THE "QF" METHOD FOR FACTORING QUADRATICS

Remember our old friend $f(x) = 4x^3 - 5x^2 - 7x + 2$.
Let's say we want to factor this completely over **C**.

From Part B on the Rational Zero Test, we found a list of candidates for rational zeros. It turned out that 2 was, in fact, a zero. Therefore, by the Factor Theorem, $(x-2)$ was a factor of $f(x)$. After performing Synthetic Division, we found that:

$$4x^3 - 5x^2 - 7x + 2 = (x-2)\cdot(4x^2 + 3x - 1)$$

Trial-and-error can be used to factor the quadratic factor, but this method makes some people nervous. There is a more systematic alternative offered to us by the Quadratic Formula (QF). Ordinarily, we factor before finding zeros, but we will reverse that here.

Use the QF to find the zeros of $4x^2 + 3x - 1$; in other words, solve $4x^2 + 3x - 1 = 0$.
Observe: $a = 4$, $b = 3$, and $c = -1$.

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(3) \pm \sqrt{(3)^2 - 4(4)(-1)}}{2(4)} \\ &= \frac{-3 \pm \sqrt{25}}{8} \\ &= \frac{-3 \pm 5}{8} \end{aligned}$$

$$x = \frac{-3+5}{8} = \frac{2}{8} = \frac{1}{4} \qquad \text{or} \qquad x = \frac{-3-5}{8} = \frac{-8}{8} = -1$$

The zeros of $4x^2 + 3x - 1$ are $\frac{1}{4}$ and -1.

Remember that the leading coefficient of $4x^2+3x-1$ was $a_n=4$.
An LFT Form of $4x^2+3x-1$ is, therefore:

$$4x^2+3x-1=4\left(x-\frac{1}{4}\right)(x+1)$$

However, factorizations over **Z** tend to be more useful in simplifications, so we will distribute the "4" through the $\left(x-\frac{1}{4}\right)$ factor.

Warning: You may distribute the "4" through one of the other factors, but not both!!

We obtain:

$$4x^2+3x-1=(4x-1)(x+1)$$

This is the kind of factorization we are typically used to, and it often helps us in simplification problems.

Example

Simplify $\dfrac{x+1}{4x^2+3x-1}$.

Solution

$$\begin{aligned}\frac{x+1}{4x^2+3x-1}&=\frac{\cancel{x+1}^{\,1}}{(4x-1)\cancel{(x+1)}_{\,1}},\quad x\neq -1\\ &=\frac{1}{4x-1},\quad x\neq -1\end{aligned}$$

By the way, we must complete the original problem! We had to factor $f(x)=4x^3-5x^2-7x+2$ over **C**.

Don't forget the $(x-2)$ factor that we obtained earlier:

$$\begin{aligned}4x^3-5x^2-7x+2&=(x-2)(4x^2+3x-1)\\ &=(x-2)(4x-1)(x+1)\end{aligned}$$