CH. 4

4.1/4.2 : REVIEW

Ⓐ e^x, $\ln x$

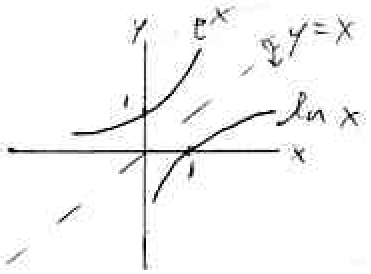
What is e?

e is a # (natural base)
$e \approx 2.718$

$f(x) = e^x$
Natural exponential func.

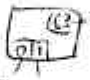
What's the inverse of e^x?

$\Rightarrow f^{-1}(x) = \ln x$, or $\log_e x$
inverse
" " logarithmic " "



Domain $\xleftrightarrow{\text{switch}}$ Range

Seattle's King Bill



\$$2^{63}$ $\approx 9 \times 10^{18}$

\$1 \$2
\$$2^n$ on square $\boxed{n}$

what does that tell you about values of e^x (not family values.)
Domain of $\ln x$?

e^x always ⊕ (Graph entirely above x-axis.)
We only $\ln$ (⊕) (" to right of y-axis.)

Ⓑ Evaluating logs

$2^{\text{what?}} = 8$

Ex $\log_2 8 = \boxed{3}$

bec. $2^{\boxed{3}} = 8$

logs are exponents

Need there for $\frac{d}{dx}(\ )|_{x=...}$

$\underline{Ex}$ $\ln 1 = \log_e 1 = \boxed{0}$

bec. $e^{\boxed{0}} = 1$

$\underline{Ex}$ $\ln e = \log_e e = \boxed{1}$

$\underline{Ex}$ $\ln e^4 = \log_e e^4 = \boxed{4}$ (In previous Exs., $1 = e^0$, $e = e^1$.)

"undo"

$\underline{Ex}$ $e^{\ln 4} = \boxed{4}$

undo

from Inverse Properties

$\underline{Ex}$ $\ln 0 = \log_e 0$ $\boxed{\text{undefined}}$

$e^{\boxed{?}} = 0$

ⓒ Applics

Exp. Growth: Population

Cont. Comp. Interest

P = principal, or present value

r = annual nominal interest rate (decimal)

4% nominal

After t years,

Future value (FV) $= Pe^{rt}$

$$P = \$1 \xrightarrow[t=1]{r=.04\ (4\%)} FV \approx \$1.041$$

APR (Annual %age Rate) $\approx 4.1\%$

Doubling Time: $\$1 \to \$2$ in $t = \frac{\ln 2}{r} = \frac{\ln 2}{.04} \approx 17.3$ yrs.

or $\$1000 \to \$2000$, (Rule of 70: $t \approx \frac{70}{r \text{ as } \%}$)

Exp. Decay: Radioactive, C-14 (Shroud of Turin - p. 269)

See "The World's Worst Currency." $\approx$ 1320 AD

Extra for 4ippee.
Buy Enron stock
p. 280 (2nd)
1993 Yugo.
> 1M% infl.
500M dinar note
worth \$5 → $\frac{1}{10\text{m}}$ ¢
in 1 mo.
34 (3rd)
Dating Older Women

Shroud of Turin
2nd: 298
3rd: 264
4.2, 32
1988 - 668 = 1320 AD

4.3: D_x

A $\ln$ ☆

$$\boxed{D_x(\ln x) = \frac{1}{x}} \quad (x>0)$$

Ex $f(x) = \frac{x}{\ln x}$

What rule do we use?

a) Find $f'(x)$

$$f'(x) = \frac{(\ln x)D_x(x) - (x)D_x(\ln x)}{(\ln x)^2} \quad \left(\begin{matrix}\text{Quotient}\\ \text{Rule}\end{matrix}\right)$$

$$= \frac{(\ln x)(1) - (x)(\frac{1}{x})}{(\ln x)^2}$$

Many thought yes

$$= \boxed{\frac{\ln x - 1}{(\ln x)^2}} \qquad (\ln x)^2 \neq 2\ln x$$

Don't strike!

b) Find $f'(e)$

$$f'(e) = \frac{\ln e - 1}{(\ln e)^2}$$

$$= \frac{1-1}{(1)^2}$$

$$= \boxed{0}$$

(Horiz. tan line)

$$\boxed{\begin{array}{l} D_x[\ln f(x)] = \frac{1}{f(x)} \cdot \underbrace{f'(x)}_{\text{tail}} = \frac{f'(x)}{f(x)} \\ \\ D[\ln \text{☆}] = \frac{1}{\text{☆}} D(\text{☆}) \end{array}}$$ $(f(x)$ diff'e, $>0)$

Dracula ln blah

Could do w/out ln props., but harder!!

<u>Ex</u> $f(x) = \ln \sqrt[4]{5x^2+3}$

ⓐ Find $f'(x)$

Power (Smackdown) Rule: $\ln M^p$ if $M>0$

$$f(x) = \ln(5x^2+3)^{1/4}$$
$$= \frac{1}{4}\ln(5x^2+3)$$

Do not smack down

Do not split

$$f'(x) = \frac{1}{4} \cdot \frac{1}{5x^2+3} \overbrace{D_x(5x^2+3)}^{=10x}$$

$$= \frac{\cancel{10}^{5}x}{\cancel{4}_{2}(5x^2+3)}$$

$$= \boxed{\frac{5x}{10x^2+6}}$$

Again, no striking! $(5x, 10x^2)$

ⓑ Approx. $f'(2)$ (to 3 dec. places)

$$f'(2) = \frac{5(2)}{10(2)^2+6} \begin{array}{l} \leftarrow 10 \\ \leftarrow 46 \end{array}$$ $(\div$ last!!)

If I had said, "find $f'(2)$," you'd give $\frac{5}{23}$. Give exact answers, unless you're told to approx.

$$\boxed{\approx 0.217}$$

Up to 9

(B) $e^{☆}$

$$\boxed{D_x(e^x) = e^x}$$

$$\boxed{D_x[e^{f(x)}] = e^{f(x)} \cdot \underbrace{f'(x)}_{\text{tail}} \qquad D[e^{☆}] = e^{☆} D(☆)}$$ (f(x) diff'e)

Ex $D_x(e^{x^4+\ln x}) = e^{x^4+\ln x} \cdot D_x(x^4 + \ln x)$

$= \boxed{e^{x^4+\ln x}\left(4x^3 + \frac{1}{x}\right)}$

Ex $D_x(e^{10x}) = \boxed{\underbrace{10}_{\text{tail}} e^{10x}}$

Ex $D_x(e^{-\frac{x}{7}}) = D_x(e^{-\frac{1}{7}x})$

$= \boxed{-\frac{1}{7}e^{-\frac{1}{7}x}}$

For constant k,

If $y = e^{kx}$

$\Rightarrow y' = k\underbrace{e^{kx}}_{=y}$

$y' = ky$

Rate of change is proportional to the current amt. (Ch. 9)

always same fraction (multiple) of

Explains exp. growth in pop, & accts.

<u>Ex</u> On Feb. 1, 2000, you deposited \$3000 at 3% annual interest compounded continuously. How fast does your account grow on Feb. 1, 2007?

A(7) gives amt. after 7 years. I don't want A(7), I want

$$\text{Let } A(t) = \text{your amt. after } t \text{ years}$$
$$= Pe^{rt}$$
$$= 3000e^{.03t}$$

Feb. 1, 2007 $\Leftrightarrow t = 7$
Find $A'(7)$.

$$A'(t) = 3000(.03e^{.03t})$$
$$= 90e^{.03t}$$

At UCSD - common mistake!!

$$A'(7) = 90e^{\overbrace{(.03)(7)}^{e^{.15t}}}$$
$$= 90e^{.21}$$
$$\approx 111.03 \left(\frac{\$}{yr.}\right)$$

~~e ^ .03 * 7~~ $\Rightarrow e^{.03} \cdot 7$ Not what I want!!

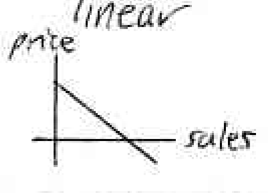
$\boxed{e^x}$ button

$$\boxed{\text{On Feb. 1, 2007, your account grows at the rate of about \$111.03 per year.}}$$

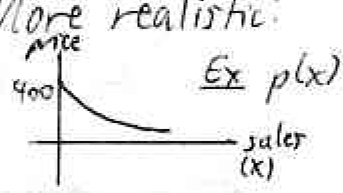
<u>More Econ</u> (Price funcs.)

In 65-66 (3rd) not assigned

linear



More realistic:



<u>Ex</u> $p(x) = 400e^{-0.3x}$
always > 0

You can't get something for nothing. Tribbles

<u>Note</u> $D_x(2^x) = 2^x \ln 2$
NOT $x2^{x-1}$ (Power Rule doesn't work for this exp'l func.)

(4.4): RELATIVE RATES

Who bugs you more? The music co. or the computer company?

	Given: Current Price	Given: Rate of ↗	Relative Rate of ↗
CDs	\$20	\$2/yr.	$\frac{2}{20} = 0.1$ (10%/yr.)
Computers	\$2000	\$100/yr.	$\frac{100}{2000} = 0.05$ (5%/yr.)

What's an expr. for relative

$f(t) =$	$f'(t) =$	$\frac{f'(t)}{f(t)} =$
amt. at time t	[absolute] rate of change of f	relative

Potatoes, caviar

Good for comparisons!

One way to find $\frac{f'(t)}{f(t)}$ is to find $D_t[\ln \overbrace{f(t)}^{\text{If} > 0}]$.
Easier?

Why?

$$D_t[\ln f(t)] = \frac{1}{f(t)} \cdot f'(t)$$
$$= \frac{f'(t)}{f(t)}$$

Ex $f(t) = e^{-t}\sqrt{t^2+2}$ (t in years)

ⓐ Find the relative rate of change.
ⓑ " " at $t=3$.

If do $\frac{f'(t)}{f(t)}$ ← May be hard to find! Instead, let's

Expand $\ln f(t)$

$$\ln f(t) = \ln e^{-t}\sqrt{t^2+2}$$

Do you know how to expand

<u>Note</u> $\ln(MN) = \ln M + \ln N \quad (M, N > 0)$

log(product) = sum of logs

<u>Also</u> $\ln\left(\frac{M}{N}\right) = \ln M - \ln N$

<u>Warning</u> $\ln(M+N)$

can't split

$$= \ln e^{-t} + \ln\sqrt{t^2+2}$$

Power Rule

$$= -t \quad + \ln(t^2+2)^{\frac{1}{2}}$$

$$= -t \quad + \tfrac{1}{2}\ln(t^2+2)$$

<u>D_t</u>

$$D_t[\ln f(t)] = -1 + \cancel{\tfrac{1}{2}}\,\frac{1}{t^2+2}\cdot \cancel{2}t$$

$$= \boxed{-1 + \frac{t}{t^2+2}}$$

Answer to (a)

(b) <u>At $t=3$</u>

$$= -1 + \frac{3}{(3)^2+2}$$

Interpret

$$\approx -0.727$$

Enron stock

$\boxed{\text{Decrease of } \approx 72.7\% \text{ per year}}$

(Optional)

Elasticity of Demand

= Relative rate of change of demand / Relative rate of change of price

How sensitive is demand to price changes?

rubber band, underwear
if stretch, there's a reaction

If $>1 \Rightarrow$ Demand is elastic. (Demand is sensitive to price changes.)

Why can this help?
price war - gas stations "Texaco, Mobil," etc...
ignores costs

You should lower prices, bec. the ↗ in demand will make up for it.

Boats
Baumol — not Denny's
Restaurant meals: 1.6

Ex luxuries (Restaurant meals: 1.6 — If prices ↗ 10% ⇒ demand ↘ 16%)
close subs available (Tide vs. Cheer)
price war? (Mobil vs. Texaco)

Proven in your book
But profit also depends on... cost!!

If $= 1 \Rightarrow$ You're maximizing revenue.

If $< 1 \Rightarrow$ (Demand is) inelastic.

You should raise prices, bec. it won't drive away too many people.

Final Jeopardy?

Cereal prices?
Lucky Charms vs. Miniwheats
not close subs

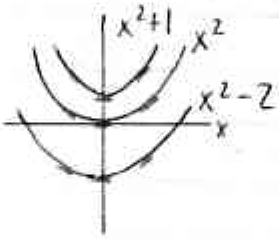
Ex potatoes (U.S. chief source of Vit. C!)
Huge harvests ⇒ ↓ price
⇒ ↓ rev. (not enough new demand)

Interesting, taking deriv.

5.1: ANTIDERIVATIVES (ADs) and INDEFINITE $\int$s

What's the deriv. of x^2? If I tell you $2x$ is the deriv. of some func., does it have to be x^2?

(A) ADs



$$\xrightarrow{D_x} 2x \qquad \xleftarrow[\text{Find ADs}]{\text{NOW}}$$

Wouldn't you agree the slope situation is the same

Given the deriv. family, how do we reconstruct "the original func."? Actually, we have a whole family of ADs.

Given: $f(x) = 2x$
Then, $F(x) = x^2 + C$ gives all the ADs of f.

C: arbitrary constant

$$\boxed{F'(x) = f(x)}$$

(B) Indefinite $\int$s

Symbol. Book.

It's because of this C that we call these indefinite. We'll flow to "Ax", family of funcs. Definite $\int$ → #

$$\int \underbrace{2x}_{f(x)} \underbrace{dx} = \underbrace{x^2}_{F(x)} + C$$

$\int$: integral sign
$f(x)$, the integrand
dx: says x is the variable of integration
$F(x)$, an AD of f
C: constant of integration

And we CAN'T w.r.t. we want!

(Evaluating the $\int$
Integrating $f(x)$)

$$\boxed{\text{Form: } \int f(x)\,dx = F(x) + C}$$

$F(x)$: an AD of f

Ⓒ Basic Rules

Deriv of x is 1
∫ of 1 is

$D_x(x) = 1$ $\qquad$ $\int 1\,dx = \int dx = x + C$

$D_x(3x) = 3$ $\qquad$ $\int 3\,dx = 3x + C$

D_x to ✓

Power Rule for ∫

what if we want
∫ $x^3\,dx$?
How do we adjust?
Not sing
Don't need $\frac{c}{4}$

$D_x(x^4) = 4x^3 \Rightarrow \int 4x^3\,dx = x^4 + C$ $\qquad$ (Berresford) "Sing at sight"

$\left(D_x\left(\frac{x^4}{4}\right) = \frac{1}{4}\cdot 4x^3 = x^3 \right) \Rightarrow \int x^3\,dx = \frac{x^4}{4} + C$ ← Don't need $\frac{c}{4}$.

Berresford ∫:
$\frac{1}{n+1}x^{n+1} + C$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad \text{if } n \neq -1$$

$$\frac{x^{\text{add } 1}}{\text{copy}} + C$$

∫ is Linear, like D_x, lim

If I'm D_x a long poly. -

Can ∫ term-by-term

$\int c\, f(x)\,dx$

constant factor

① Exs

Ex $\int (x^7 + 2x^4 - 1)\,dx$

$= \int x^7 dx + \int 2x^4 dx - \int 1\,dx$ (can skip)

$= \frac{x^8}{8} + 2\left(\frac{x^5}{5}\right) - x + C$

$= \boxed{\frac{1}{8}x^8 + \frac{2}{5}x^5 - x + C}$

Can ✓

Ex $\int \left(\frac{3}{x^6} + \frac{2}{\sqrt{x}} + 7\cdot\sqrt[4]{x^3}\right) dx$

① Rewrite ⇒ Powers, ≈ Poly.

$= \int (3x^{-6} + 2x^{-1/2} + 7x^{3/4})\,dx$

② $\int$ (+C)

$= 3\left(\frac{x^{-5}}{-5}\right) + 2\left(\frac{x^{1/2}}{1/2}\right) + 7\left(\frac{x^{7/4}}{7/4}\right) + C$

③ Simplify, ÷ by a frac ⇒ · by its recip.

$= -\frac{3}{5}x^{-5} + 2(2)x^{1/2} + 7\left(\frac{4}{7}\right)x^{7/4} + C$

$= \boxed{-\frac{3}{5x^5} + 4\sqrt{x} + 4\cdot\sqrt[4]{x^7} + C}$ (Am I asking for positive exponents? Radical form?)

__Ex (#24)__ $\int (1-7w)\cdot\sqrt[3]{w}\ dw$

$$= \int (1-7w)\, w^{1/3}\ dw$$

Algebra!

$$= \int \left(w^{1/3} - 7w^{\overbrace{4/3}^{1+\frac{1}{3}}}\right) dw$$

$$= \frac{w^{4/3}}{4/3} - 7\left(\frac{w^{7/3}}{7/3}\right) + C$$

$$= \frac{3}{4} w^{4/3} - 7\left(\frac{3}{7}\right) w^{7/3} + C$$

$$= \boxed{\frac{3}{4} w^{4/3} - 3w^{7/3} + C}$$

__Ex__ $\int 2\ dw = 2w + c$

Ⓔ Solving Differential Eqs.

	$\xrightarrow{D_x}$	
$f(x)$		$f'(x)$
R, C, P		Marginals
Amt.		Rate of change
Position $s(t)$		Velocity $v(t)$
	$\xleftarrow{\int}$	

∫ to reconstruct.
You need an
<u>initial condition</u> (IC)

<u>Ex</u> The marginal cost func. is $MC(x) = 3x^2 + x$ (\$/doll), where x = # dolls produced. Fixed costs are \$2000.

<u>ⓐ Find the cost func., $C(x)$.</u>

<u>① Setup</u>

Solve the diff. eq.

$$MC(x) \text{ or } C'(x) = 3x^2 + x$$

Translate into mathem notation

subject to the IC
$$C(0) = \underbrace{2000}_{\text{fixed}}$$

(When 0 units are produced, the <u>only</u> costs are fixed costs.)

∫ at sight
Your advisor: You have \$20,000+K fired!!

② $\int$ to find the general sol'n

$$\int C'(x)\,dx = \int (3x^2 + x)\,dx$$
$$C(x) = x^3 + \frac{x^2}{2} + \underbrace{K}_{\text{We're using "C" for Cost}}$$

③ Use the IC to solve for K, and Find the particular sol'n

IC: $C(0) = 2000$
When $x = 0 \Rightarrow C(x) = 2000$,
Plug in

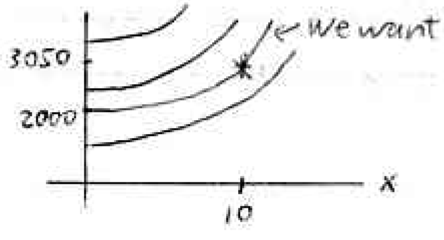
$$2000 = (0)^3 + \frac{(0)^2}{2} + K$$
$$K = 2000$$

$$\boxed{C(x) = x^3 + \frac{x^2}{2} + 2000}$$

ⓑ How much does it cost to produce 10 dolls?

$$C(10) = (10)^3 + \frac{(10)^2}{2} + 2000$$
$$= \boxed{\$3050}$$

Gen sol'n (family of funcs.)



(5.2): $\int$, $e^{☆}$, ln

(A) $e^{☆}$

$D_x(e^x) = e^x$

$\Rightarrow \boxed{\int e^x\,dx = e^x + C}$

Ex $D_x(e^{4x}) = \underbrace{4}_{\text{tail}}e^{4x}$

$\Rightarrow \int 4e^{4x}\,dx = e^{4x} + C$

$\Rightarrow \int e^{4x}\,dx = \frac{e^{4x}}{4} + C$

When you D_x e^{kx}, you mult. by k; when you $\int$, you ÷ by k

$$\boxed{\begin{array}{l}\text{For constant } k, \\ \int e^{kx}\,dx = \frac{e^{kx}}{k} + C\end{array}}$$

Remember, $D_x(e^{kx}) = ke^{kx}$ (We mult. by k.) $\int$ reverses D_x; we ÷.

Ex $\int e^{-10x}\,dx = \frac{e^{-10x}}{-10} + C$

$= \boxed{-\frac{1}{10}e^{-10x} + C}$

Ex $\int 4e^{\frac{3x}{7}}\,dx = \int 4e^{\frac{3}{7}x}\,dx$

$= 4\left(\frac{e^{\frac{3}{7}x}}{\frac{3}{7}}\right) + C$

$= 4\left(\frac{7}{3}\right)e^{\frac{3}{7}x} + C$

$= \boxed{\frac{28}{3}e^{\frac{3}{7}x} + C}$

Up to 11

2nd, 3rd

Ex (#40)

A biotech investment, originally worth \$20,000, grows continuously at the rate of $1000e^{0.10t}$ \$/year, where t = # yrs. since the investment was made.

(a) Find a formula for the value of the investment after t years.

(1) Setup

Call this $V(t)$.

What is $1000e^{0.10t}$ Translate into mathem. notation.

Solve
$$V'(t) = 1000e^{0.10t}$$

What is 20,000?

subject to
$$V(0) = 20,000$$

(2) Gen. Sol'n

$$\int V'(t)\,dt = \int 1000e^{0.10t}\,dt$$
$$V(t) = 1000\left(\frac{e^{0.10t}}{0.10}\right) + C$$
$$\div \tfrac{1}{10} \Leftrightarrow \cdot 10$$
$$V(t) = 10,000e^{0.10t} + C$$

(3) Partic. Sol'n

When $t = 0 \Rightarrow V(t) = 20,000$

$$20{,}000 = 10{,}000\underbrace{e^{0.10(0)}}_{=e^0 \; =1} + C$$

Huh?

$$20{,}000 = 10{,}000 + C$$
$$C = 10{,}000 \qquad (\underline{\text{not}}\ 20{,}000\ !!)$$

$$\boxed{V(t) = 10{,}000e^{0.10t} + 10{,}000}$$

(b) Find the value of the investment after 7 years.

$$V(7) = 10{,}000e^{\overbrace{0.10(7)}^{0.15t}} + 10{,}000$$
$$= 10{,}000e^{0.70} + 10{,}000$$
$$\boxed{\approx \$30{,}137.53}$$

What's another kind of ? you can ask. You're an investor.

C: Not in book When will the investment be worth $50,000?

Solve $V(t) = 50{,}000$ for t.

$$10{,}000e^{0.10t} + 10{,}000 = 50{,}000$$
$$10{,}000e^{0.10t} = 40{,}000$$
$$e^{0.10t} = 4$$

What kills off e as a base?

$$\underset{\text{undo}}{\ln e}^{0.10t} = \ln 4$$
$$0.10t = \ln 4$$
$$t = \frac{\ln 4}{0.10}$$

Your toolure will have to wait...

Up to 11

$t \approx 13.9$

$\boxed{\text{After about 13.9 years}}$

(B) ln

$\int \ln x\, dx$ - 122

Review Power Rule for $\int$

Ex $\int x^2 dx = \frac{x^3}{3} + C$

$\int x^n dx = \frac{x^{n+1}}{n+1} + C$ if $n \neq -1$

oh, +C

NO: $\int x^{-1} dx = \cancel{\frac{x^0}{0}} + C$

What is $\int x^{-1} dx$, or $\int \frac{1}{x} dx$?

4.3 $D_x(\ln x) = \frac{1}{x}$

$\Rightarrow$ $\boxed{\int \frac{1}{x} dx = \ln x + C \text{ if we have the restriction } x > 0}$

if not, say:

$\boxed{\int \frac{1}{x} dx = \ln |x| + C}$ Write this, unless you're given

(x never 0)

Ex $\int \frac{4+x}{3x} dx = \int \left(\frac{4}{3x} + \frac{x}{3x}\right) dx$ (Algebra)

$= \int \left(\frac{4}{3} \cdot \frac{1}{x} + \frac{1}{3}\right) dx$

$= \boxed{\frac{4}{3} \ln |x| + \frac{1}{3} x + C}$

CH. 4, 5.1, 5.2

Ch. 2 D_x Rules
Prod., Quot., Gen. Power, Linearity

4.1 Cont. Comp. Interest
Future Value = Pe^{rt}

4.2 ln Props:

$\ln 1 = 0$ (i.e., $\log_e 1 = \boxed{0}$ $e^{0} = 1$)

$\ln e = 1$

$\ln e^{\star} = \star$

$\ln M^P$

$\ln MN = \ln M + \ln N$

$\ln \frac{M}{N} = \quad -$

$M, N > 0$

4.3 $D_x(\ln x) = \frac{1}{x}$

$\underbrace{D(\ln \star)}_{\text{Props.?}} = \frac{1}{\star} \underbrace{D(\star)}_{\text{tail from Chain Rule}}$

$x, \star > 0$

$D_x(e^x) = e^x$

$D(e^{\star}) = e^{\star} \underbrace{D(\star)}_{\text{tail}}$

4.4 Relative rate of change of f

$$= \frac{f'(t)}{f(t)} \text{ or } D_t \underbrace{[\ln \overset{>0}{\widetilde{f(t)}}]}_{\text{Props.?}}$$

Ex in %/yr.

5.1, 5.2

Deriv. of what is 4 (wrt x)?

$D_x(e^{kx}) = e^{kx} \cdot k$

Indefinite $\int$s

$\int 4\,dx = 4x + C$

$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$

$\int x^{-1} dx$ or $\int \frac{1}{x} dx = \ln|x| + C$

$\int e^x dx = e^x + C$

$\int e^{kx} dx = \frac{e^{kx}}{k} + C$

$\int$ linear
Rewrite ⇒ Powers, ≈ Poly.
Algebra

Solving Diff. Eqs. subject to IC

Do an Ex

Reconstruct $f(x)$ from $f'(x)$, IC.
1. Setup
2. $\int$ ⇒ find Gen. Sol'n
3. Use IC to solve for C or K
   ⇒ find Partic. Sol'n

Use $f(x)$ to answer ?s
Eval $f(\#)$
Solve an [Exp'l] eq.
Ex $10{,}000\underbrace{e^{0.10t}}_{\text{Solve for}} + 10{,}000 = 50{,}000$
Use ln

Word probs.
Interpret
Units