REVIEW 5.3-5.6

(5.3): Definite Is and Areas

FTC

who cares if not cont. at 0

Ex $\int_e^7 \frac{1}{x}\,dx$ (cont. on $[e,7]$ ✓)

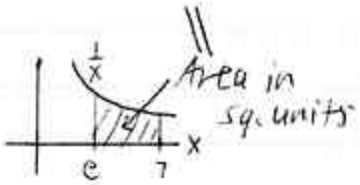
$= [\ln|x|]_e^7$ (can drop $x>0$ on $(e,7]$)

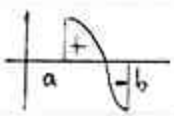
$= [\ln 7] - [\ln e]$

$= \boxed{\ln 7 - 1}$

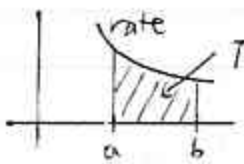
May need algebra to rewrite.
Find an AD of $\frac{1}{x}$.
Don't need + C

[Eval at top #] - [Eval at bot #]
(May need)



Note 

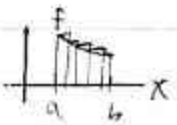
Word Probs.



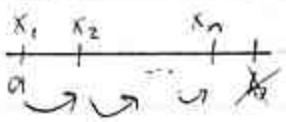
Total accum. (change) from a to b

Approx. $\int_a^b f(x)\,dx$ Using Left Riemann Sums

Given: n = # rects.



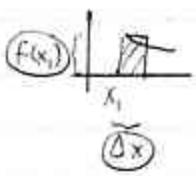
① Rect. width $\Delta x = \frac{b-a}{n}$

② Find 

Keep $+\Delta x$ until you get n #s.

③ Left R. Sum

= (Area of 1st rect.)
+(2nd)
⋮
+(nth)

$= f(x_1)\Delta x$ 

$+ f(x_2)\Delta x$
⋮
$+ f(x_n)\Delta x$

As $n \to \infty$, → Exact

Don't have to be same width - you can modify an approach

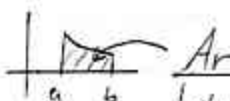
Note If given a table, can't use FTC, but can

x	f(x)
1	20
2	30
⋮	⋮

(5.4) f_{av}

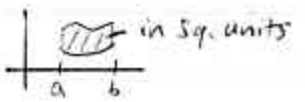
Sum Total / Input Size

$$= \frac{\int_a^b f(x)\,dx}{b-a}$$



$$\frac{\text{Area}}{\text{Interval Length}}$$

Area Bet. Curves



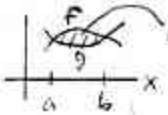
in sq. units

May need

$$\int_a^b [(\text{top}) - (\text{bot})]\,dx$$

Graph or test  to see who's who

I'll give it no intersects.



find bounded area

To find a, b, solve $f(x) = g(x)$ for x.

Who's on top? Test 

(5.6) u-Subs

$\int f(x)\,dx$

Let u = inside, exp., denom.; its deriv. in $\int$ up to const. factor
du = (deriv.) dx (May need to factor)

Templates: $\int u^n\,du$, $\int e^u\,du$, $\int \frac{du}{u}$
(Algebra?)

Ex $\int \frac{x^3}{x^4+1}\,dx$

$u = x^4+1$
$du = 4x^3\,dx$ (can put in const. factor)

$= \frac{1}{4}\int \frac{4x^3}{x^4+1}\,dx$ (Compensate)

$= \frac{1}{4}\int \frac{du}{u}$
$= \frac{1}{4}\ln|u| + C$ (+C)
$= \frac{1}{4}\ln|x^4+1| + C$ (Go back to x.) (→x)
(can drop $x^4+1 > 0$)

Definite Ints: $\int_a^b f(x)\,dx$

$x = a \Rightarrow u =$
b
No: (+C), (→x)

(Word Probs.)

(Phase 1)
or Work out $\int f(x)\,dx$ 1st.
$\int_a^b f(x)\,dx = [F(x)]_a^b$ (stick w/ x)
(Phase 2) (an ∞ from Phase 1)

REVIEW 7.1-7.3

(7.1) $f(x, y, \ldots)$

Domain
Evaluate
Word probs.

(7.2) $f_x, f_y, \ldots$
$\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \ldots$

To find, treat other var. as #.
Evaluate

$f_{xx}, f_{xy}, f_{yx}, f_{yy}$ or $\frac{\partial^2 f}{\partial x^2}, \frac{\partial^2 f}{\partial y \partial x}, \frac{\partial^2 f}{\partial x \partial y}, \frac{\partial^2 f}{\partial y^2}$

usu. =

Word probs.

(7.3) Optimization

Finding CPs
Solve $\begin{cases} f_x = 0 \\ f_y = 0 \end{cases}$
$D = AC - B^2$
f_{xx} f_{yy} f_{xy}

Classify
If $D > 0$
If $A < 0$ ∩ CD R. Max.
If $A > 0$ ∪ CU R. Min.
If $D < 0 \Rightarrow$ neither
If $D = 0 \Rightarrow$ useless
Eval f to find extreme values.