$.
$$f'(x) = \underbrace{12x^3 - 24x^2 - 36x}_{\text{never DNE}} \overset{\text{set}}{=} 0$$
$$12x(x^2 - 2x - 3) = 0$$
$$12x(x-3)(x+1) = 0$$

CNs: 0 $\quad \cancel{3}$ (outside $(-2,2)$) $\quad -1$

Maybe R. Max/Min.
Maybe also A. Max/Min
Is King of Hill also King of world? tragic/tickets. contest
lowest of low - Limbo

Step 3 Table

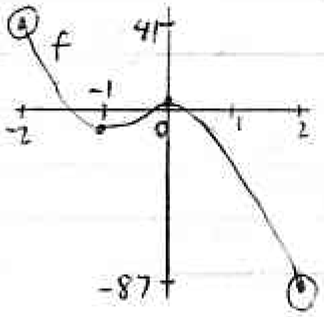
Setting up a lineup - you, what's your func. value / y-coord?...

	x	f(x)	Candidates	
	a = -2	$3(-2)^4-8(-2)^3-18(-2)^2+1=$	41	highest
CNs	-1		-6	
CNs	0		1	
	b = 2		-87	lowest

up to 11

$\boxed{\text{A. Max value is 41 (at } x=-2\text{). A. Min. value is } -87 \text{ (at } x=2\text{).}}$

Turns out



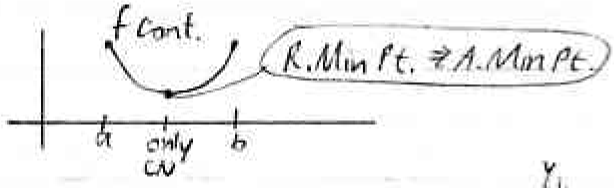
c) Shortcut

A cont. func. f can only turn around at CNs.

If there's only 1 CN,
a R. Min. Pt. ⇒ also an A. Min. Pt.
Max. Max.

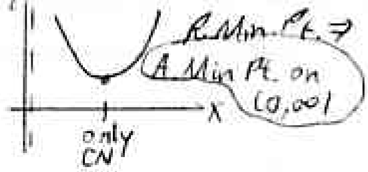
must also be...

Ex



Domain: $(0,\infty)$

True for non-closed intervals:
(where f cont.)



Up to 27

(I) Maximizing Profit (P)

So I can watch Survivor

Partial Ex You sell TVs.

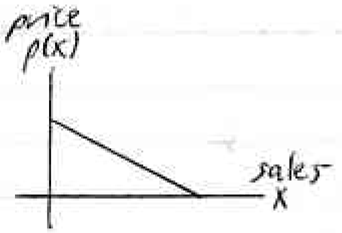
The price function

$$p(x) = 1700 - 15x \quad (\$)$$

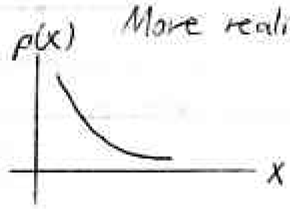
How? Models of other biz? Stats?

Note 1 $p(x)$ is the unit price (i.e., price per TV) at which you can sell x TVs.

Nitpickers: What's 0.2 of a TV?



More realistic:



↘ func.
(people buy more when TVs cheaper)

diff perspectives
demand
buy more when TVs cheaper
supply
↗ supply ↗ price

* you have to pay, can't give (Tribbles) em away

We love linear funcs., but not realistic

Note 2

If I know unit price, how can I compute total R?

$$\text{Revenue} = \begin{pmatrix}\text{Unit} \\ \text{price}\end{pmatrix} \begin{pmatrix}\text{Qty:} \\ \text{\# TVs}\end{pmatrix}$$

$$\boxed{R(x) = p(x) \cdot x}$$

Note 3

To compute P what else do we need to consider?
P = ...
big P

$$\text{Profit} = \text{Revenue} - \text{Cost}$$

$$\boxed{P(x) = R(x) - C(x)}$$

Full Ex You sell TVs. Each TV costs \$200 to make.
Fixed costs are \$1000.
The price func.

$$p(x) = 1700 - 15x \quad (\$)$$

To maximize profit,

ⓐ How many TVs should be produced?
ⓑ What should be the price of each TV?
ⓒ What is your max. profit?

What decisions do you need to make? a, b $\underbrace{}_{p(x)}$

Bottom line

Step 1: Find profit func., $P(x)$

$$R(x) = p(x) \cdot x$$
$$= (1700 - 15x)x$$
$$= 1700x - 15x^2$$

$$C(x) = 200x + 1000$$

$$P(x) = R(x) - C(x)$$
$$= (1700x - 15x^2) - (200x + 1000)$$
$$= 1700x - 15x^2 - 200x - 1000$$
$$P(x) = -15x^2 + 1500x - 1000$$

Step 2: Find CNs

$$P'(x) = -30x + 1500 \overset{set}{=} 0$$

never DNE

$$x = 50$$ in domain of P ✓ $(0, \infty)$

There's one common sense restriction $x \geq 0$, $\in \mathbb{Z}$ ok

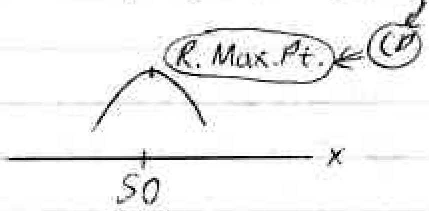
So what? → Why are we happy we just have 1 CN?

CN: 50

Step 3: Verify we get an A. Max. Pt. there

2nd DT: $P''(x) = -30$

$P''(50) = -30$ $\ominus$



50 is the <u>only</u> CN, P cont.

$\Rightarrow$ also A. Max. Pt.

ⓐ 50 TVs should be produced.

Step 4: What price?

What do I do? Book just has \$. In written English, mentions "per each TV"

$p(50) = 1700 - 15(50)$

$= \$950$ (per TV)

ⓑ Selling price should be \$950 (per TV)

Step 5: What's the max profit?

$P(50) = -15(50)^2 + 1500(50) - 1000$

$= \$36{,}500$

Up to 30

ⓒ The max. profit will be \$36,500.

(E) Piggen Problems

#30 (2nd ed.) → square

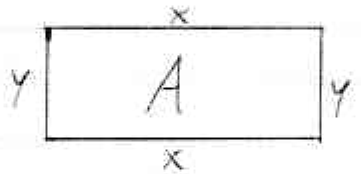
Among all rects. w/ perim. 80 yds, which one has largest area?

Ex Find the area and the dimensions of the largest rectangular piggen that uses 80 yards of fencing.

Step 1: Read!

Step 2: Diagram, Define Vars.

Let x = length
y = width
A = area



What's your objective?

Step 3: Max/Min What?

Max

$$A = xy$$ (Primary Eq.)

What info haven't we used yet? 80 yds is the what of this rect?

Step 4: Secondary Eq(s).

$$\text{Perimeter} = 80 \text{ yds}$$

$$x + y + x + y = 80$$

$$2x + 2y = 80$$

$$x + y = 40$$ Constraint

Step 5: Combine Eqs.

Book: y

Express A in terms of 1 var. (x, say)

$$A = xy$$
$$x + y = 40 \Rightarrow y = 40 - x$$

Solve for y in terms of x

$$A = x(40 - x)$$
$$A = 40x - x^2$$

or

$$A(x) = 40x - x^2$$

func. of x

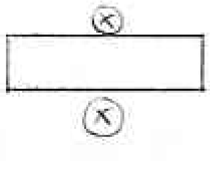
Don't use your knowledge of parabolas. Use calculus!

Step 6: Domain

Don't wrack your brain for 10 mins.

Possible values for x: $(0, 40)$

If $x = 70$, you run out of fencing!



Step 7: Optimize

Find CNs

$$A(x) = 40x - x^2$$
$$A'(x) = 40 - 2x \stackrel{\text{set}}{=} 0$$

never DNE

$$40 = 2x$$
$$x = 20 \quad \text{in } (0, 40) \checkmark$$

So what length that gives max A

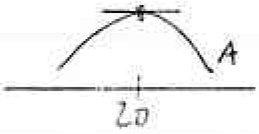
Do this! Don't appeal to common sense.

<u>Verify we get an A.Max Pt. there</u>

$A'(20)=0$, so can use

2^{nd} DT: $A''(x)=-2$

$A''(20)=-2$ ⊖ → CD → R.Max.Pt.



Concl.?

20 is the only CN, A cont.

⇒ also A.Max.Pt.

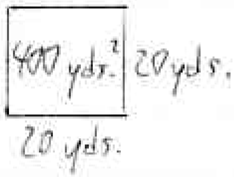
<u>Answer the ?</u>

$x = 20$ yds.

$y = 40-x$

$= 40-20$

$= 20$ yds.



What kind of rect do we have? A square is a rect!

Rects.

$A = xy$

$=(20)(20)$

$= 400$ sq. yds.

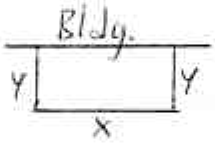
$$\boxed{\text{The largest pigpen is 20 yds. by 20 yds., and its area is 400 sq. yds.}}$$

Corners of sq. are meaningful

<u>Note</u> Even better: ≈ 509 sq. yds. circle, C = 80 yds.

You'll notice patterns; don't appeal to them on tests

<u>In HW</u>



⇒ Not a square!