

SECTION 7.5: GRAPHING INEQUALITIES, and **SECTION 7.6: LINEAR PROGRAMMING**

In [Example 2 on p.542](#), the one-variable linear inequalities $x > -2$ and $y \leq 3$ are graphed in the xy -plane.

In [Example 3 on p.542](#), the two-variable linear inequality $x - y < 2$ is graphed in the xy -plane. Two methods:

Method 1: Test Point Method

Step 1: Graph the boundary line, which separates the xy -plane into two half-planes.

- Replace the inequality symbol with “=” to obtain the equation of the boundary line.

- To figure out how to graph the line,

§ Put the equation in slope-intercept form: $y = mx + b$, or

§ Plot the intercepts. (See [Section 1.3: Notes 1.16-1.17.](#))

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If the inequality had	Then graph the line as
\leq or \geq (weak inequality)	a solid line (We include the line in the graph.)
$<$ or $>$ (strict inequality)	a dashed line (We exclude the line.)

Step 2: Decide which half-plane to shade.

- Pick a test point not on the boundary line. $(0,0)$ is usually the best choice if it doesn't lie on the line.

- If the coordinates of the test point make the inequality true, shade the half-plane containing the test point (i.e., shade “towards” the test point). Otherwise, shade the other half-plane (i.e., shade “away from” the test point).

Method 2: "Solve for y" Method

Step 1: Put the inequality in the form

$$y \left(\begin{array}{c} \text{inequality} \\ \text{symbol} \\ (<, \leq, >, \text{ or } \geq) \end{array} \right) mx + b$$

Step 2: Graph the boundary line, whose equation is $y = mx + b$.
(The box from Method 1 on solid vs. dashed applies here, too.)

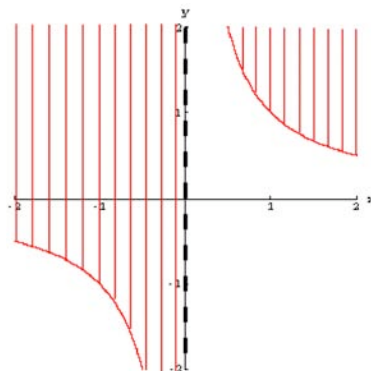
Step 3: If the inequality from Step 1 is of the form:

$$\left. \begin{array}{l} y > mx + b \\ y \geq mx + b \end{array} \right\} \text{favor "high" points, so shade **above** the boundary line}$$

$$\left. \begin{array}{l} y < mx + b \\ y \leq mx + b \end{array} \right\} \text{favor "low" points, so shade **below** the boundary line}$$

In [Example 1 on p.541](#), the nonlinear inequality $y \geq x^2 - 1$ is graphed in the xy -plane. The graph of $y = x^2 - 1$ divides the xy -plane into two regions. To be safe, because the inequality is nonlinear, we use a test point in **each** of the two regions to see if none, one, or both of the regions should be shaded. Notice that the inequality favors “high” points.

Warning: If we graph the inequality $y \geq \frac{1}{x}$, the graphs of $y = \frac{1}{x}$ **and** $x = 0$ are used to separate the xy -plane into regions. Observe that $\frac{1}{x}$ is undefined when $x = 0$.



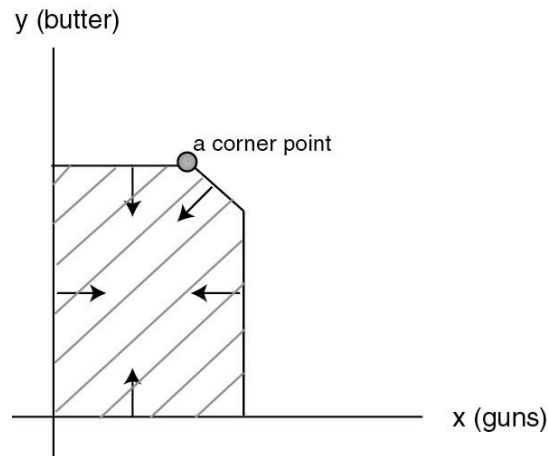
Systems of inequalities are discussed on [pp.543-547](#). As with systems of equations, we want to locate all points that represent solutions to **all** the inequalities in the system. In other words, we want to locate the points where **all** the graphs of the inequalities intersect (or “overlap”).

Linear programming is the key application of systems of linear inequalities.

Let's say we want to produce guns and/or butter. (In a way, this was the issue facing President Lyndon Johnson when he was fighting both the Vietnam War and the War on Poverty.) Given limited budget and/or physical constraints, what are our production options? That is, what combinations of guns and butter can we produce? The possible combinations make up the feasible region shaded below; this feasible region represents the solution set of a system of linear inequalities (corresponding to the constraints).

The inequalities must be weak inequalities; i.e., they must contain \leq or \geq symbols, as opposed to $<$ or $>$ symbols.

Notice that we have the nonnegativity constraints $x \geq 0$ and $y \geq 0$.



What is the "best" production strategy? We first need an objective function that assigns a value to each of the points in the feasible region; these values are a measure of "goodness" or "badness," such as profit or cost. We want the point in the feasible region that has the maximum or minimum such value. This point represents our optimal solution – that is, our best production strategy. If everything's linear (the objective function and the constraints), then the optimal solution will lie at a corner point (or take up some boundary) of the feasible region. These ideas extend to higher dimensions! See [Notes 7.40](#).

Example

Objective function $z = f(x, y)$

$$z = 4x + 3y, \text{ where:}$$

z represents profit that we want to maximize
 x and y represent quantities of two products

Constraints

$$x \geq 0$$

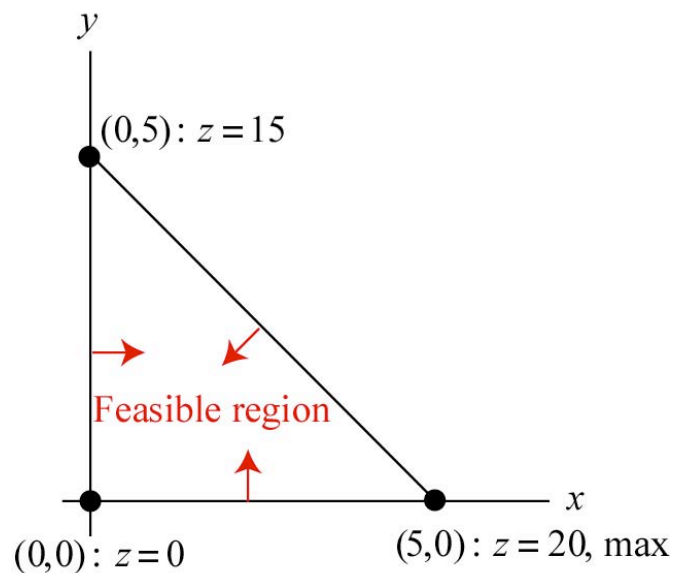
$$y \geq 0$$

$x + y \leq 5$ (Total production cannot exceed 5 pounds, for example.)

Feasible region

The optimal solution must occur at a corner point.

That corner point turns out to be $(5, 0)$.



How has linear programming been used?

- In scheduling, transportation, and resource allocation applications during World War II
- In the famous Berlin Airlift (1948-1949) during the Soviet blockade of Berlin
- In the 1980s, American Airlines cut costs 10% using linear programming techniques.
- In pricing and supply management
- It's the basis for 10% to 25% of all scientific computation today.

Oh, and that urban legend about that brilliant graduate student who solved two problems he saw on the blackboard, thinking that they were homework problems (they were actually as-yet unsolved research problems) – that's true. That student was George Dantzig, who invented the simplex method in 1947. It is the key algorithm used in linear programming. The simplex method allows us to efficiently travel along the edges of a multidimensional polytope bounding the feasible region.

See [Larson, p.554](#) to see a photo of Dantzig, who died in May 2005.

Sources:

Larson, “linear programming” from *Encyclopedia Britannica*,
Stanford Report (May 25, 2005) link:

<http://news-service.stanford.edu/news/2005/may25/dantzigobit-052505.html>