In terms of graphs D_x are slopes

(5.3): DEFINITE $\int$s and AREAS

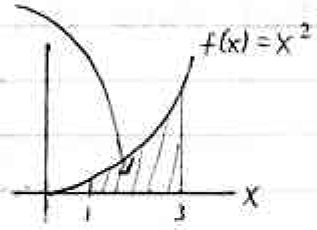
(A) Intro

Ex $\int_1^3 x^2\,dx$ = the definite integral of x^2 from 1 to 3

upper limit

lower limit

= Area



bec. $x^2 \geq 0$ on $[1,3]$

$\overset{\text{turns out}}{=} 8\frac{2}{3}$ or $8.\bar{6}$

what do you think
Dummy var,
replace him
w/another dummy,
nothing changes,
like airport
security

Ex $\int_1^3 t^2\,dt = 8.\bar{6}$, also

In Ch. 5,

An indefinite $\int$ gives a family of funcs.

Ex $\int x^2\,dx = \frac{x^3}{3} + C$

gives us...

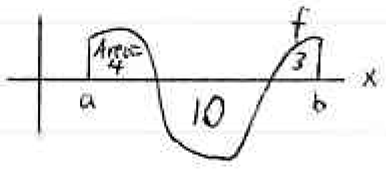
A definite $\int$ gives a #.

Ex $\int_1^3 x^2\,dx = 8.\bar{6}$

What if $f < 0$ sometimes?

What do you think
we do w/10
contribute
a neg. # to
the def. $\int$

Ex

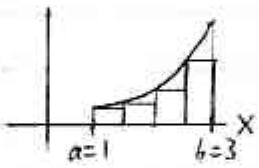


$\int_a^b f(x)\,dx = 4 - 10 + 3$
$= -3$

(B) Approximating Areas, Definite $\int$s Using Riemann Sums ($f \geq 0$)

Ex Approx. $\int_1^3 x^2\,dx$ using a Left Riemann Sum with $n=4$ rectangles of the same "width."
(i.e., Approx. the area under the curve $f(x)=x^2$ from $a=1$ to $b=3$.)

Step 1: Find Δx (delta), the rect. width.



n=4 rects.

$\underbrace{\qquad}$ width of interval $= b-a$ ($=2$, here)

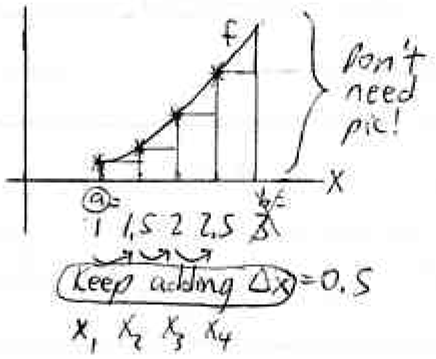
width of each rect. $= \Delta x = \frac{b-a}{n}$

Here, $\Delta x = \frac{3-1}{4} = \frac{2}{4} = \frac{1}{2}$ or 0.5

Step 2: Find the left breakpoints $(x_1, x_2, \dots, x_n)$

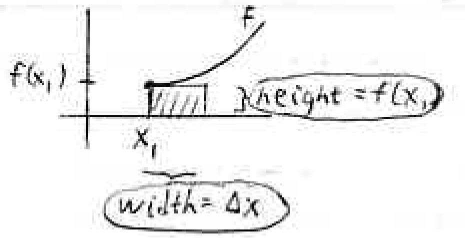
In a Left Riemann Sum,
the left corners of each rect.
lie on the curve and the x-axis.

x_i: where do we start? fenceposts



Note What is the area of the 1^{st} rect.?

If this is x_1, what's this y-coord?



func. value at x_1
= y-coord. of pt. (at x_1)
= height of rect.

what's the area of the 2nd

Area of 1^{st} rect. $= f(x_1) \cdot \Delta x$

Step 3: Find the Left Riemann Sum

$$= \text{sum of the areas of the rects.}$$

$$= \underbrace{f(x_1)\cdot\Delta x}_{1st} + \underbrace{f(x_2)\cdot\Delta x}_{2nd} + \underbrace{f(x_3)\cdot\Delta x}_{3rd} + \underbrace{f(x_4)\cdot\Delta x}_{4th}$$

$$= [f(1)][0.5] + [f(1.5)][0.5] + [f(2)][0.5] + [f(2.5)][0.5]$$

$f(x) = x^2$

$$= [(1)^2][0.5] + [(1.5)^2][0.5] + [(2)^2][0.5] + [(2.5)^2][0.5]$$

$$= 0.5 + 1.125 + 2 + 3.125$$

$$= \boxed{6.75}$$

2nd: do 1,3

How come we have an underest. of the true value?

Book: All Δx_i =

$$\text{Area} \approx 6.75 \text{ square units}$$

$$\int_1^3 x^2\,dx \approx 6.75 \quad (\text{Exact: } 8.\overline{6})$$

n = # rects. (assume same width)

As $n \to \infty$,
Approxs. $\to$ Exact

$$\boxed{\begin{array}{l} \text{If } f \text{ cont. on } [a,b], \\ \displaystyle\int_a^b f(x)\,dx = \lim_{n\to\infty} [\text{Left Riemann Sums}] \end{array}}$$ (Def'n)

Ⓒ An Easier Way to Find $\int_a^b f(x)\,dx$: Fundamental Thm. of Calculus (FTC)

$$\boxed{\begin{array}{l} \text{If } f \text{ cont. on } [a,b], \\ \displaystyle\int_a^b f(x)\,dx = [\underset{\text{Find an AD of } f}{F(x)}]_a^b \\ \qquad = \underbrace{F(b)}_{\text{Eval at } b} - \underbrace{F(a)}_{\text{Eval at } a} \end{array}}$$

<u>Ex</u> $$\int_1^3 \underbrace{x^2}_{\text{cont. on } [1,3] \checkmark}\,dx = \left[\underbrace{\frac{x^3}{3}}_{\text{an AD of } x^2}\right]_1^3$$ Don't need "+C"!!

$$= \underbrace{\left[\frac{(3)^3}{3}\right]}_{(\text{Eval. at } 3)} - \underbrace{\left[\frac{(1)^3}{3}\right]}_{(\text{Eval. at } 1)}$$

$$= 9 - \frac{1}{3}$$

$$= \boxed{8\tfrac{2}{3} \text{ or } 8.\overline{6}}$$

<u>Ex</u> Find the area under $f(x) = \underbrace{2 + \frac{1}{\sqrt[3]{x}}}_{\geq 0 \text{ on } [1,8], \text{ so}}$
from $x = 1$ to $x = 8$.

$$\text{Area} = \int_1^8 \underbrace{\left(2 + \frac{1}{\sqrt[3]{x}}\right)}_{\substack{\text{Rewrite} \\ \text{Cont. on } [1,8] \checkmark}} dx$$

$$= \int_1^8 \underbrace{\left(2 + x^{-1/3}\right)}_{\text{Find an AD (S)}} dx$$

$$= \Big[\underbrace{2x + \frac{x^{2/3}}{2/3}}_{\text{Rewrite}} \Big]_1^8$$

$$= \left[2x + \frac{3}{2} x^{2/3} \right]_1^8$$

what's easier to do 1st? ^{2} or $\sqrt[3]{\ }$

$$= \left[2x + \frac{3}{2} \left(\sqrt[3]{x}\right)^2 \right]_1^8$$

$$= \underbrace{\left[2(8) + \frac{3}{2}\left(\sqrt[3]{8}\right)^2 \right]}_{\text{Eval. at 8}}$$
$$- \underbrace{\left[2(1) + \frac{3}{2}\left(\sqrt[3]{1}\right)^2 \right]}_{\text{Eval. at 1}}$$

$$= \left[16 + \frac{3}{2}(2)^2 \right]$$
$$- \left[2 + \frac{3}{2}(1)^2 \right]$$

$$= \left[16 + \frac{3}{2}(4) \right] - \left[2 + \frac{3}{2} \right]$$

$$= 22 - \frac{7}{2}$$

$$= \frac{44}{2} - \frac{7}{2}$$

2nd: Up to 69
3rd: Up to 75

$$= \boxed{\frac{37}{2} \text{ or } 18\frac{1}{2} \text{ square units}}$$

D Word Probs.

Idea $\int_a^b f'(t)\,dt \overset{"[f(t)]_a^b"}{=} f(b) - f(a)$

$$\boxed{\begin{aligned} \int_a^b (\text{rate})\,dt &= \begin{pmatrix}\text{amt. at}\\ t=b\end{pmatrix} - \begin{pmatrix}\text{amt. at}\\ t=a\end{pmatrix} \\ &= \text{total (net) change from } a \text{ to } b \\ &\quad \text{"total accumulation"} \end{aligned}}$$

Book

Ex (#80 2nd Ed., #86 3rd Ed.)

(Shortened)

A car will sell at the rate of $\frac{30}{t}$ cars per month, where t is in months, and $t=1$ corresp. to Jan. 1. Find the # of cars that will be sold from Jan. 1 to May 1.
$\quad t=1 \qquad\qquad t=5$

No IC here
Not like we're selling a ton of cars on Jan. 1

$$\int_1^5 \underbrace{\frac{30}{t}}_{\text{cont. on } [1,5]\ \checkmark} dt$$

$$= \int_1^5 30 \cdot \frac{1}{t}\,dt$$

$$= \left[30 \ln |t|\right]_1^5$$

can drop
$t>0$ on $[1,5]$

$$= 30 \ln 5 - 30 \underbrace{\ln 1}_{=0}$$

$$= 30 \ln 5$$

$$\boxed{\approx 48 \text{ cars}}$$