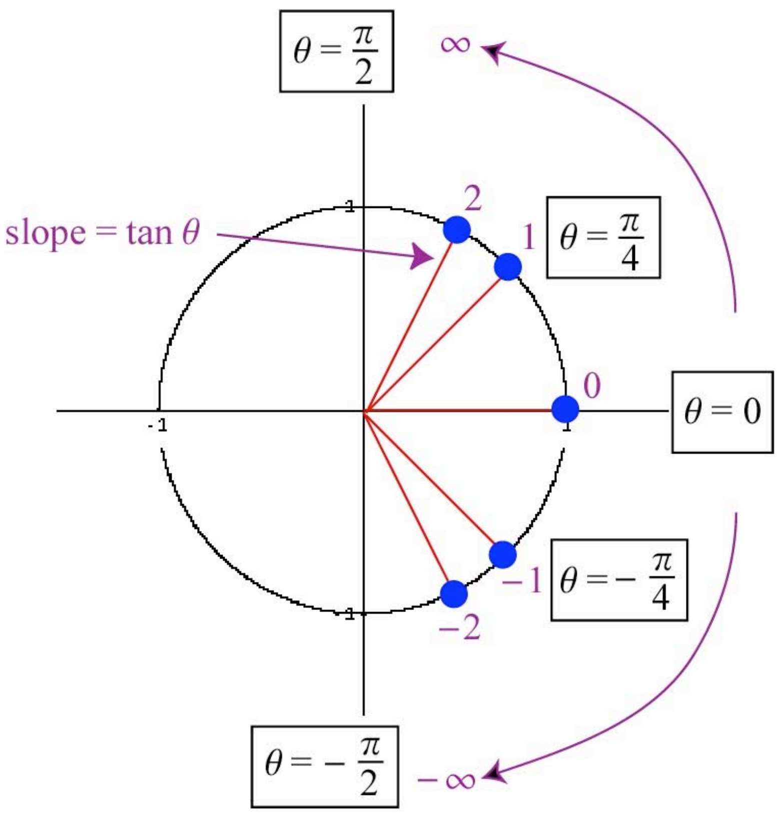
SECTION 4.6: GRAPHS OF OTHER TRIG FUNCTIONS

PART A : GRAPH $f(\theta) = \tan\theta$

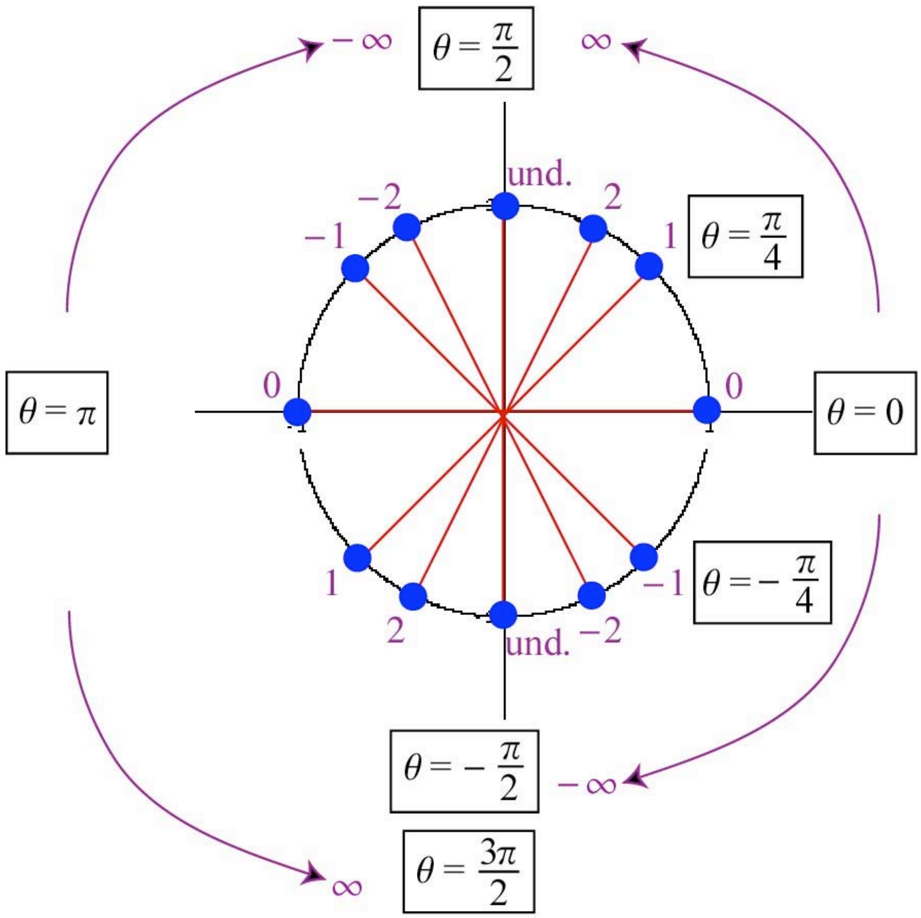
We begin by tracing the slope of the terminal side of the standard angle θ as θ increases from 0 towards $\frac{\pi}{2}$ and as it decreases from 0 towards $-\frac{\pi}{2}$.

The information on slopes is in purple in the figure below.

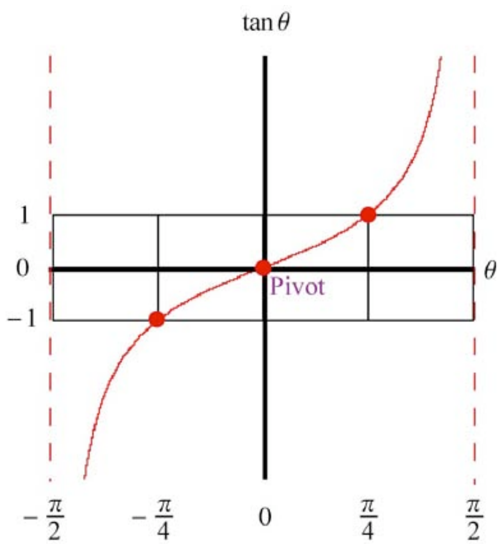


We obtain one cycle of the graph of $f(\theta) = \tan\theta$. The period is π, not 2π.
Why?

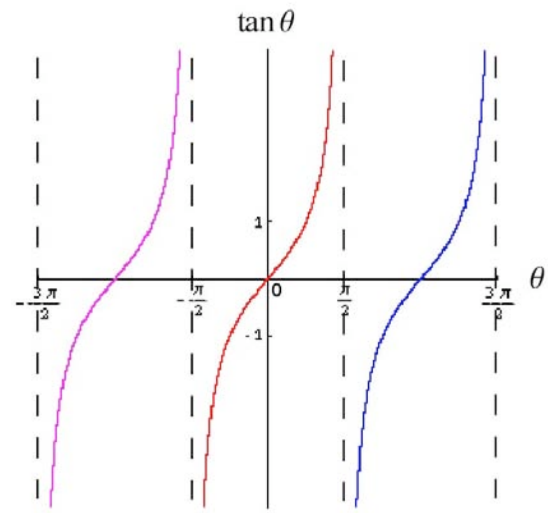
Observe that the behavior of f on the interval $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ is identical to its behavior on the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$:



One cycle:



Three cycles (not framed):



You can see that $f(\theta) = \tan\theta$ is odd due to the symmetry about the origin. Observe that the vertical asymptotes (VAs) naturally divide the graph into cycles.

"Framing" One Cycle

The setup for the frame of a tan or cot function has a number of differences from our setup in the last section:

• (Warning!) When we graph a tan function, the central point, not the "left-center" point, of the frame will be our pivot; it is typically a point on the graph. (When we graph a cot function, the "left-center" point will again be our pivot; it is typically not a point on the graph.) For now, the pivot is the point $(0, 0)$.

• The left and right edges of the frame correspond to vertical asymptotes (VAs).

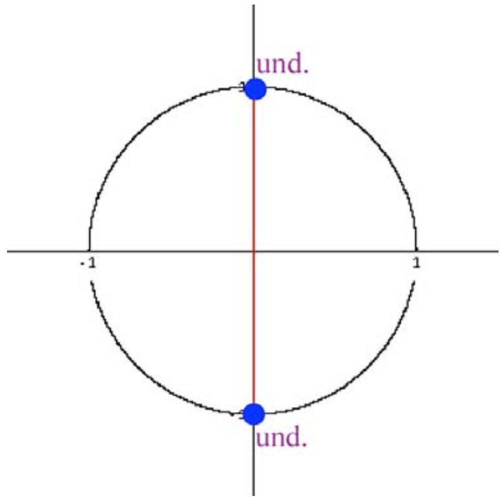
> Warning: When you are told to graph a trig function, you are expected to draw in the vertical asymptotes (if any) as dashed lines.

• The graph of a cycle will stretch beyond its frame and approach these asymptotes.

• The cycles are "snakes." There are no maximum or minimum points. The "midpoint" of a cycle (which is the pivot here), is still an inflection point where the cycle changes curvature (concavity).

• There are only three "key points" on the graph (instead of five) that lie on the gridlines.

PART B: DOMAIN, RANGE, AND VERTICAL ASYMPTOTES (VAs) FOR $f(\theta) = \tan\theta$



From both the Unit Circle and the graphs we've just seen, observe that, if θ is real,

$$\tan\theta \text{ is undefined} \quad \Leftrightarrow \quad \theta = \frac{\pi}{2} + \pi n \quad \left(\text{i.e., } \theta = \underbrace{(2n+1)}_{\text{an odd integer}} \frac{\pi}{2} \right) \text{ for some integer } n.$$

When you see the πn, think "half revolutions."

The VAs of the graph appear at the values of θ mentioned above.

Remember that $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$. Observe that the values of θ mentioned above are the zeros of $\cos\theta$; the x-coordinate is 0 at the corresponding intersection points on the Unit Circle.

If $f(\theta) = \tan\theta$, then the domain of f is:

$$\left\{ \theta \;\middle|\; \theta \neq \frac{\pi}{2} + \pi n \quad \left(\text{i.e., } \theta \neq \underbrace{(2n+1)}_{\text{an odd integer}} \frac{\pi}{2} \right) \text{ for all integers } n \right\}$$

Because a slope can be any real number, …

If $f(\theta) = \tan\theta$, then the range of f is **R**.

This explains why our snake cycles are "infinitely tall."

PART C: GRAPH $f(\theta) = \cot\theta$

How can we use the graph for $\tan\theta$ to obtain the graph for $\cot\theta$?

$$\cot\theta = \tan\left(\frac{\pi}{2} - \theta\right) \qquad \text{(by the Cofunction Identities)}$$

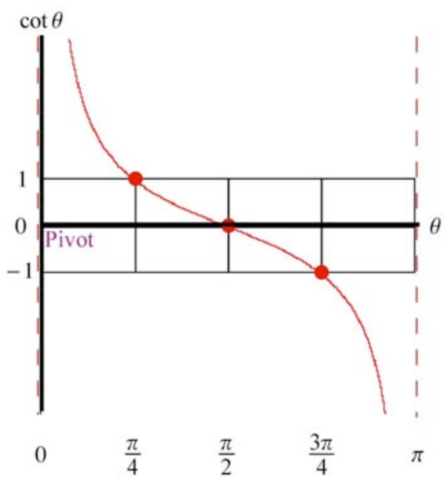
$$= \tan\left[-\left(\theta - \frac{\pi}{2}\right)\right] \qquad \text{(by the "Switch Rule" for Subtraction)}$$

$$= -\tan\left(\theta - \frac{\pi}{2}\right) \qquad \text{(because tan is an odd function)}$$

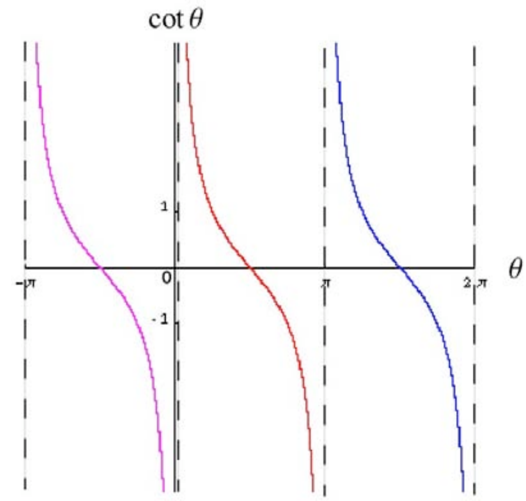
We can obtain the graph for $\cot\theta$ by:

- Taking the graph of $\tan\theta$,
- Shifting it to the right by $\frac{\pi}{2}$ units (it turns out that shifting to the left by $\frac{\pi}{2}$ units also works), and
- Reflecting the resulting graph about the θ-axis.

One cycle:



Three cycles (not framed):



You can see that $f(\theta) = \cot\theta$ is odd due to the symmetry about the origin.
We also know that the reciprocal of an odd function is also odd, and we know that the $\tan\theta$ function is odd.

Observe that the VAs naturally divide the graph into cycles.

Warning: When we are framing a cycle for a cot function, the pivot is the "left-center" point of the frame, just as when we were graphing sin and cos functions. The central point of the frame is the pivot only when we are graphing tan functions.

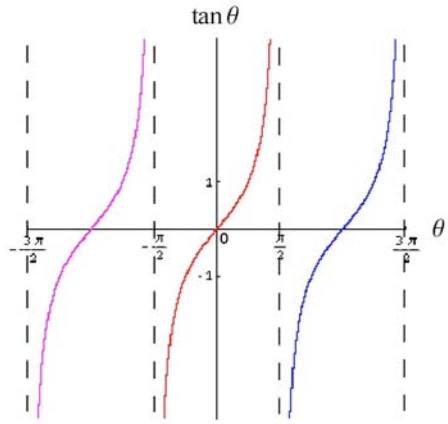
How does the graph for $\cot\theta$ compare with the graph for $\tan\theta$?

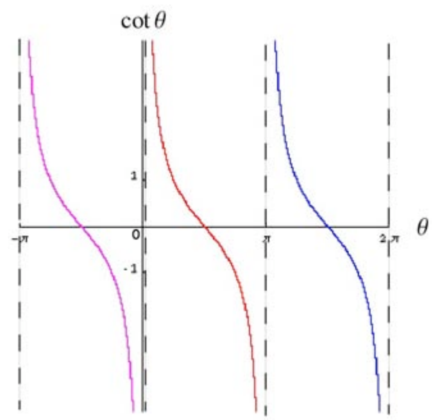
Because of the shift, the VAs are shifted $\frac{\pi}{2}$ units to the right (or to the left; either perspective works).

Because of the reflection, the snake cycles "fall" instead of "rise."

Properties of Graphs of Pairs of Basic Reciprocal Trig Functions

(See how these apply to the $\tan\theta$ (top) and $\cot\theta$ (bottom) functions.)





• One function is 0 in value $\Leftrightarrow$ The other is undefined.

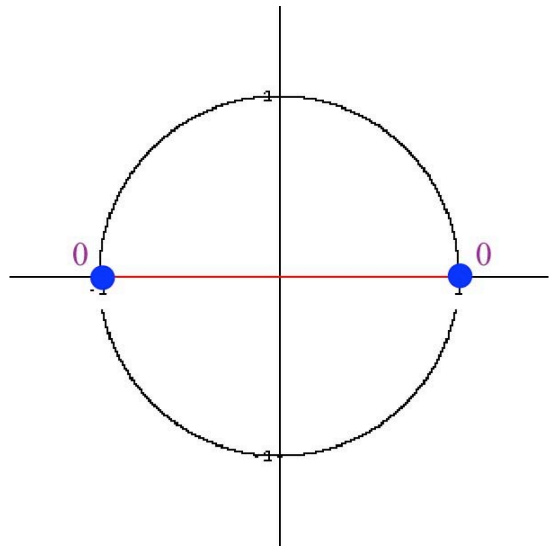
• Otherwise, their values have the same sign.
(In particular, they are positive in value in Quadrant I.)

• If you collect the asymptotes from both graphs, then, between them, one function increases $\Leftrightarrow$ The other decreases.

• They have the same period.

PART D: DOMAIN, RANGE, AND VERTICAL ASYMPTOTES (VAs) FOR $f(\theta) = \cot\theta$

It will help to notice when $\tan\theta = 0$:



$\cot\theta$ is undefined $\Leftrightarrow$ $\tan\theta = 0$ $\Leftrightarrow$ $\theta = \pi n$ for some integer n

When you see the πn, think "half revolutions."

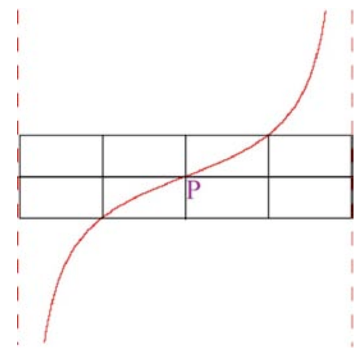
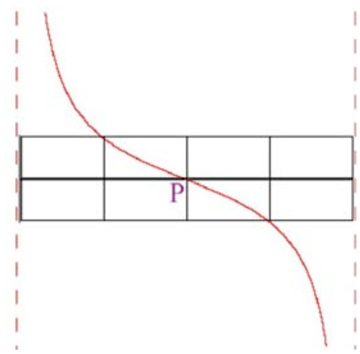
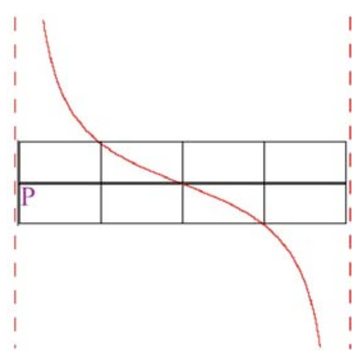
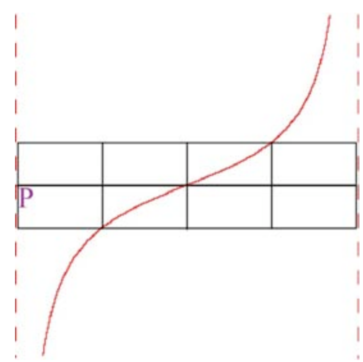
The VAs of the graph appear at the values of θ mentioned above.

Remember that $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$. Observe that the values of θ mentioned above are the zeros of $\sin\theta$; the y-coordinate is 0 at the corresponding intersection points on the Unit Circle.

If $f(\theta) = \cot\theta$, then the domain of f is: $\{\theta \mid \theta \neq \pi n \text{ for all integers } n\}$,   and the range of f is $\mathbf{R}$ (just as for $\tan\theta$).

PART E: CYCLE SHAPE AND OUR FRAME METHOD

Use the following Cycle Grid:

	$a > 0$	$a < 0$
tan		
cot		

The "P"s are reminders that the pivot for a tan frame is at the center (not the "left-center") of the frame.

PART F: EXAMPLES

We now consider the forms:

$$y = a\tan\left[b(x-p)\right] + d \text{, and}$$
$$y = a\cot\left[b(x-p)\right] + d$$

PCAPI still applies, but with the following modifications:

- Use the cycle shape determined by the Cycle Grid on the previous page.

- tan and cot graphs technically have no "amplitude." Nevertheless, we will informally say that "Amplitude" $= |a|$, because it helps us label the frame in an expected way.

- The period is given by $\dfrac{\pi}{b}$, not $\dfrac{2\pi}{b}$. We assume $b > 0$; otherwise, it is $\dfrac{\pi}{|b|}$.

- If you are drawing a cycle for a tan graph, remember that the pivot (p, d) is the central point of the frame. When writing the x-coordinates for the frame, you will be moving both to the right and to the left of the pivot. The x-coordinates you will label will be:

$$x = p - (2 \cdot \text{Inc.})$$
$$x = p - \text{Inc.}$$
$$x = p$$
$$x = p + \text{Inc.}$$
$$x = p + (2 \cdot \text{Inc.})$$