(5.6): "u" SUBS

Mention Ave, Rad Prof. Make you so powerful $e^{\#}$, ln #

Chain Rule for D_x
$\approx$ u Subs for $\int$

<u>Idea</u> $\displaystyle\int \begin{pmatrix}\text{Ugly}\\ \text{in } x\end{pmatrix} dx = \int \begin{pmatrix}\text{Nice}\\ \text{in } u\end{pmatrix} du$

(A) Indefinite $\int$s

<u>Ex</u> $\int (x^3+1)^{10}\, x^2\, dx$

In PRINCIPLE Bin. Thm? w/ grandma

Books always start "differential"

Mult. out? YUK! (Binomial Thm.)

Let $u = x^3+1$

$\dfrac{du}{dx} = 3x^2$ (technically inappropriate to say we're mult. by dx)

$du = 3x^2\,dx$ a "Differential form"

x^3 the doesn't count

We want to replace $3x^2 dx$ with du.
Put in 3

What makes up for the 3 Cancel w/ (≈ CTS) Make our own luck like dating

$\int (x^3+1)^{10}\, x^2\, dx$

$= \frac{1}{3} \int (x^3+1)^{10} \cdot \underbrace{3x^2\, dx}_{du}$

to <u>compensate</u> for 3

$= \frac{1}{3}\int u^{10}\, du$ (Nice!)

What? + C (Put it?) Am I done? I give apples you give me worms

$= \frac{1}{3}\cdot\frac{u^{11}}{11} + C$

$= \frac{1}{33}u^{11} + C$

<u>Go back to x.</u>

If we had mult. out or Bin. Thm, we'd have to factor to get + C ⇒ some flexibility

$= \boxed{\frac{1}{33}(x^3+1)^{11} + C}$ Can D_x to ✓

You took me on faith here...

Strategies for Picking u

cooked up

$$\int \underbrace{(x^3+1)}_{u}{}^{10} x^2\,dx$$

non-0 (goes w/out saying)

① Its deriv. is (maybe) also in $\int$,
except for a constant factor.

② u^{10} nice

It's buried

③ Common: $u=$ "inside", exp., denom.
Templates: $\int u^n\,du$, $\int e^u\,du$, $\int \frac{du}{u}$

Another Way (not in book)

x^2dx is Bill

$$\int (x^3+1)^{10} \underbrace{x^2dx}_{\text{"Kill"}}$$

$$u = x^3+1$$

$$du = 3x^2\,dx \Rightarrow \tfrac{1}{3}\,du = \underbrace{x^2dx}$$

Replace "Kill" with

$$= \int u^{10}\cdot \tfrac{1}{3}du$$

$$= \tfrac{1}{3}\int u^{10}\,du$$

⋮ (same as before)

Ex $\int \underbrace{(x^3+1)^{10}}_{u^{10}} \underbrace{x\ dx}_{\text{"Kill"}}$

~~Try $u = x^3+1$~~

~~$du = 3x^2\,dx$~~

(5)- Fancier tech
You can't put in an extra x

For a basic u-sub, "Kill" and du
can differ by (at most) a constant factor.

7-8:
Can do 13-27
but we have more relevant Ex.

Ex $\int \sqrt{y-3}\, dy$

$$u = y-3$$
$$du = 1\,dy \text{ or } dy$$

$$= \int \sqrt{u}\, du$$
$$= \int u^{1/2}\, du$$

$$= \frac{u^{3/2}}{3/2} + C$$

$$= \frac{2}{3} u^{3/2} + C$$
$$= \boxed{\frac{2}{3}(y-3)^{3/2} + C}$$

Ex $\int \frac{dx}{(4x-1)^3}$

$$u = 4x-1$$
$$du = 4\, dx$$

$$= \frac{1}{4} \int \frac{4\, dx}{(4x-1)^3}$$

$$= \frac{1}{4} \int \frac{du}{u^3}$$

$$= \frac{1}{4} \int u^{-3}\, du$$
$$= \frac{1}{4}\left(\frac{u^{-2}}{-2}\right) + C$$
$$= -\frac{1}{8} u^{-2} + C$$

3rd: Can do 13-27 $= \boxed{-\frac{1}{8}(4x-1)^{-2} + C \quad \text{or} \quad -\frac{1}{8(4x-1)^2} + C}$

Very cooked up!

<u>Ex</u> $\int (x^2+6x)^7(x+3)\,dx$

$$u = x^2+6x$$
$$du = (2x+6)\,dx$$

I need to kill (x+3)dx How can I rewrite?

$$du = 2(x+3)\,dx$$

$$= \frac{1}{2}\int \underbrace{(x^2+6x)^7}_{u^7}\cdot \underbrace{2(x+3)\,dx}_{du}$$

$$= \frac{1}{2}\int u^7\,du$$
$$= \frac{1}{2}\cdot\frac{u^8}{8}+C$$
$$= \frac{1}{16}u^8+C$$
$$= \boxed{\frac{1}{16}(x^2+6x)^8+C}$$

3rd. i. Can do 13-45 but we have more relevant Exr.

8.5: 5 times the piece in front

<u>Ex</u> $\int 7x^3e^{-2x^4}\,dx$

$$u = -2x^4$$
$$du = -8x^3\,dx$$

recip

$$= 7\left(-\frac{1}{8}\right)\int \underbrace{-8x^3e^{-2x^4}\,dx}_{du}$$

$$= -\frac{7}{8}\int e^u\,du$$
$$= -\frac{7}{8}e^u+C$$

$$= \boxed{-\frac{7}{8}e^{-2x^4}+C}$$

Earlier ln prob later. (B) find indef $\int$

Ex $\displaystyle\int \frac{x^2-3}{x^3-9x}\,dx$

$$u = x^3 - 9x$$
$$du = (3x^2-9)\,dx$$
$$du = 3(x^2-3)\,dx$$

$$= \frac{1}{3}\int \frac{3(x^2-3)\,dx}{x^3-9x} \qquad (3(x^2-3)\,dx = du,\quad x^3-9x = u)$$

$$= \frac{1}{3}\int \frac{du}{u}$$
$$= \frac{1}{3}\ln|u| + C$$
$$= \boxed{\frac{1}{3}\ln\underbrace{|x^3-9x|}_{\text{Can be } <0,\text{ so don't drop } |\,|} + C}$$

$-x - \frac{1}{2}\ln|1-x| + \frac{1}{2}\ln|1+x|$

M151, not 121
Long ÷, partial frac
3rd: 4pt to 45

Ex $\displaystyle\int \frac{x^2}{1-x^2}\,dx$ ($x^2\,dx$ = "KILL")

$$u = 1-x^2$$
$$du = -2x\,dx$$ (Need x^2, not x, to kill "KILL")

Can't do basic u-sub.

NO!: $\frac{1}{x}\int \cdots dx$

Never use something in as a "compensator" outside $\int$.

≈ #37: Indef. ∫

(B) Definite ∫s

No PFs in 122?

I almost put 1 →

Bracing you for 122

Ex $\displaystyle\int_2^3 \underbrace{\frac{x}{1-x^2}}_{\text{cont. on } [2,3]\ \checkmark} dx$

$u = 1 - x^2$

$du = -2x\, dx$

New limits

$x = 2 \Rightarrow u = 1 - (2)^2$

$\Rightarrow u = -3$

$x = 3 \Rightarrow u = 1 - (3)^2$

$\Rightarrow u = -8$

Good reminder

$$= \left(-\frac{1}{2}\right)\int_{x=2}^{x=3} \frac{\overbrace{-2x\, dx}^{du}}{\underbrace{1-x^2}_{u}}$$

What's strange here? Don't technically need "u =" Areas same! $-x \Rightarrow -u$

$$= -\frac{1}{2}\int_{u=-3}^{u=-8} \frac{du}{u}$$

OK if lesser # is on top!

or $-\frac{1}{2}$ inside

$$= -\tfrac{1}{2}\Big[\ln|u|\Big]_{u=-3}^{u=-8}$$

Don't go back to x.

$$= -\tfrac{1}{2}\left(\ln|-8| - \ln|-3|\right)$$

or $\ln\sqrt{\frac{3}{8}}$

$$= \boxed{-\tfrac{1}{2}\left(\ln 8 - \ln 3\right) \quad \text{or} \quad -\tfrac{1}{2}\ln\tfrac{8}{3}}$$

Improper ∫s in book: Change limits immed.

Another Way (helps in 122): Phase 1 Do Indefinite ∫ 1st: $\displaystyle\int \frac{x}{1-x^2}\,dx = \ldots \underset{u=1-x^2}{=} -\frac{1}{2}\int\frac{du}{u} = -\frac{1}{2}\ln|u| + C = -\frac{1}{2}\ln|1-x^2| + C$ } All sub work here!

You have to do the $1-x^2$ compos., anyway

Phase 2 $\Rightarrow \displaystyle\int_2^3 \frac{x}{1-x^2}\,dx \overset{\text{FTC}}{=} -\frac{1}{2}\Big[\ln|1-x^2|\Big]_2^3 = -\frac{1}{2}\left(\ln 8 - \ln 3\right)$