In terms of graphs ∂_x are slopes

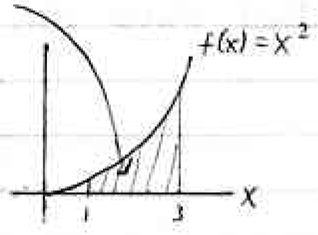
(5.3): DEFINITE $\int$s and AREAS

Ⓐ Intro

Ex

$$\int_{\underset{\nwarrow\ \underline{\text{lower limit}}}{1}}^{\overset{\underline{\text{upper limit}}}{3}} x^2\,dx = \text{the } \underline{\text{definite integral}} \text{ of } x^2 \text{ from } 1 \text{ to } 3$$

$= \text{Area}$



bec. $x^2 \geq 0$ on $[1,3]$

$$\overset{\text{turns out}}{=} 8\tfrac{2}{3} \text{ or } 8.\overline{6}$$

what do you think
Dummy var,
→ replace him
w/another dummy,
nothing changes;
like airport
security

Ex $\int_1^3 t^2\,dt = 8.\overline{6}$, also

In Ch. 5,

An $\underline{\text{indefinite } \int}$ gives a family of funcs.

$\underline{\text{Ex}}$ $\int x^2\,dx = \frac{x^3}{3} + C$

gives us...

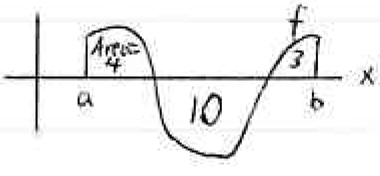
A $\underline{\text{definite } \int}$ gives a #.

$\underline{\text{Ex}}$ $\int_1^3 x^2\,dx = 8.\overline{6}$

What if $f < 0$ sometimes?

What do you think we do w/10
contribute
a neg. # to
the def. $\int$

Ex

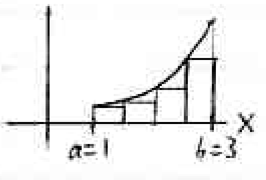


$$\int_a^b f(x)\,dx = 4 - 10 + 3$$
$$= -3$$

Ⓑ Approximating Areas, Definite $\int$s Using Riemann Sums $(f \geq 0)$

Ex Approx. $\int_1^3 x^2\,dx$ using a Left Riemann Sum with $n=4$ rectangles of the same "width." (i.e., Approx. the area under the curve $f(x)=x^2$ from $a=1$ to $b=3$.)

Step 1: Find $\underset{\text{delta}}{\Delta} x$, the rect. width.



n=4 rects.

width of interval
$= b-a$
$(= 2, \text{ here})$

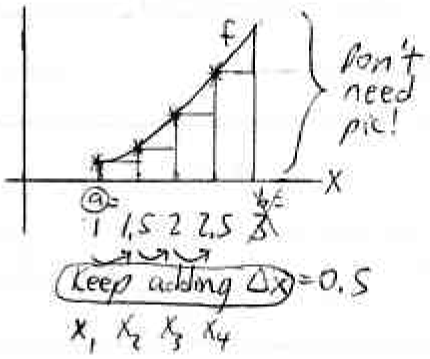
$$\boxed{\text{width of each rect.} = \Delta x = \frac{b-a}{n}}$$

Here, $$\boxed{\begin{aligned}\Delta x &= \frac{3-1}{4}\\ &= \frac{2}{4}\\ &= \frac{1}{2} \text{ or } 0.5\end{aligned}}$$

<u>Step 2</u>: Find the left breakpoints $(x_1, x_2, \ldots, x_n)$

In a <u>Left</u> Riemann Sum,
the <u>left</u> corners of each rect.
lie on the curve and the x-axis.

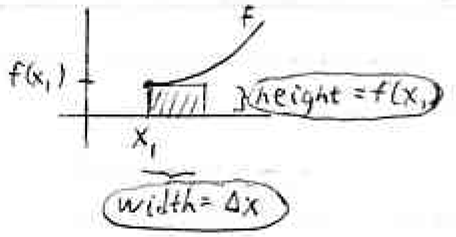
x_i: where do we stargaze? fenceposts



Don't need pic!

<u>Note</u> What is the area of the 1^{st} rect.?

If this is x_1, what's this y-coord?



$$\left(\begin{aligned} &\text{func. value at } x_1 \\ &= \text{y-coord. of pt. (at } x_1\text{)} \\ &= \text{height of rect.} \end{aligned}\right)$$

what's the area of the 2nd

$$\boxed{\underset{\text{of 1st rect.}}{\text{Area}} = f(x_1)\cdot\Delta x}$$

<u>Step 3</u>: Find the Left Riemann Sum

$$\begin{aligned}
&= \text{sum of the areas of the rects.} \\
&= \underbrace{f(x_1)\cdot\Delta x}_{1st} + \underbrace{f(x_2)\cdot\Delta x}_{2nd} + \underbrace{f(x_3)\cdot\Delta x}_{3rd} + \underbrace{f(x_4)\cdot\Delta x}_{4th} \\
&= [f(1)][0.5] + [f(1.5)][0.5] + [f(2)][0.5] + [f(2.5)][0.5] \\
&= [(1)^2][0.5] + [(1.5)^2][0.5] + [(2)^2][0.5] + [(2.5)^2][0.5] \\
&= 0.5 + 1.125 + 2 + 3.125 \\
&= \boxed{6.75}
\end{aligned}$$

$f(x) = x^2$

2nd: Do 1, 3

How come we have an underest. of the true value?

Prob: All Δx_i =

Area $\approx$ 6.75 square units

$\int_1^3 x^2\,dx \approx 6.75$ (Exact: $8.\bar{6}$)

n = # rects. (assume same width)

As $n \to \infty$,
Approxs. → Exact

$$\boxed{\begin{array}{l}\text{If } f \text{ cont. on } [a,b], \\ \quad \int_a^b f(x)\,dx = \lim\limits_{n\to\infty} [\text{Left Riemann Sums}]\end{array}}$$ (Def'n)

Ⓒ An Easier Way to Find $\int_a^b f(x)\,dx$: Fundamental Thm. of Calculus (FTC)

$$\boxed{\begin{array}{l}\text{If } f \text{ cont. on } [a,b], \\ \quad \int_a^b f(x)\,dx = [\overset{\uparrow}{F}(x)]_a^b \quad \text{Find an AD of } f \\ \quad\quad = \underbrace{F(b)}_{\text{Eval at } b} - \underbrace{F(a)}_{\text{Eval at } a}\end{array}}$$

Ex $\int_1^3 \underbrace{x^2}_{\text{cont. on } [1,3] \checkmark}\,dx = \left[\underbrace{\frac{x^3}{3}}_{\text{an AD of } x^2}\right]_1^3$ Don't need "+C"!!

$= \underbrace{\left[\frac{(3)^3}{3}\right]}_{\text{(Eval. at 3)}} - \underbrace{\left[\frac{(1)^3}{3}\right]}_{\text{(Eval. at 1)}}$

$= 9 - \frac{1}{3}$

$= \boxed{8\frac{2}{3} \text{ or } 8.\bar{6}}$

LS-8
5.3

Ex Find the area under $f(x) = \underbrace{2 + \frac{1}{\sqrt[3]{x}}}_{\geq 0 \text{ on } [1,8], \text{ so}}$
from $x = 1$ to $x = 8$.

$$\text{Area} = \int_1^8 \underbrace{\left(2 + \frac{1}{\sqrt[3]{x}}\right)}_{\substack{\text{Rewrite} \\ \text{Cont. on } [1,8] \checkmark}} dx$$

$$= \int_1^8 \underbrace{\left(2 + x^{-1/3}\right)}_{\text{Find an AD (1)}} dx$$

$$= \Big[\underbrace{2x + \frac{x^{2/3}}{2/3}}_{\text{Rewrite}} \Big]_1^8$$

$$= \left[2x + \frac{3}{2}x^{2/3} \right]_1^8$$

What's easier to do 1st? ^{2} or $\sqrt[3]{}$

$$= \left[2x + \frac{3}{2}\left(\sqrt[3]{x}\right)^2 \right]_1^8$$

$$= \underbrace{\left[2(8) + \frac{3}{2}\left(\sqrt[3]{8}\right)^2 \right]}_{\text{Eval. at 8}}$$

$$\ominus \underbrace{\left[2(1) + \frac{3}{2}\left(\sqrt[3]{1}\right)^2 \right]}_{\text{Eval. at 1}}$$

$\nwarrow$ $\nearrow$

$$= \left[16 + \frac{3}{2}(2)^2 \right] - \left[2 + \frac{3}{2}(1)^2 \right]$$

$$= \left[16 + \frac{3}{\underset{1}{\cancel{2}}}(\overset{2}{\cancel{4}}) \right] - \left[2 + \frac{3}{2} \right]$$

$$= 22 - \frac{7}{2}$$

$$= \frac{44}{2} - \frac{7}{2}$$

$$= \boxed{\frac{37}{2} \text{ or } 18\tfrac{1}{2} \text{ square units}}$$

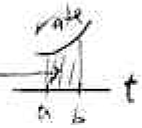
2nd: Up to 69
3rd: Up to 75

Ⓓ Word Probs.

$$\text{Idea} \quad \int_a^b f'(t)\,dt \overset{\text{"}[f(t)]_a^b\text{"}}{=} f(b) - f(a)$$

$$\boxed{\begin{aligned}\int_a^b (\text{rate})\,dt &= \begin{pmatrix}\text{amt. at}\\ t=b\end{pmatrix} - \begin{pmatrix}\text{amt. at}\\ t=a\end{pmatrix}\\ &= \text{total (net) change from a to b}\\ &\quad \text{"total accumulation"}\end{aligned}}$$

Book



Ex (#80, #86)
2nd Ed. 3rd Ed.

(Shortened)

A car will sell at the rate of $\frac{30}{t}$ cars per month, where t is in months, and t=1 corresp. to Jan. 1. Find the # of cars that will be sold from Jan. 1 to May 1.

$t=1$ $t=5$

No IC here
Not like we're
selling a ton
of cars on Jan. 1

$$\underbrace{\int_1^5 \frac{30}{t}\,dt}_{\text{cont. on }[1,5]\ \checkmark}$$

$$= \int_1^5 30 \cdot \frac{1}{t}\,dt$$

$$= \left[30 \ln \underbrace{|t|}_{\substack{\text{can drop}\\ t>0 \text{ on } [1,5]}}\right]_1^5$$

$$= 30 \ln 5 - 30 \underbrace{\ln 1}_{=0}$$

$$= 30 \ln 5$$

$$\boxed{\approx 48 \text{ cars}}$$