(5.4) MORE APPLICS OF DEFINITE Is

(A) Average Value of f (f_{av})

Prelim Ex

Quiz scores :) ?
How would I avg. your scores if final broke down?

Average of 4, 5, and 9

$$= \frac{4+5+9}{3} \quad \begin{array}{l} \leftarrow \text{Sum} \\ \leftarrow \text{Input Size (\# of \#s)} \end{array}$$

$$= \boxed{6}$$

Idea

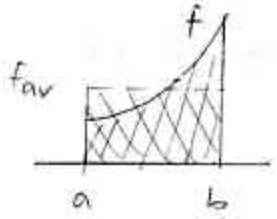
$$\underbrace{4+5+9}_{18} = \underbrace{6+6+6}_{18}$$

level off Communism
$18 to give to 3 kids
Like Price is Right range game - raise bar until get same total/area.

Same area (18)

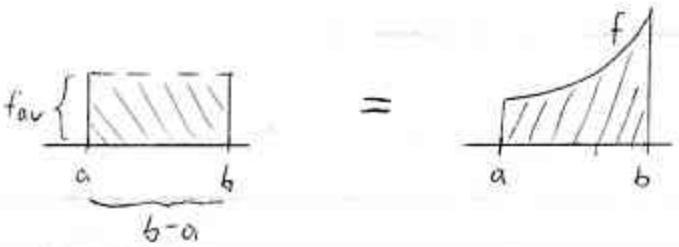
How do we find f_{av} for a func. f on $[a,b]$?
cont. on $[a,b]$

Ex



We need

We're merely extending the area principle from the discrete case.



f_{av} can we indicate the area using math notation?

$$(f_{av})(b-a) = \int_a^b f(x)\,dx$$

$$f_{av} = \frac{\int_a^b f(x)\,dx}{b-a}$$

← Continuous Sum (Area)

← Input Size (Interval Length)

Ex Find f_{av} of $f(x) = e^{2x}$ on $[-1, 2]$.

$$f_{av} = \frac{\int_{-1}^{2} e^{2x}\,dx}{2-(-1)}$$

$$= \frac{\left[\frac{e^{2x}}{2}\right]_{-1}^{2}}{3} \quad \left(\text{Can pull out } \tfrac{1}{2}\right)$$

$$= \frac{\frac{e^{2(2)}}{2} - \frac{e^{2(-1)}}{2}}{3}$$

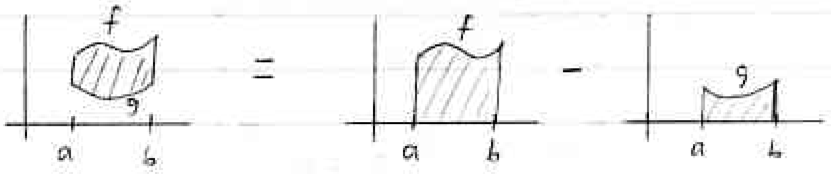
$$= \frac{\frac{1}{2}\left(e^4 - e^{-2}\right)}{3}$$

$$\boxed{\approx 9.077}$$

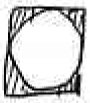
2nd 3rd Up to 29

(B) Area Between Curves (No Intersections)

Like scanning lengths of line segments → ... $\int$ as cont. sum.
"Fxx" idea too intense? How can I describe the area using math?



(Like Math 95:



Shaded area = Area of square − Area of circle)

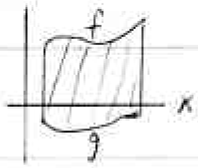
$$= \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

↓ Linearity

$$= \int_a^b [f(x) - g(x)]\,dx$$

"top − bot."

Works even if

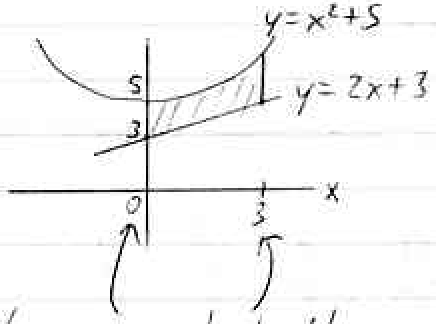


Ex (2nd, 3rd #38)

They say "parabola" "line"
Book omits:

(a) Sketch $y = x^2 + 5$ and $y = 2x + 3$ on the same graph. (Hint: They never intersect.)

What's the point? $y = x^2 \rightarrow y = x^2 + 5$
Does the line go up or down?



(b) Find the area bet. them from $x = 0$ to $x = 3$.

$$\text{Area} = \int_0^3 [(x^2+5) - (2x+3)]\,dx$$

top − bot.

$$= \int_0^3 [x^2 + 5 - 2x - 3]\,dx$$

$$= \int_0^3 [x^2 - 2x + 2]\,dx$$

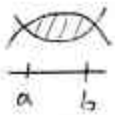
$$= \left[\frac{x^3}{3} - x^2 + 2x\right]_0^3$$

$$= \left[\frac{(3)^3}{3} - (3)^2 + 2(3)\right] - [0]$$

2nd: Up to 37
3-3'

$$= \boxed{6 \text{ square units}}$$

ⓒ Area Between Curves (w/ Intersections)



Ex (2nd, 3rd #44)

Find the area bounded by the curves $y = 3x^2 - x - 1$ and $y = 5x + 8$.

We're setting up a Def. I What I'm I should we ask?

$$\int_a^b (\text{top} - \text{bot.})\,dx$$

(what are a, b?) (which is which?)

Intersection pt. corresp. to solns of a... system!

① To find a, b, find where the curves intersect

Solve $\begin{cases} y = 3x^2 - x - 1 \\ y = 5x + 8 \end{cases}$

$y = y$

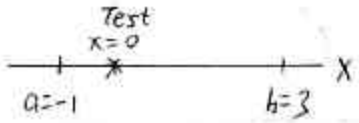
$$3x^2 - x - 1 = 5x + 8$$
$$3x^2 - 6x - 9 = 0$$
$$3(x^2 - 2x - 3) = 0$$
$$3(x - 3)(x + 1) = 0$$

$x = 3$ $\quad$ $x = -1$

Who's a?

② Who's on top?

What's a convenient



(We have continuity for both.)
(Don't test 0 if 0 not in $(a, b]$.)

$y = 3x^2 - x - 1$ (parabola)
$= 3(0)^2 - (0) - 1$ at $x = 0$
$= \boxed{-1}$ (can skip)

$y = 5x + 8$ (line)
$= 5(0) + 8$ at $x = 0$
$= \boxed{8}$

or $y = 3x^2 - x - 1$
↑ lead. coeff. > 0, so parabola opens up. ✓
Line intersects it twice. ⇒
~~✓~~ Line must be on top of bounded region!

③ ∫

$$\text{Area} = \int_{-1}^{3} [\underbrace{(5x+8)}_{\text{top}} - \underbrace{(3x^2 - x - 1)}_{\text{bot.}}]\,dx$$

$$= \int_{-1}^{3} [5x + 8 - 3x^2 + x + 1]\,dx$$

$$= \int_{-1}^{3} [-3x^2 + 6x + 9]\,dx$$

$$= \left[-x^3 + 6\left(\frac{x^2}{2}\right) + 9x\right]_{-1}^{3}$$

$$= \left[-x^3 + 3x^2 + 9x\right]_{-1}^{3}$$

$$= [-(3)^3 + 3(3)^2 + 9(3)] - [-(-1)^3 + 3(-1)^2 + 9(-1)]$$

$$= [27] - [-5]$$

$$= \boxed{32 \text{ square units}}$$

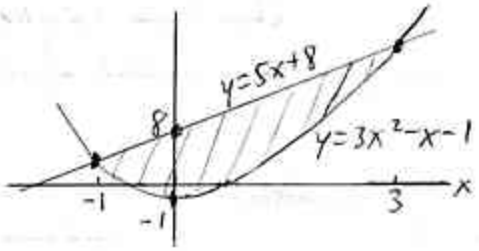
If curious

Use $5x+8$ to compute $\int_{-1}^{3}$

In fact we found y-intr. ($x=0$).

Like a developing photo up to 45 (2nd, 3rd)

Turns out



Ex (2nd, 3rd #56)

(Labor Contracts)

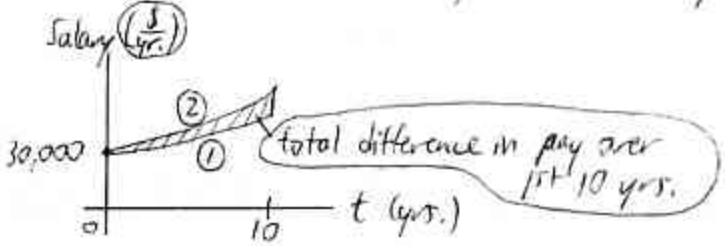
① Employer wants workers' salary (pay rate) to be $30,000e^{0.04t}$ per year.

② Union wants it to be $30,000e^{0.08t}$ per year.

Book: $

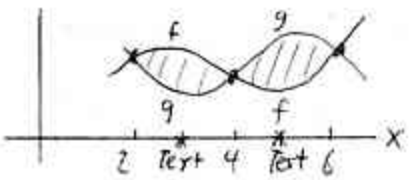
Which is on top ① or ②

What employer wants or what union wants?



$$\int_0^{10} (30{,}000e^{0.08t} - 30{,}000e^{0.04t})\,dt \approx \$90{,}709.32$$

Harder



What what?

$$\text{Area} = \int_2^4 (f-g)\,dx + \int_4^6 (g-f)\,dx$$