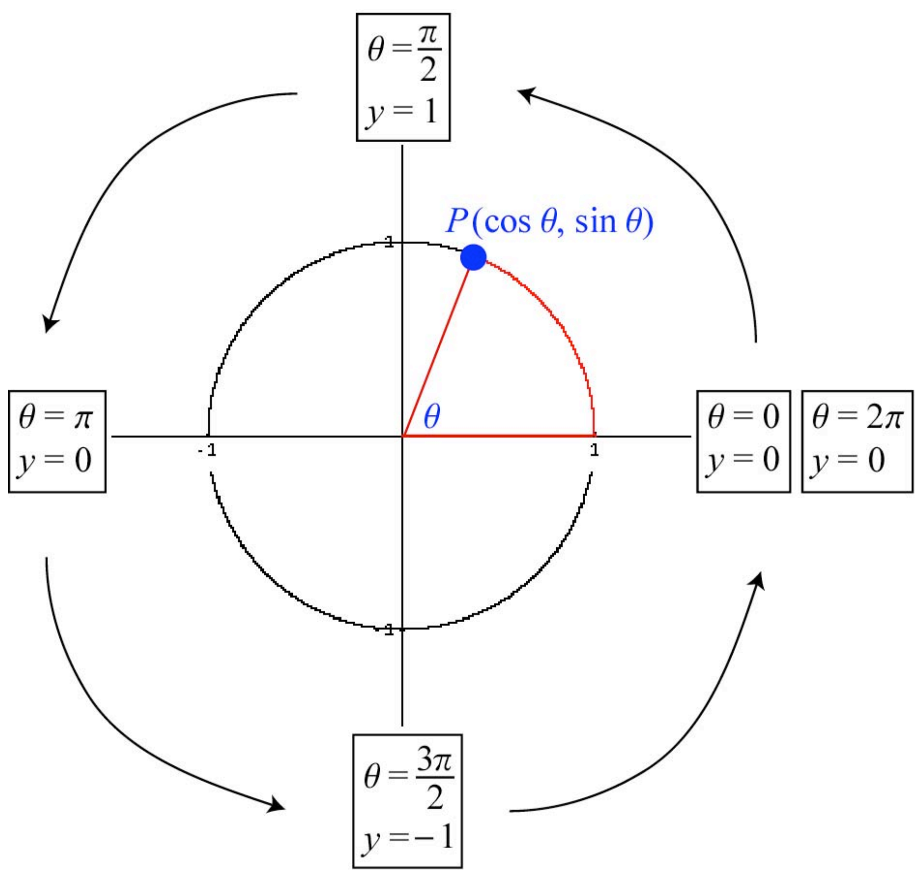
# SECTION 4.5: GRAPHS OF SINE AND COSINE FUNCTIONS

## PART A : GRAPH $f(\theta) = \sin\theta$

Note: We will use $\theta$ and $f(\theta)$ for now, because we would like to reserve $x$ and $y$ for discussions regarding the Unit Circle.

We use radian measure (i.e., real numbers) when we graph trig functions.

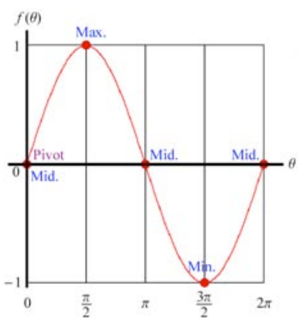
To analyze $\sin\theta$, begin by tracing the $y$-coordinate of the blue intersection point as $\theta$ increases from 0 to $2\pi$.



We obtain one cycle of the graph of $f(\theta) = \sin\theta$. A cycle is the smallest part of a graph (on some interval) whose repetition yields the entire graph. Because such a cycle can be found, $f$ is called a periodic function.

The period of $f(\theta) = \sin\theta$ is $2\pi$. This is because coterminal angles have the same basic trig values (Think: Retracing the Unit Circle), and the $y$-coordinate of the blue intersection point never exhibits the same behavior twice on the $\theta$-interval $[0, 2\pi)$.
To construct the entire graph of $f(\theta) = \sin\theta$, draw one cycle every $2\pi$ units along the $\theta$-axis.

"Framing" One Cycle (Inspired by Tom Teegarden)



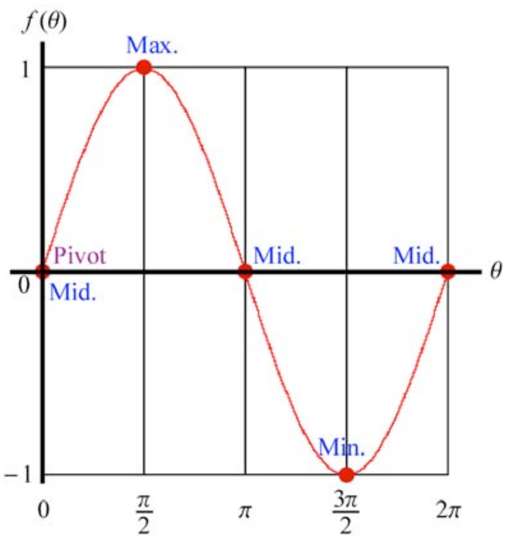

(enlarged on the next page)

We can use a frame to graph one or more cycles of the graph of $f(\theta) = \sin\theta$, including the cycle from $\theta = 0$ to $\theta = 2\pi$. The five "key points" on the graph that lie on the gridlines correspond to the maximum points, the minimum points, and the "midpoints" (here, the $\theta$-intercepts). The midpoints may also be thought of as inflection points, points where the graph changes curvature (you will locate these kinds of points in Calculus). For example, observe that the "curvature" of the cycle changes from concave down to concave up at the central point of the frame.

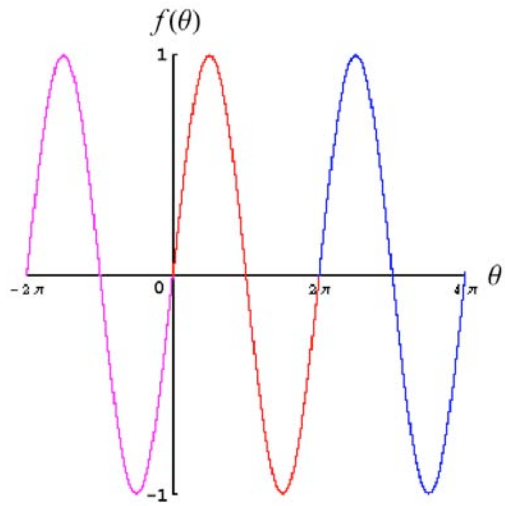
Let's call the "left-center" point of the frame the "pivot." When we graph a sine function, the pivot of the frame will typically be a point on the graph. This will not be true of cosine functions.

It looks cleaner if we place coordinates on the bottom and to the left of the overall frame. (We don't want our writing to interfere with the actual graph.) In this class, we will superimpose the coordinate axes on our graph, even if they would ordinarily be far from the frame; in this sense, we may not be drawing to scale. For the purposes of analysis, it is the relative positions of the coordinate axes to the frame that matter.

One cycle:




Three cycles (not framed):

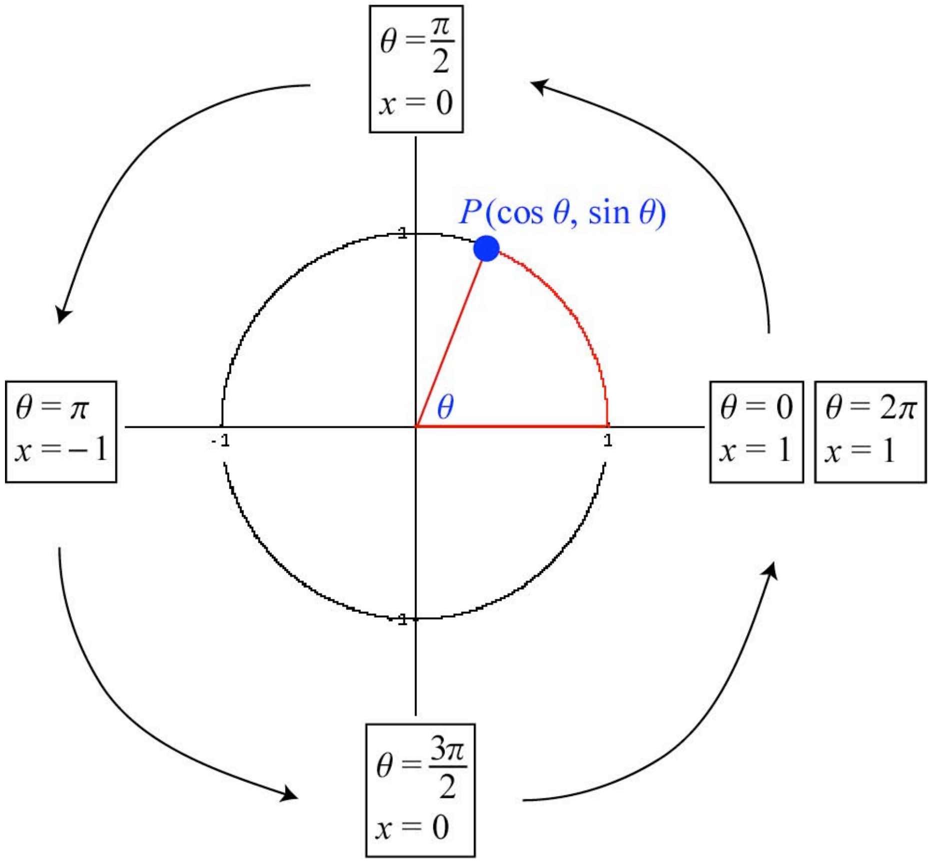



From the graph on the right, you can see that $f(\theta) = \sin\theta$ is odd due to the symmetry about the origin.

Warning: In the graph on the left, the key $\theta$-coordinates correspond to our most "famous" quadrantal angles. This is not always the case for more general sine functions, however!

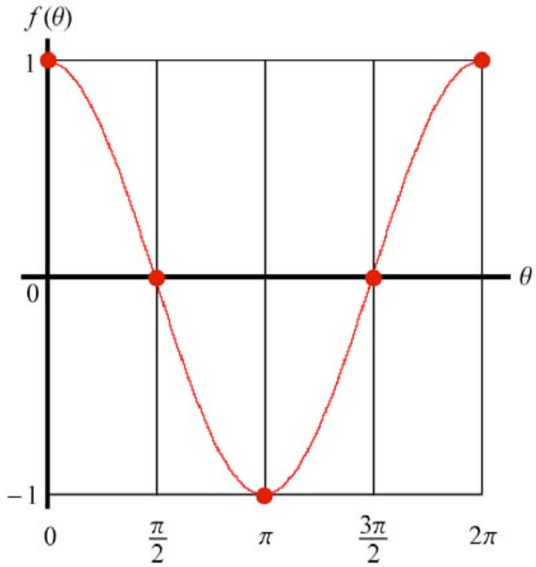
## PART B : GRAPH $f(\theta) = \cos\theta$

This time, we trace the *x*-coordinate of the blue intersection point as $\theta$ increases from 0 to $2\pi$.

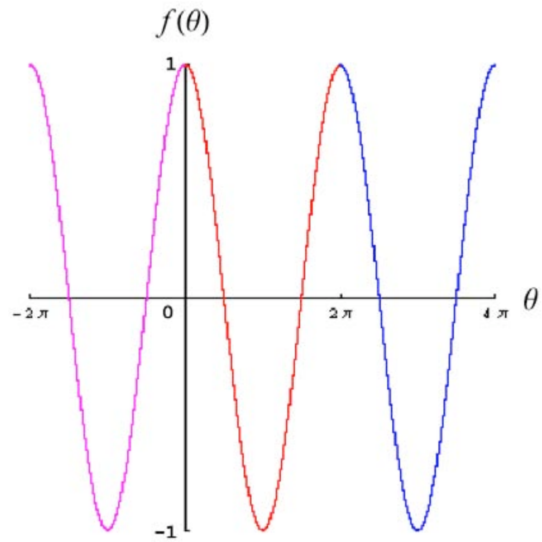



We obtain one cycle of the graph of $f(\theta) = \cos\theta$. Again, the period is $2\pi$.

One cycle ("curvy V"):




Three cycles (not framed):




From the graph on the right, you can see that $f(\theta) = \cos\theta$ is even due to the symmetry about the vertical coordinate axis.

In fact, the graph of $f(\theta) = \cos\theta$ is simply a horizontally shifted version of the $\sin\theta$ graph. Any cycle of the $\cos\theta$ graph looks exactly like some cycle of the $\sin\theta$ graph if you graph them in the same coordinate plane.

Technical Note: In fact, $\cos\theta = \sin\left(\theta + \frac{\pi}{2}\right)$, which means that the $\cos\theta$ graph can be obtained by simply taking the $\sin\theta$ graph and shifting it to the left by $\frac{\pi}{2}$ units. Consider the behavior of the *y*-coordinate (corresponding to sin) of the blue intersection point on the Unit Circle if you start at $\theta = 0$ and increase $\theta$. The *x*-coordinate (corresponding to cos) of the point exhibits the same behavior if you start at $\theta = -\frac{\pi}{2}$ and increase $\theta$. Also remember the Cofunction Identity $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$.

## PART C: DOMAIN AND RANGE

From both the Unit Circle and the graphs we've just seen, observe that:

> If $f(\theta) = \sin\theta$ or $\cos\theta$, then:
>
> The domain of $f$ is **R**, and
> The range of $f$ is $[-1, 1]$.

## PART D: CYCLE SHAPE, AMPLITUDE, AND RANGE

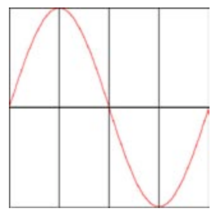
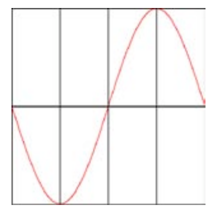
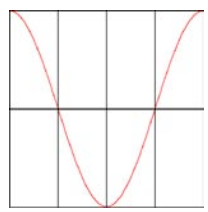
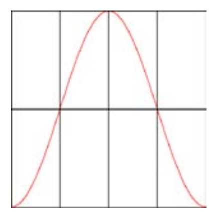
Recall the issue of transformations in Section 1.6.
(It may be more profitable to look at examples as opposed to reading the following blurb.)

In Sections 4.5 and 4.6, we assume that *a* is a nonzero real number.

> Let *G* be the graph of $y = f(x)$.
>
> If $a > 0$, then the graph of $y = a \cdot f(x)$ is:
> a **vertically stretched** version of *G* if $a > 1$
> a **vertically squeezed** version of *G* if $0 < a < 1$
>
> (See Notes 1.62-1.63.)
>
> If $a < 0$, then we take the graph of $y = |a| \cdot f(x)$ and reflect it about the *x*-axis.
>
> (See Notes 1.59-1.60.)

Cycle Shape and our Frame Method

If we want to draw one cycle of the graph of $f(x) = a\sin x$ or $f(x) = a\cos x$ using the Frame Method we will discuss later, then we will draw the "cycle shape" given to us by the following Cycle Grid:

| | $a > 0$ | $a < 0$ |
|---|---|---|
| sin |  |  |
| cos |  |  |

All four cycle shapes above appear on the complete graph of $f$, but we want the one suited for our Frame Method.

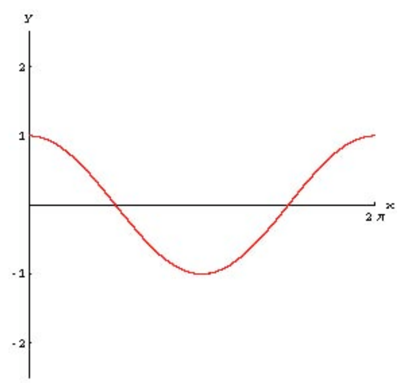
Amplitude

If $f(x) = a\sin x$ or $f(x) = a\cos x$, then $|a|$ = the amplitude of its graph. It is half the "height" of the graph.

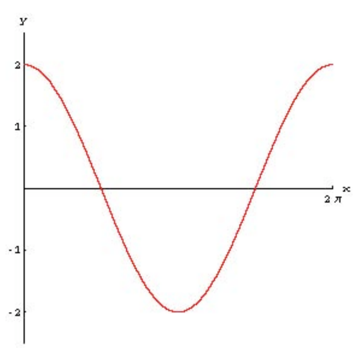
Examples

The graphs below are not framed, but they are well suited for comparative purposes.

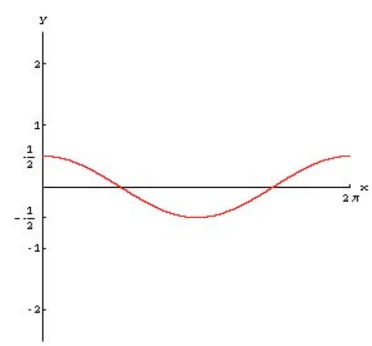
Graph of [one cycle of] $y = \cos x$:



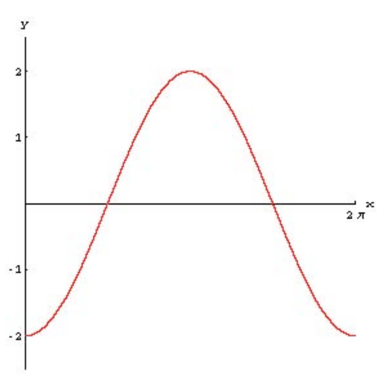

Graph of $y = 2\cos x$:




Graph of $y = \frac{1}{2}\cos x$:




Graph of $y = -2\cos x$:




Observe that the amplitude for both $y = 2\cos x$ and $y = -2\cos x$ is 2, and the range of the corresponding functions is $[-2, 2]$; the domain of both is still **R**.